

Name Solutions Mailbox _____

ECE-320 Linear Control Systems

Winter 2015, Exam 1

No calculators or computers allowed, you may leave your answers as fractions.

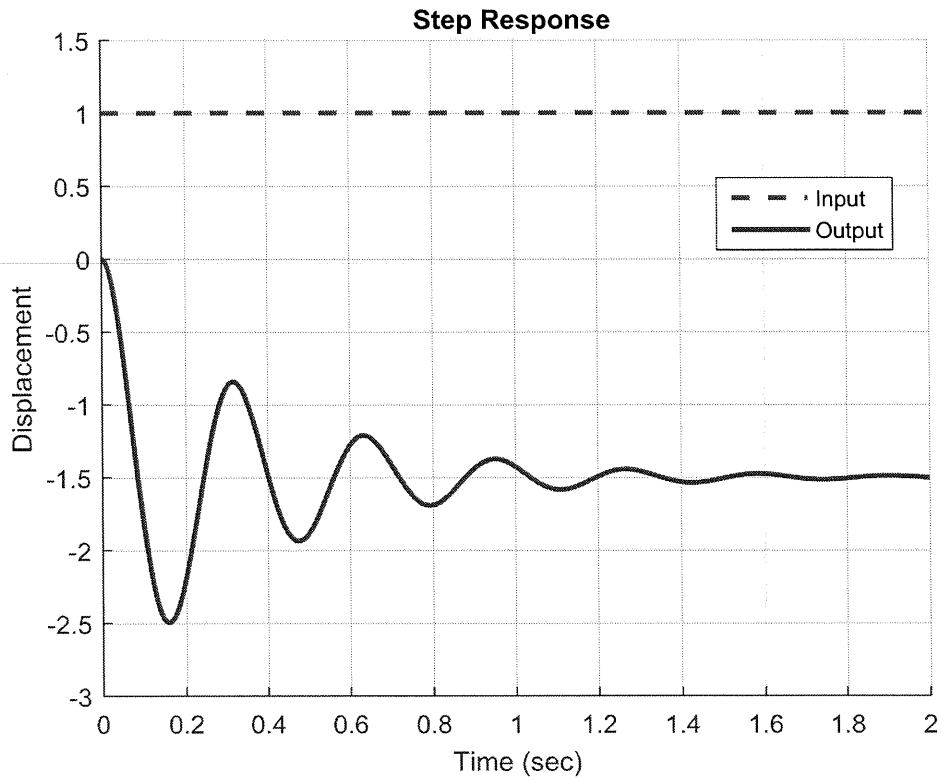
All problems are worth 3 points unless noted otherwise.

Total _____ **/100**

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Problems 1-3 refer to the unit step response of a system, shown below



