$\boxed{\text{Solutions}}$

# ECE-320 Linear Control Systems

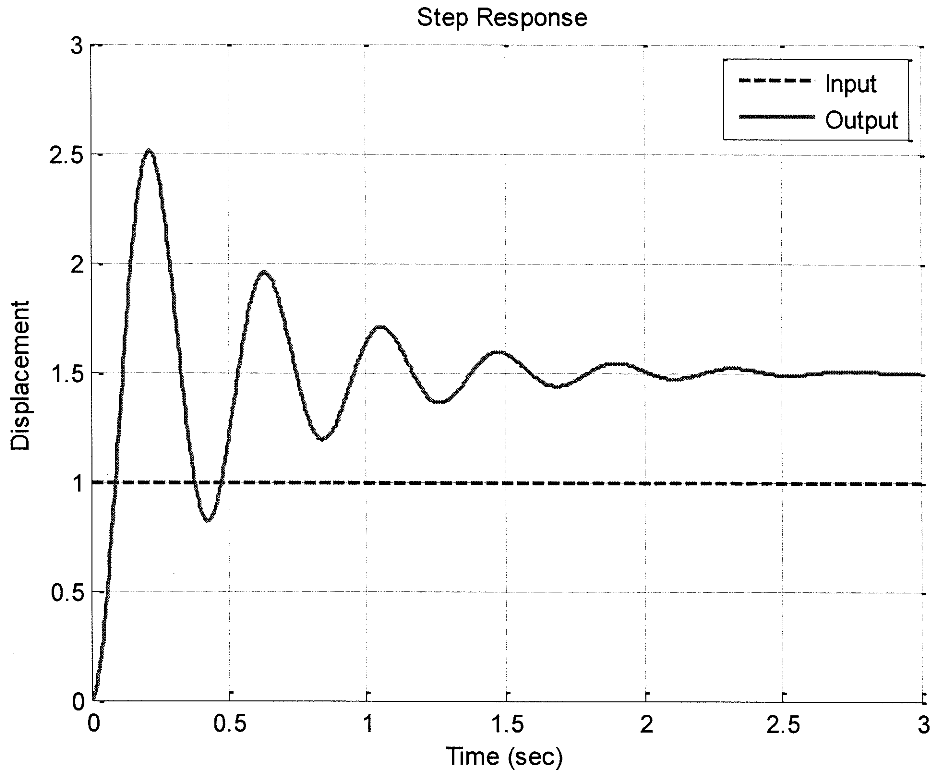
## Winter 2014, Exam 1

**No calculators or computers allowed, you may leave your answers as fractions.**

**All problems are worth 3 points unless noted otherwise.**

**Total _______/100**

Problems 1-3 refer to the **unit step response** of a system, shown below

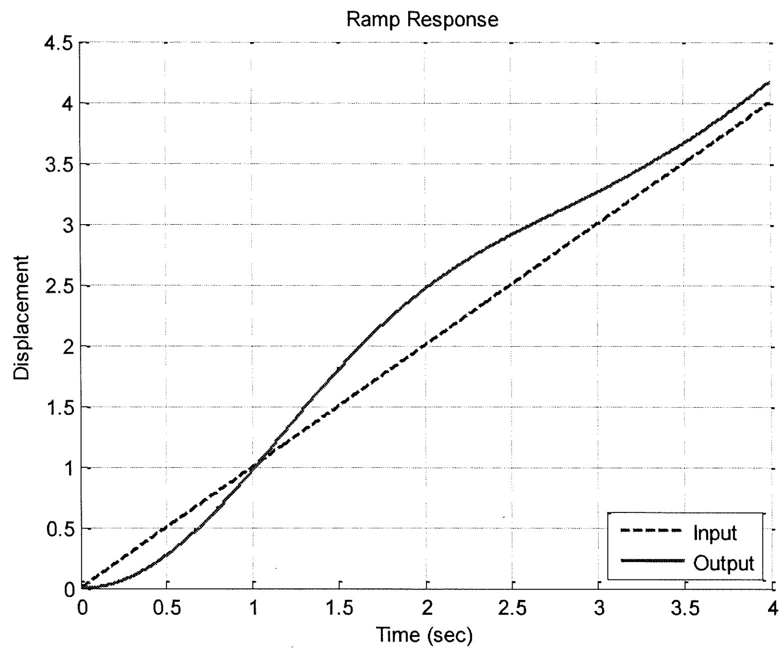



**1)** Estimate the **steady state error** $1 - 1.5 = \boxed{-0.5}$

**2)** Estimate the **percent overshoot** $\frac{2.5 - 1.5}{1.5} \times 100\% = \frac{1}{1.5} \times 100\% = \boxed{67\%}$

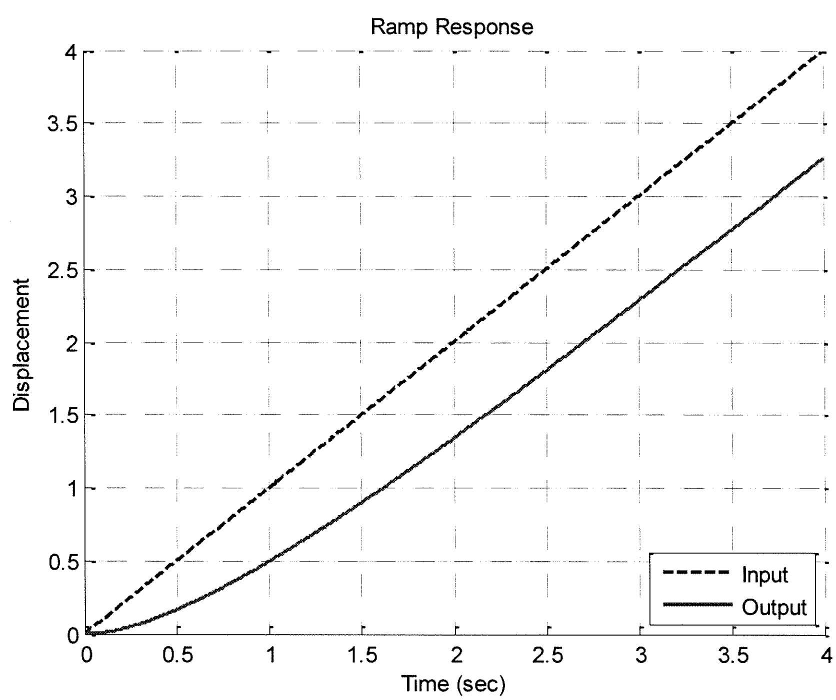
