

**ECE-320,  
Quiz #4**

For your ease, assume the form of convolution  $y(n) = \sum_{k=-\infty}^{k=\infty} x(k)h(n-k)$  in all of the following problems.

1) The finite summation  $S_N = \sum_{k=0}^N a^k$  is equal to

- a)  $\frac{1-a^N}{1-a}$    b)  $\frac{1-a^{N-1}}{1-a}$    c)  $\frac{1-a^{N+1}}{1-a}$    d)  $\frac{1+a^{N+1}}{1-a}$    e) none of these

2) The finite summation  $S = \sum_{k=-1}^{N+2} a^k$  is equal to

- a)  $a^{-1} \frac{1-a^{N+3}}{1-a}$    b)  $a^1 \left( \frac{1-a^{N+4}}{1-a} \right)$    c)  $a^{-1} \left( \frac{1-a^{N+4}}{1-a} \right)$    d)  $a^{-1} \left( \frac{1-a^{N-4}}{1-a} \right)$    e) none of these

3) For a discrete time system,  $\delta(0)$  is equal to

- a) 0   b) 1   c)  $\infty$    d) it is not defined

4) If an LTI system with impulse response  $h(n) = 4^{n-1}u(n-1)$  has input  $x(n) = \delta(n)$ , the output of the system is

- a)  $y(n) = 4^{n-1}u(n-1)\delta(n)$    b)  $y(n) = 4^{n-1}u(n)$    c)  $y(n) = 4^{n-1}u(n-1)$    d) none of these

5) If an LTI system with impulse response  $h(n) = 3^{n+1}u(n)$  has input  $x(n) = 3\delta(n-1)$ , the output of the system is

- a)  $y(n) = 3^{n+1}u(n-1)$    b)  $y(n) = 3^n u(n-1)$    c)  $y(n) = 3^n u(n)$    d) none of these

6) If an LTI system with impulse response  $h(n) = 2^{n-1}u(n-1)$  has input  $x(n) = 2\delta(n-1)$ , the output of the system is

- a)  $y(n) = 2^{n-2}u(n-2)$    b)  $y(n) = 2^n u(n-2)$    c)  $y(n) = 2^{n-1}u(n-2)$    d) none of these

7) If an LTI system with impulse response  $h(n) = 3\delta(n-1)$  has input  $x(n) = 2\delta(n-1)$ , the output of the system is

- a)  $y(n) = 3 \times 2u(n-2)$    b)  $y(n) = 3 \times 2\delta(n-1)$    c)  $y(n) = 3 \times 2\delta(n-2)$    d) none of these

8) If an LTI system with impulse response  $h(n) = 3^n u(n)$  has input  $x(n) = u(n)$ , the output of the system is

- a)  $y(n) = 3^n u(n)$    b)  $y(n) = 3^{n+1} u(n)$    c)  $y(n) = \frac{1-3^{n+1}}{1-3} u(n)$    d)  $y(n) = \frac{1-3^{n-1}}{1-3} u(n)$    e) none of these

9) If an LTI system with impulse response  $h(n) = 3^n u(n)$  has input  $x(n) = 2^n u(n)$ , the output of the system is

- a)  $y(n) = 3^n 2^n u(n)$    b)  $y(n) = 3^n \frac{1-\left(\frac{2}{3}\right)^{n+1}}{1-\frac{2}{3}} u(n)$    c)  $y(n) = 2^n \frac{1-\left(\frac{3}{2}\right)^{n+1}}{1-\frac{3}{2}} u(n)$

- d)  $y(n) = \left[ \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} \right] \left[ \frac{1-\left(\frac{1}{3}\right)^{n+1}}{1-\frac{1}{3}} \right] u(n)$    e) none of these

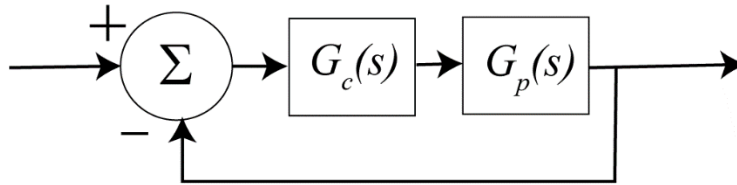
10) The sum  $S = \sum_{k=0}^{\infty} a^k$  will converge provided

- a)  $|a| > 1$    b)  $|a| < 1$

11) If the sum  $S = \sum_{k=0}^{\infty} a^k$  converges, it is equal to

- a)  $\frac{1}{1+a}$    b)  $\frac{1}{1-a}$    c)  $\frac{a}{1-a}$    d)  $\frac{a}{1+a}$    e) none of these

For the problems 12-16, assume the closed loop system below and assume  $G_p(s) = \frac{3}{(s+2)(s+3)}$



For each of the following problems sketch the root locus, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary.

**12)** Assume the proportional controller  $G_c(s) = k_p$

**13)** Assume the integral controller  $G_c(s) = \frac{k_i}{s}$

**14)** Assume the PI controller  $G_c(s) = \frac{k(s+5)}{s}$

**15)** Assume the PD controller  $G_c(s) = k(s+6)$

**16)** Assume the PID controller  $G_c(s) = \frac{k(s+6+2j)(s+6-2j)}{s}$

**Root Locus Construction**

**Once each pole has been paired with a zero, we are done**

1. *Loci Branches*

$$\underset{k=0}{\text{poles}} \rightarrow \underset{k=\infty}{\text{zeros}}$$

Continuous curves, which comprise the locus, start at each of the  $n$  poles of  $G(s)$  for which  $k = 0$ . As  $k$  approaches  $\infty$ , the branches of the locus approach the  $m$  zeros of  $G(s)$ . Locus branches for excess poles extend to infinity.

The root locus is **symmetric** about the real axis.

2. *Real Axis Segments*

The root locus includes all points along the real axis to the left of an odd number of poles plus zeros of  $G(s)$ .

3. *Asymptotic Angles*

As  $k \rightarrow \infty$ , the branches of the locus become asymptotic to straight lines with angles

$$\theta = \frac{180^\circ + i360^\circ}{n - m}, i = 0, \pm 1, \pm 2, \dots$$

until all  $(n - m)$  angles not differing by multiples of  $360^\circ$  are obtained.  $n$  is the number of poles of  $G(s)$  and  $m$  is the number of zeros of  $G(s)$ .

4. *Centroid of the Asymptotes*

The starting point on the real axis from which the asymptotic lines radiate is given by

$$\sigma_c = \frac{\sum_i p_i - \sum_j z_j}{n - m}$$

where  $p_i$  is the  $i^{\text{th}}$  pole of  $G(s)$ ,  $z_j$  is the  $j^{\text{th}}$  zero of  $G(s)$ ,  $n$  is the number of poles of  $G(s)$  and  $m$  is the number of zeros of  $G(s)$ . This point is termed the *centroid of the asymptotes*.

5. *Leaving/Entering the Real Axis*

When two branches of the root locus leave or enter the real axis, they usually do so at angles of  $\pm 90^\circ$ .