1) Estimate the steady state error

$$1 - (-1.5) = 2.5 = e_{ss}$$

2) Estimate the percent overshoot

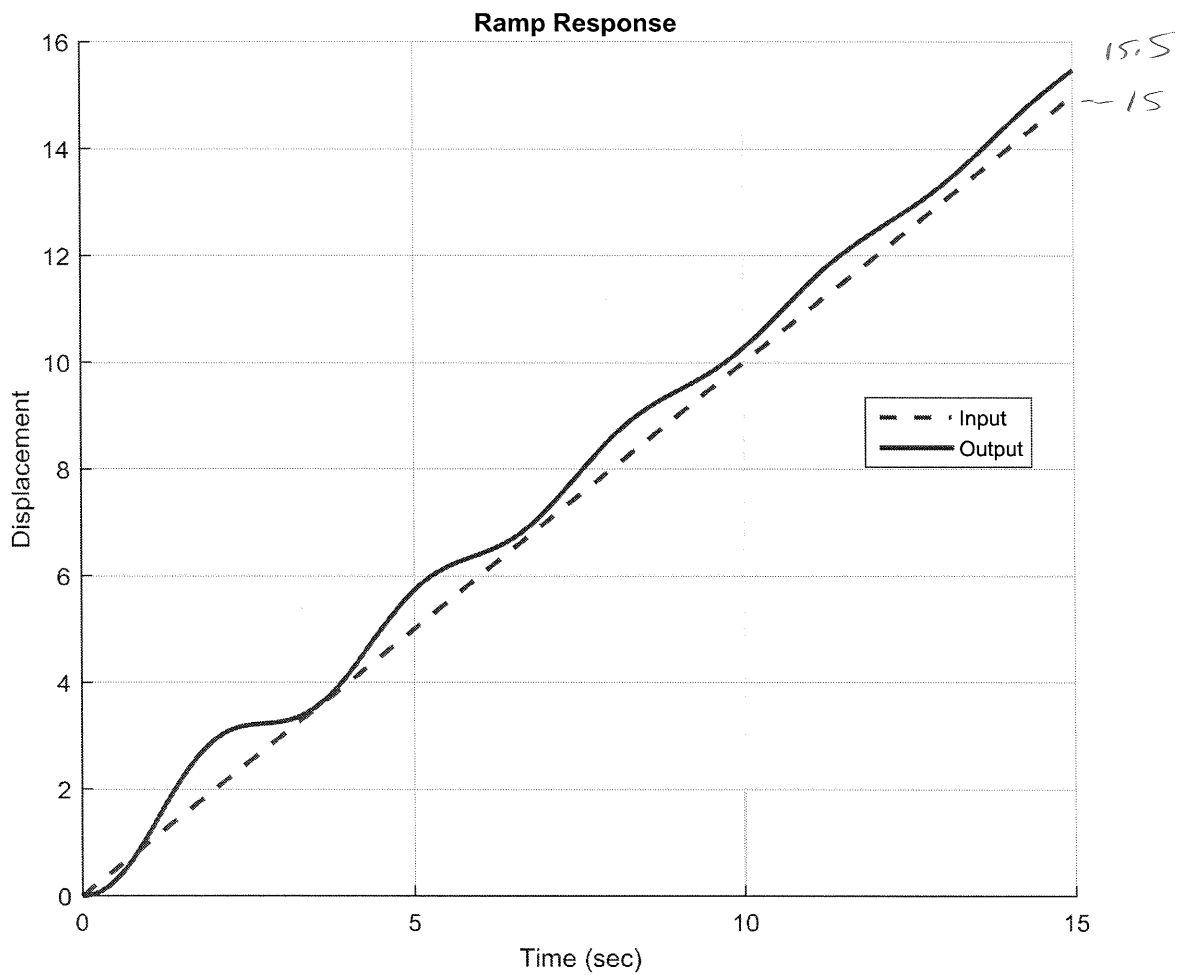
$$\frac{-2.5 - (-1.5)}{-1.5} \times 100\% = 67\%$$

3) Estimate the static gain

$$K(1) = -1.5 \quad K = -1.5$$

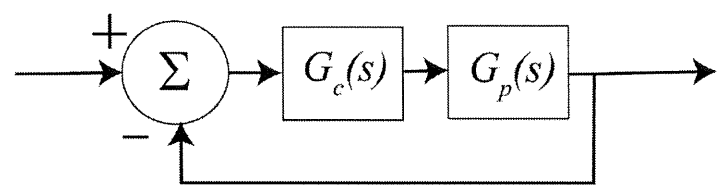
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4) Estimate the steady state error for the ramp response of the system shown below:



$$e_{ss} = 15 - 15.5 = -0.5 = e_{ss}$$

5) (9 points) For this problem assume the following unity feedback system



with $G_p(s) = \frac{5}{(s+2)(s+3)}$ and $G_c(s) = 3$

a) Determine the position error constant K_p $K_p = \frac{15}{2} = \frac{5}{2} = K_p$

b) Estimate the steady state error for a unit step using the position error constant.

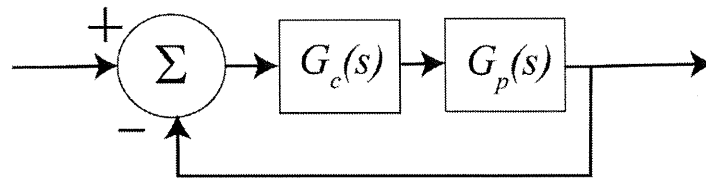
$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+5/2} = \frac{2}{7} = e_{ss}$$

c) Determine the velocity error constant K_v $K_v = 0$

d) Estimate the steady state error for a unit ramp using the velocity error constant.

$$e_{ss} = \frac{1}{K_v} = \infty = e_{ss}$$

6) (9 points) For this problem assume the following unity feedback system



with $G_p(s) = \frac{2}{(s+3)(s+4)}$ and $G_c(s) = 5(s+6)$

a) Determine the position error constant K_p

$$K_p = 5$$

b) Estimate the steady state error for a unit step using the position error constant.