**3)** Estimate the **static gain** $K(1) = 1.5$ $\boxed{K = 1.5}$

**4)** Estimate the steady state error for the ramp response of the system shown below:



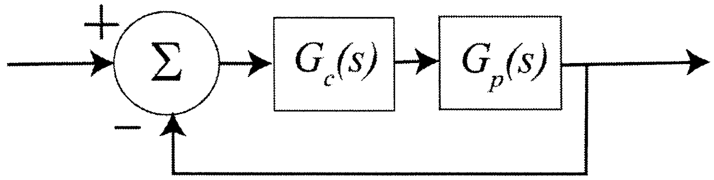

$4 - 4.2 = \boxed{-0.2}$

**5)** Estimate the steady state error for the ramp response of the system shown below:




$4 - 3.25 = \boxed{0.75}$

6) **(10 points)** For this problem assume the following unity feedback system




with $G_p(s) = \dfrac{2}{(s+1)(s+2)}$ and $G_c(s) = 2(s+1)$

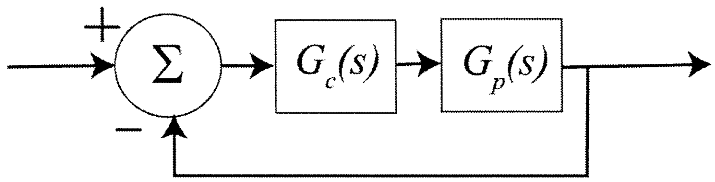
**a)** Determine the position error constant $K_p$ $\boxed{K_p = 2}$