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{6} = e_{ss}$$

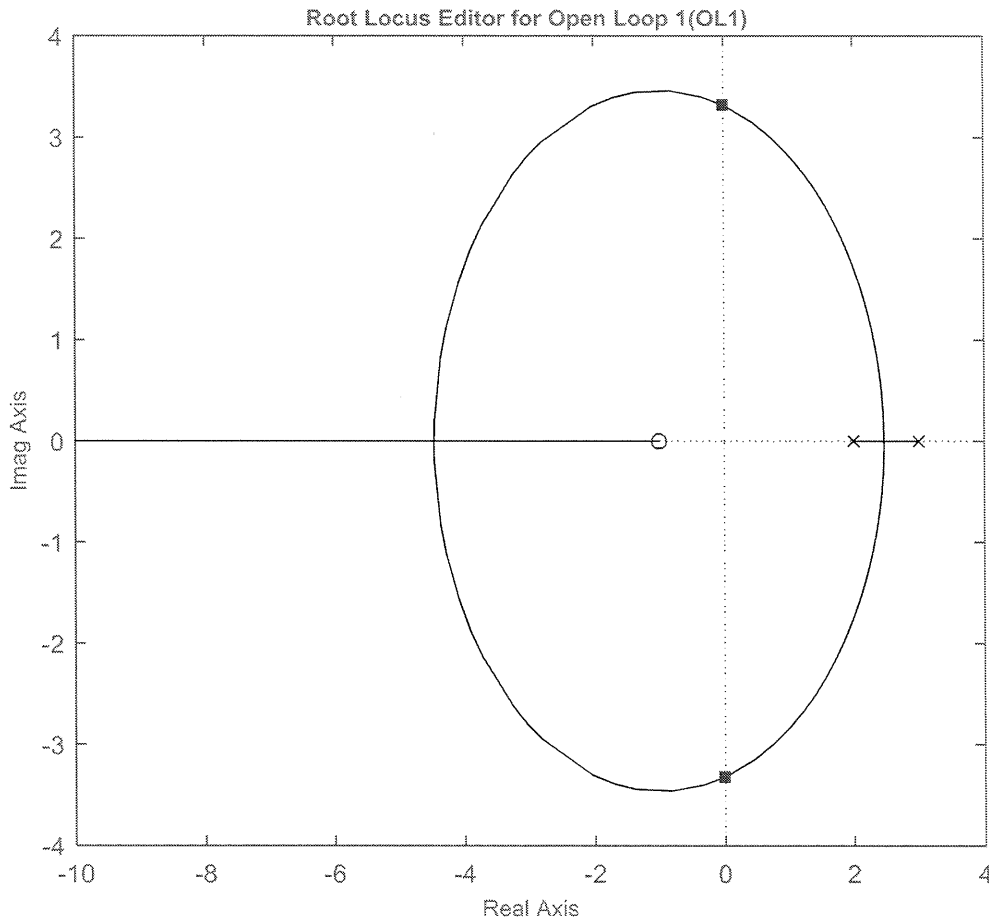
c) Determine the velocity error constant K_v

$$K_v = 0$$

d) Estimate the steady state error for a unit ramp using the velocity error constant.

$$e_{ss} = \infty$$

Problems 7-9 refer to the following root locus plot (from *sisotool*)



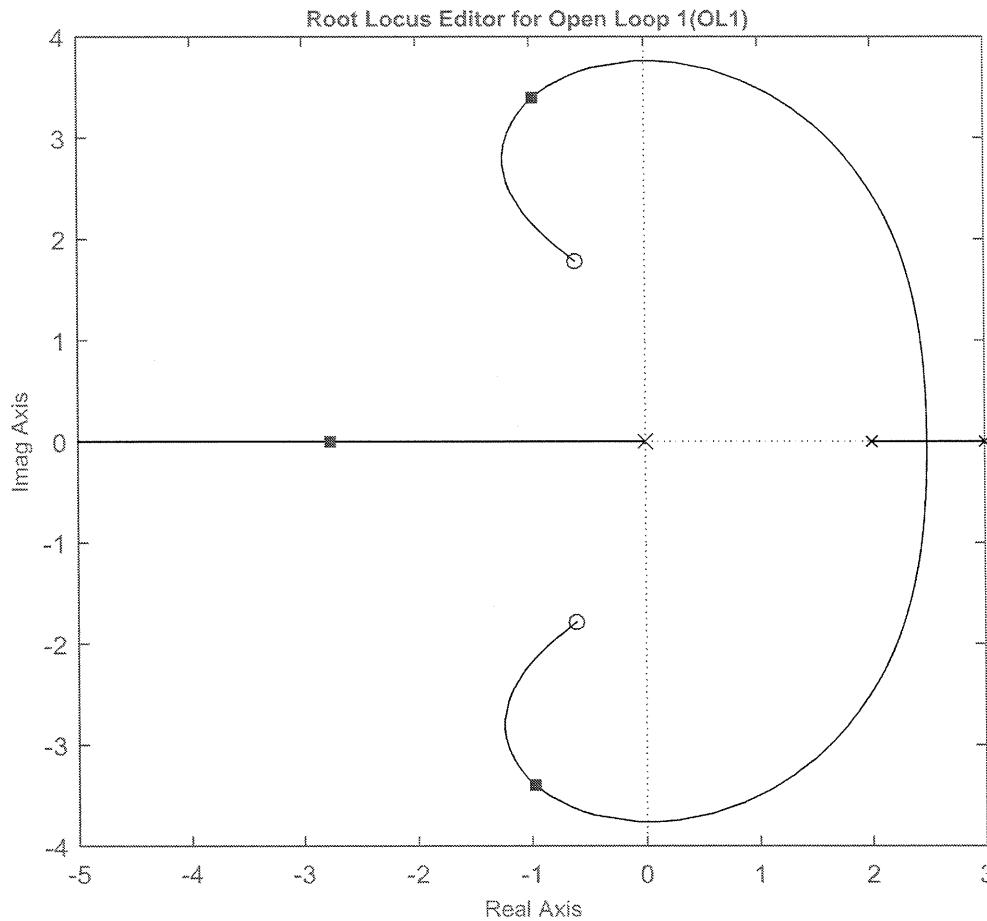
7) Is it possible for -3 to be a closed loop pole for this system ? (Yes or No)

8) When the gain is approximately 5.0 the closed loop poles are as shown in the figure. If we want the system to be stable what conditions do we need to place on the gain k ?

$$K > 5.0$$

9) Is this a type one system? (Yes or No)

Problems 10 -12 refer to the following root locus plot (from *sisotool*)



10) When $k = 3.5$ the poles are as they are shown in the figure. Estimate the closed loop poles.

$$\boxed{-3, -1 \pm j3.5}$$

11) Estimate the settling time as the gain $k \rightarrow \infty$

$$\boxed{T_s \approx \frac{4}{0.8}}$$

12) Is this a type one system? (Yes or No)