**b)** Estimate the steady state error for a unit step using the position error constant. $e_{ss} = \dfrac{1}{1+K_p} = \boxed{\dfrac{1}{3}}$

**c)** Determine the velocity error constant $K_v$ $\boxed{K_v = 0}$

**d)** Estimate the steady state error for a unit ramp using the velocity error constant. $\boxed{e_{ss} = \infty}$

**7) (10 points)** For this problem assume the following unity feedback system




with $G_p(s) = \dfrac{3}{(s+2)(s+4)}$ and $G_c(s) = \dfrac{2(s+1)}{s^2}$

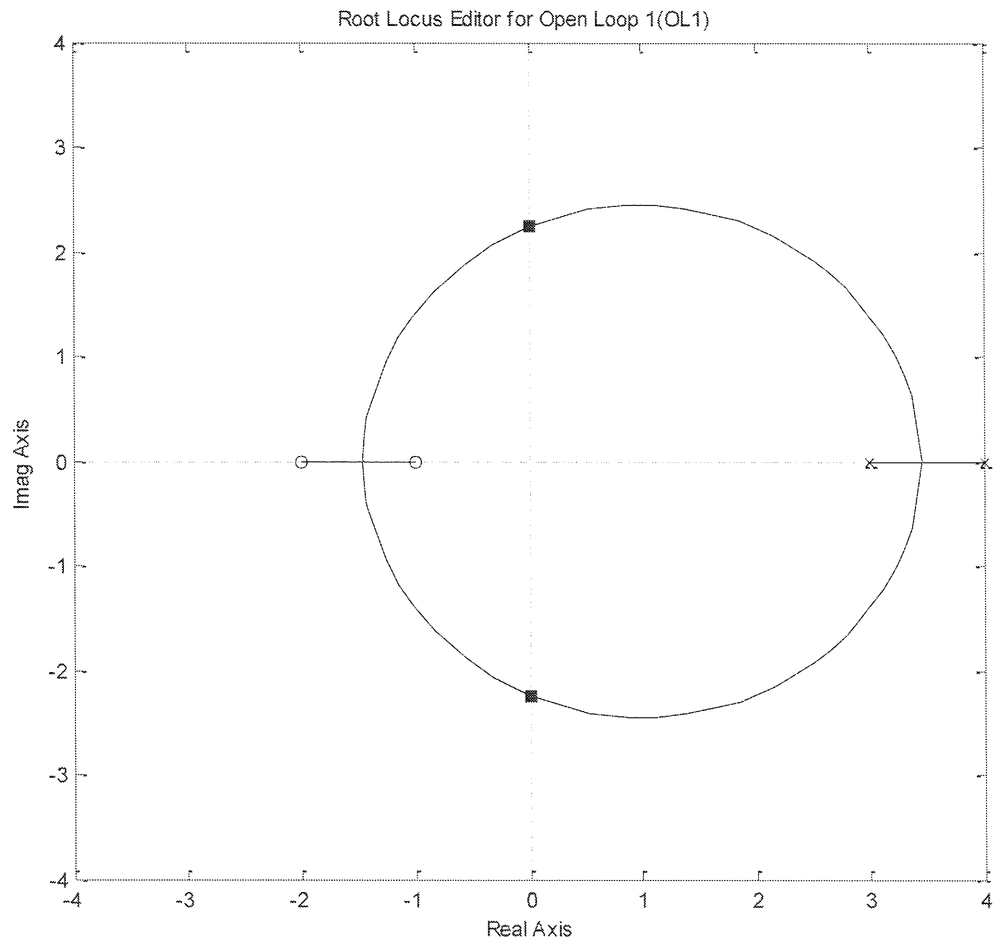
**a)** Determine the position error constant $K_p$ $\boxed{K_p = \infty}$

**b)** Estimate the steady state error for a unit step using the position error constant. $\boxed{e_{ss} = 0}$

**c)** Determine the velocity error constant $K_v$ $\boxed{K_v = \infty}$

**d)** Estimate the steady state error for a unit ramp using the velocity error constant. $\boxed{e_{ss} = 0}$

Problems 8-10 refer to the following root locus plot (from sisotool)