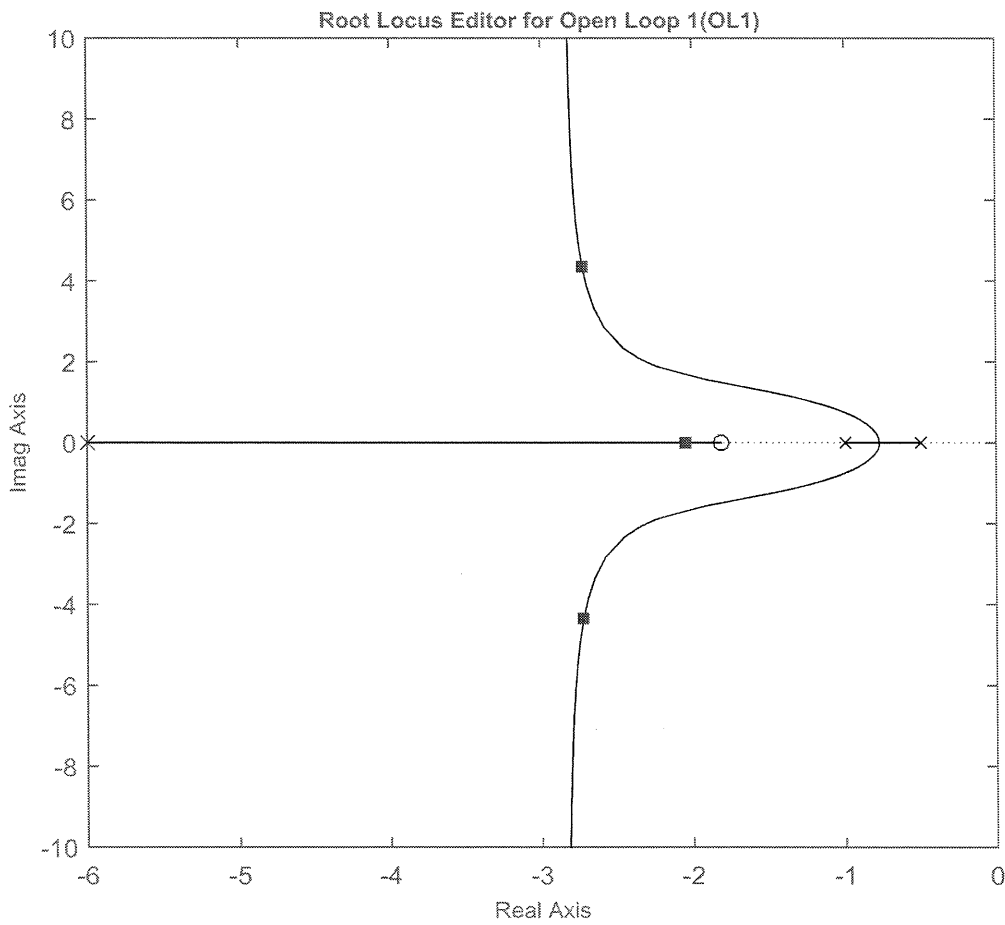
13) For the following controller $G_c(s) = \frac{2(s^2 + 4s + 6)}{s}$, determine k_p , k_i , and k_d

$$K_p = 8$$

$$K_i = 12$$

$$K_d = 2$$

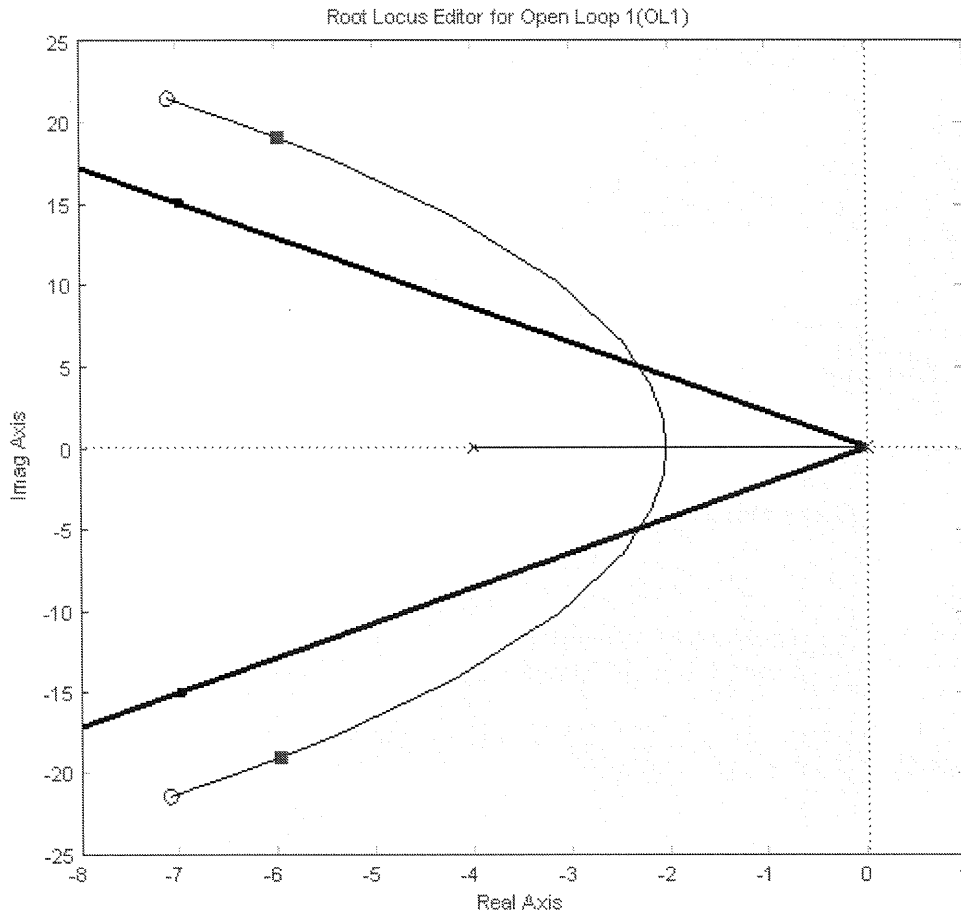
14) Estimate the settling time for the system with the following root locus plot (from *sisotool*)



$$T_s = \frac{4}{2}$$

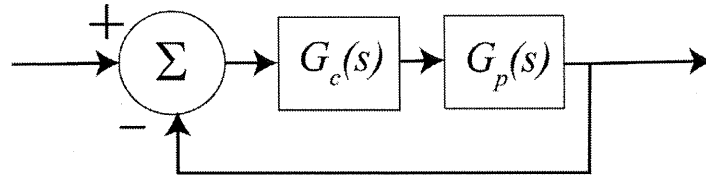
15) (5 points) Consider a system with plant $G_p(s) = \frac{1}{s+4}$ and controller $G_c(s) = \frac{3.6(s^2 + 14.2s + 507)}{s}$.

This information was entered into *sisotool*, as well as the constraint that the percent overshoot should be less than 20%. The corresponding root locus plot is shown below. With this information, are you guaranteed that the unit step response of the system with this root locus plot will have percent overshoot greater than or equal to 20%? Explain why you answered the way you did.



No, this is not an ideal 2nd order system and the constraints are only guaranteed to work for ideal 2nd order systems.

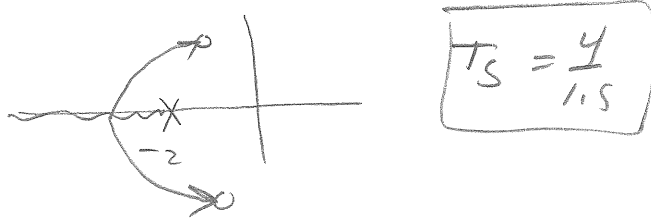
16) (14 points) For this problem assume the closed loop system below.



For each of the following problems you should sketch the root locus to answer the following questions. (You **will not** be graded on your root locus sketches, just your answers.)

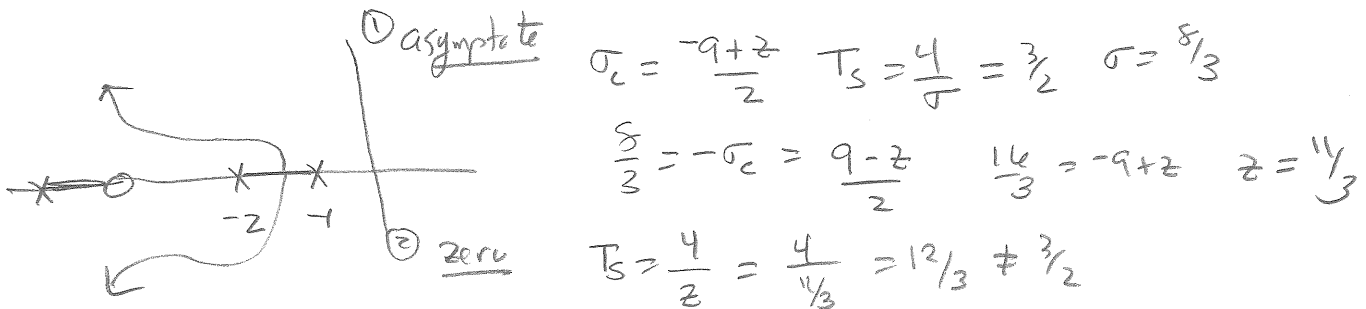
a) Assuming the plant is $G_p(s) = \frac{3}{(s+2)}$ and we are using the PID controller

$G_c(s) = \frac{k(s+1.5+1.5j)(s+1.5-1.5j)}{s}$, what is the settling time as $k_p \rightarrow \infty$?



b) Assuming the plant is $G_p(s) = \frac{3}{(s+1)(s+2)}$ and we are using a lead controller $G_c(s) = \frac{k(s+z)}{s+6}$,

what is the value of z so that the settling time $T_s = \frac{3}{2}$ as $k \rightarrow \infty$? (be careful of the signs here)



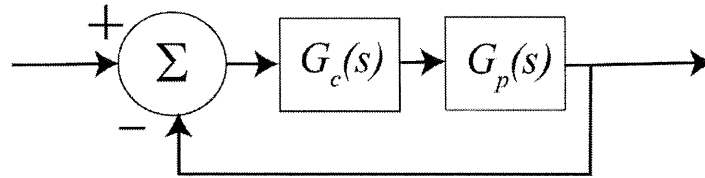
neither works!