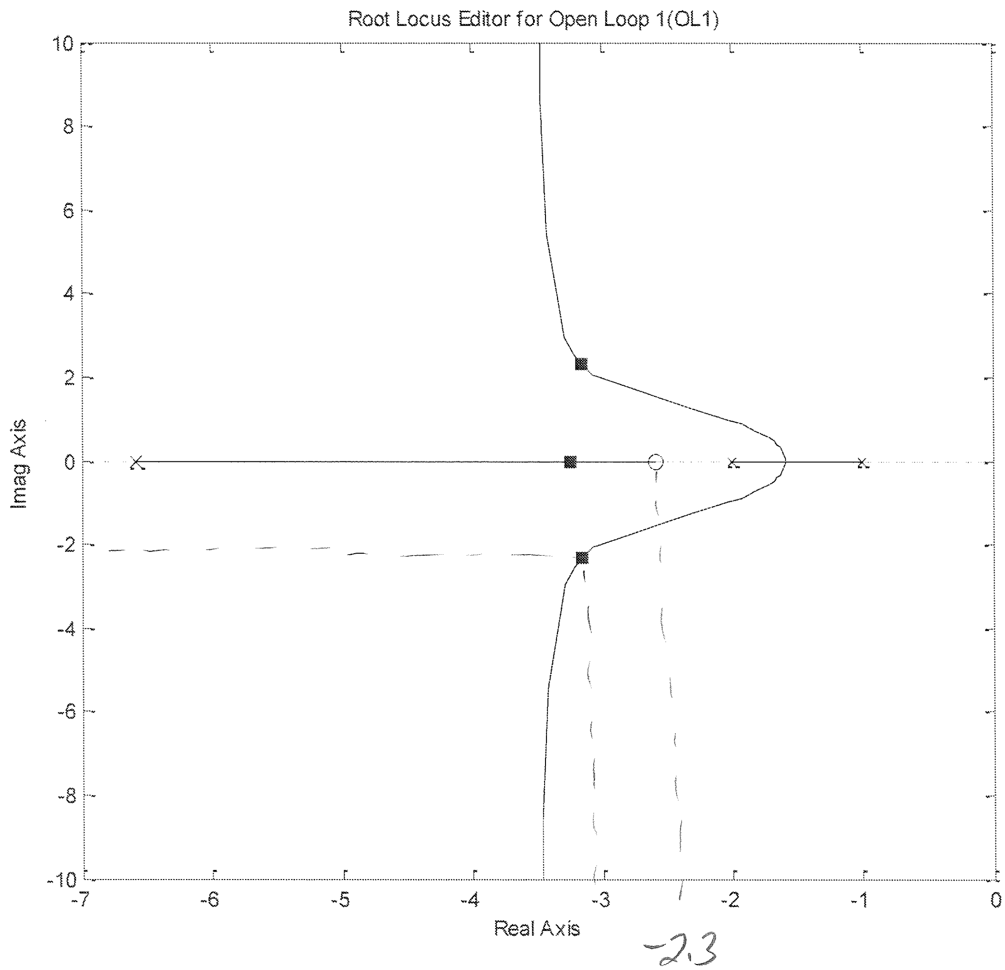

**8)** Is it possible for -1.5 to be a closed loop pole for this system ? (Yes or No)

**9)** When the gain is approximately 2.3 the closed loop poles are as shown in the figure. If we want the system to be stable what conditions do we need to place on the gain k?

K>2.3

**10)** Is this a type one system? (Yes or No)

Problems 11 -13 refer to the following root locus plot (from sisotool)




**11)** When $k = 14.1$ the poles are as they are shown in the figure. Estimate the closed loop poles.

$\boxed{-3,\ -3 \pm 2j}$

**12)** Estimate the settling time as the gain $k \to \infty$ (you can leave your answer as a fraction)

$\boxed{T_s \approx \frac{4}{2.3}}$

**13)** Is this a type one system? (Yes or No)

**14) (6 points)** For the following two PID controllers, determine $k_p$, $k_i$, and $k_d$

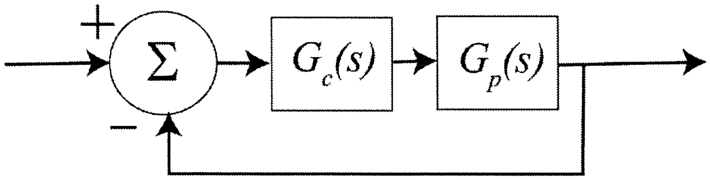
$$G_c(s) = \frac{0.4(s^2+s+5)}{s}$$

$$G_c(s) = \frac{3(s+1)(s+3)}{s}$$

$K_p = 0.4 \quad K_i = 2 \quad K_d = 0.4$

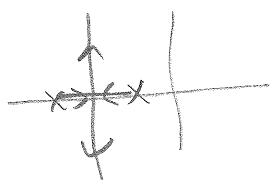
$K_p = 12 \quad K_i = 9 \quad K_d = 3$

**15) (10 points)** For this problem assume the closed loop system below and assume $G_p(s) = \dfrac{3}{(s+1)(s+2)}$




For the following problem you should sketch the root locus to answer the following questions. (You ***will not*** be graded on your root locus sketches, just your answers.)

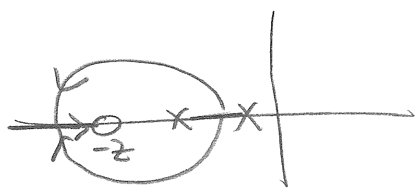
**a)** Assuming a proportional controller $G_c(s) = k_p$, what is the settling time as $k_p \to \infty$?



$\sigma_c = \frac{(-1)+(-2)}{2} = -1.5$

$\boxed{T_s = \frac{4}{1.5}}$

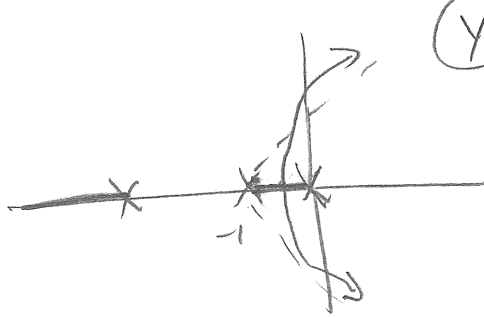
**b)** Assuming a proportional + derivative controller $G_c(s) = k(s+z)$, what is the value of $z$ so that the settling time $T_s = \frac{1}{2}$ as $k \to \infty$




$T_s = \frac{4}{z} = \frac{1}{2}$ $\quad \boxed{z = 8}$

**c)** Assuming an I controller $G_c(s) = \dfrac{k}{s}$, can the system become unstable for any value of $k$?

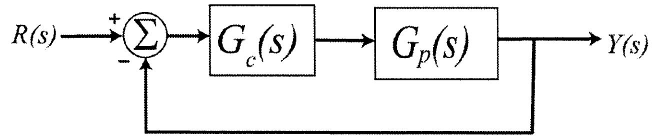
Yes




$\sigma_c = \frac{[(-1)+(-2)+0]}{3} = -1$

$\theta = \frac{180 \pm i360}{3} = \pm 60^\circ, 180^\circ$

**16) (12 points)** Consider the following simple feedback control block diagram. The plant is $G_p(s) = \dfrac{3}{s+3}$




**a)** What is the bandwidth (in rad/sec) of the plant alone (assuming there is no feedback)

$$\boxed{BW = 3 \text{ rad/sec}}$$

**b)** Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

$$\boxed{G_0(s) = \frac{3k_p}{s+3+3k_p}}$$

**c)** Assuming a proportional controller, $G_c(s) = k_p$, determine the value of $k_p$ so the bandwidth of the closed loop system is 27 rad/sec.