① zero $T_s = \frac{4}{z} = \frac{3}{2} \quad z = \frac{8}{3}$

② asymptote $\sigma_c = \frac{-9+z}{2} = \frac{-9+\frac{8}{3}}{2} = \frac{-19}{6}$

$T_s = \frac{4}{(\frac{19}{6})} = \frac{24}{19} \neq \frac{3}{2}$

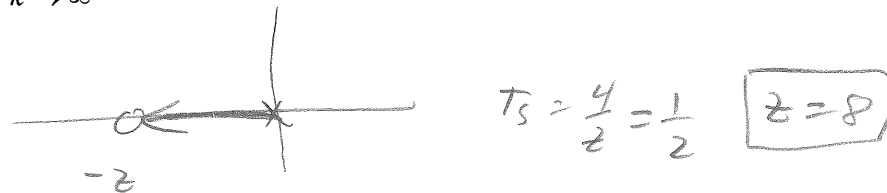
17) (9 points) For this problem assume the following unity feedback system



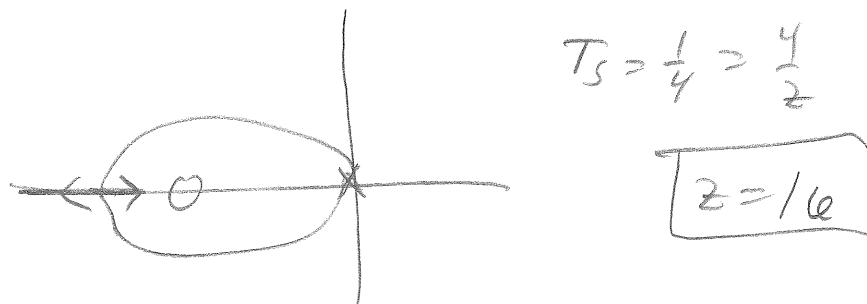
with $G_p(s) = \frac{1}{s}$

For each of the following problems you should sketch the root locus to answer the following questions. (You will not be graded on your root locus sketches, just your answers.)

a) Assuming a proportional + derivative controller $G_c(s) = k(s+z)$, what is the value of z so that the settling time $T_s = \frac{1}{2}$ as $k \rightarrow \infty$



b) Assuming a the PI controller $G_c(s) = \frac{k(s+z)}{s}$, what is the value of z so that the settling time $T_s = 0.25$ as $k \rightarrow \infty$



18) (18 points) Determine both the *impulse response* and the *unit step response* of systems with transfer functions

a) $H(s) = \frac{2}{(s+2)^2 + 1^2}$

b) $H(s) = \frac{1}{s^2}$

c) $H(s) = \frac{1}{(s+1)(s+2)}$

a) $h(t) = 2e^{-2t} \sin(t) u(t)$

$$Y(s) = \frac{2}{s[(s+2)^2 + 1]} = \frac{A}{s} + \frac{B}{(s+2)^2 + 1} + \frac{C(s+2)}{(s+2)^2 + 1}$$

$A = \frac{2}{s}$ x let $s \rightarrow \infty$ $0 = A + C$ $C = -\frac{2}{s}$

let $s = -2$ $\frac{2}{-2} = \frac{2}{s-2} + B$ $-1 = -\frac{1}{s} + B$ $B = -\frac{4}{s}$

$y(t) = \frac{2}{s} - \frac{4}{s} e^{-2t} \sin(t) - \frac{2}{s} e^{-2t} \cos(t)$

b) $h(t) = t u(t)$

$y(t) = \frac{t^2}{2} u(t)$

c) $H(s) = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$ $A = 1$ $B = -1$

$h(t) = [e^{-t} - e^{-2t}] u(t)$

$Y(s) = \frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$

$A = \frac{1}{2}$ $B = -1$ $C = \frac{1}{2}$

$y(t) = \left[\frac{1}{2} - e^{-t} + \frac{1}{2} e^{-2t} \right] u(t)$