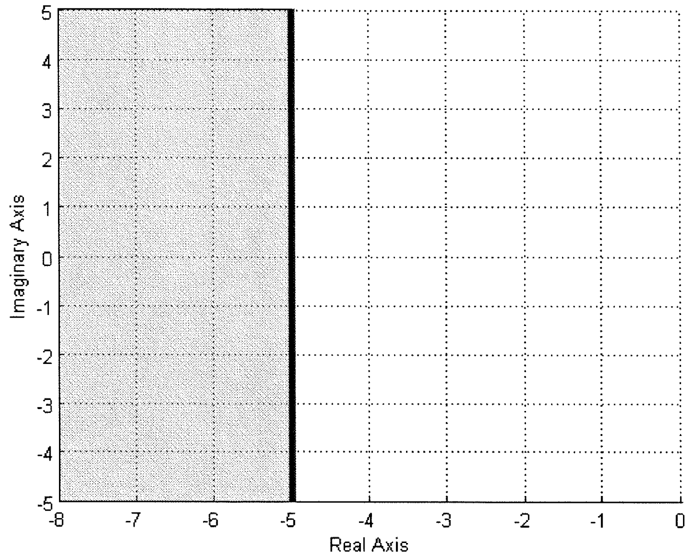
$$3 + 3k_p = 27 \quad \boxed{K_p = 8}$$

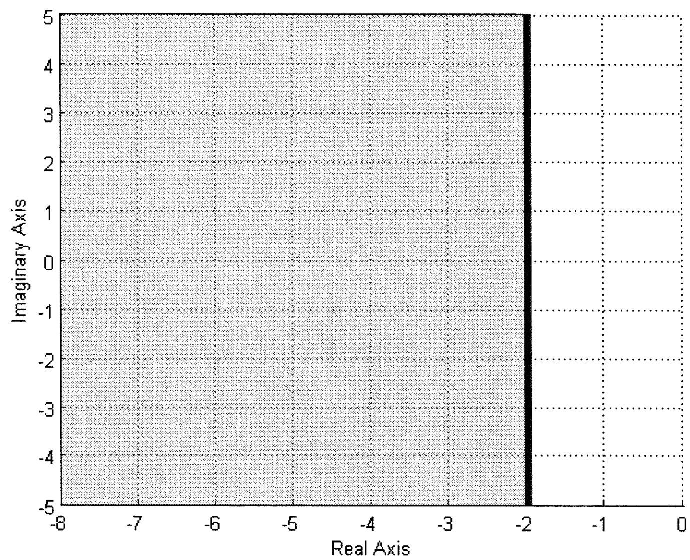
**d)** Assuming the proportional controller from problem c, determine the settling time and the steady state error for a unit step.

$$\boxed{T_s = \frac{4}{27}} \qquad e_{ss} = 1 - \frac{24}{27} = \frac{3}{27} \Rightarrow \boxed{\frac{1}{9} = e_{ss}}$$

**17)** Assuming we are allowed to place our poles only in the (dark) shaded areas, which of the following two shaded regions will in general result in a **small settling time** for our system?

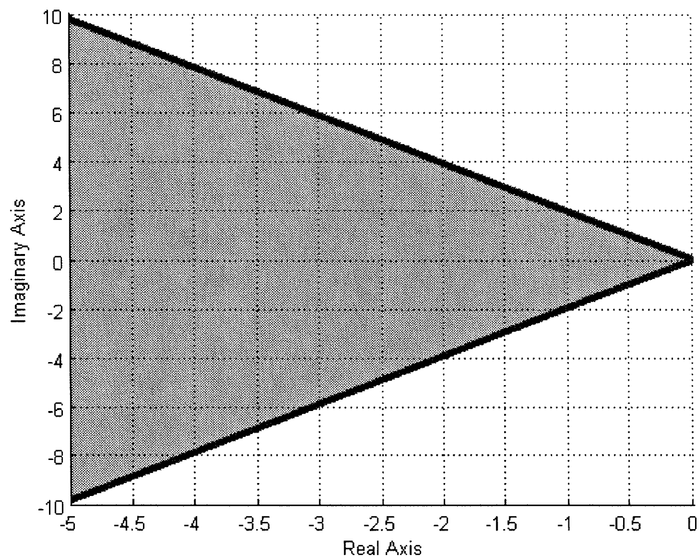
a) The region in the top figure b) the region in the bottom figure



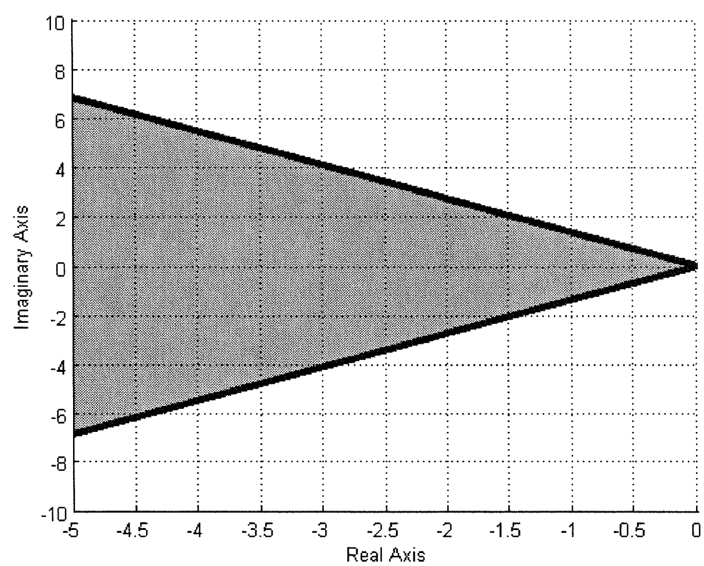



**18)** One of the shaded regions below shows the possible pole locations for a percent overshoot less than 10%, and the other shows the possible pole locations for a percent overshoot less than 20%. Which of the two graphs shows the possible pole locations for a percent overshoot less than 20%?

a) the region in the top figure  b) the region in the bottom figure




PO < 20%




PO < 10%

ccc

Name ________________________________________ Mailbox ____________

**19) (7 points)** Determine ***both*** the *impulse response* and the *unit step response* of a system with transfer function

$$H(s) = \frac{4}{(s+3)^2+2^2} = 2 \cdot \frac{2}{(s+3)^2+2^2}$$

a) $\boxed{h(t) = 2e^{-3t}\sin(2t)u(t)}$

b) $Y(s) = \dfrac{4}{s\left[(s+3)^2+2^2\right]} = \dfrac{A}{s} + B\left[\dfrac{2}{(s+3)^2+2^2}\right] + C\left[\dfrac{s+3}{(s+3)^2+2^2}\right]$

$\boxed{A = \frac{4}{13}}$ $\times s$ let $s \to \infty$ $\quad 0 = A + C$ $\boxed{C = -\frac{4}{13}}$

let $s = -3$ $\quad \dfrac{4}{(-3)(4)} = \dfrac{A}{-3} + \dfrac{B}{2}$

$4 = 4A - 6B \qquad \dfrac{4-4A}{-6} = B = \dfrac{4(1-A)}{-6} = \dfrac{4\left(\frac{9}{13}\right)}{-6}$

$\boxed{B = -\frac{6}{13}}$

$$\boxed{y(t) = \left[\frac{4}{13} - \frac{6}{13}e^{-3t}\sin(2t) - \frac{4}{13}e^{-3t}\cos(2t)\right]u(t)}$$

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**20) (6 points)** Determine ***both*** the *impulse response* and the *unit step response* of a system with transfer function

$$H(s) = \frac{s}{(s+1)^2}$$

a) $H(s) = \dfrac{s}{(s+1)^2} = \dfrac{A}{s+1} + \dfrac{B}{(s+1)^2}$ $\quad B = -1$

$\times s$, let $s \to \infty$ $\quad 1 = A$

$H(s) = \dfrac{1}{s+1} - \dfrac{1}{(s+1)^2}$

$$\boxed{h(t) = (1-t)e^{-t}u(t)}$$

b) $Y(s) = \dfrac{1}{(s+1)^2}$ $\quad \boxed{y(t) = te^{-t}u(t)}$