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ECE-320 Linear Control Systems

Winter 2013, Exam 2

You may only use your computer on the sisotool problem.

You may only use Matlab on this problem.

Problem 1	**_______/25**
Problem 2	**_______/25**
Problem 3-9	**_______/21**
Problem 10	**_______/29**
Total	**_______/100**

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1) (25 points) For impulse response $h(n)=\left(\frac{1}{2}\right)^{n-1}u(n+1)$ and input $x(n)=\left(\frac{1}{4}\right)^{n+1}u(n-1)$, determine the system output by evaluating the convolution sum $y(n)=\sum_{k=-\infty}^{\infty}h(n-k)x(k)$

Note: you do not have to simplify your answer, but you must remove all sums and include a unit step function of some sort.

$$y(n)=\sum_{k=-\infty}^{\infty}\left(\frac{1}{2}\right)^{n-k-1}u(n-k+1)\left(\frac{1}{4}\right)^{k+1}u(k-1)$$

$u(n-k+1)=1$ for $n-k+1\geq 0$

$n+1\geq k$

$u(k-1)=1$ for $k-1\geq 0$

$k\geq 1$

$$y(n)=\sum_{k=1}^{n+1}\left(\frac{1}{2}\right)^{n-1}\left(\frac{1}{2}\right)^{-k}\left(\frac{1}{4}\right)^{k}\left(\frac{1}{4}\right)^{1}$$

$n+1\geq 1$

$n\geq 0$

$$=\left(\frac{1}{2}\right)^{n-1}\frac{1}{4}\sum_{k=1}^{n+1}\left(\frac{1}{2}\right)^{k}$$

let $l=k-1$ $\quad l+1=k$

$$y(n)=\left(\frac{1}{2}\right)^{n-1}\frac{1}{4}\sum_{l=0}^{n}\left(\frac{1}{2}\right)^{l+1}=\left(\frac{1}{2}\right)^{n-1}\frac{1}{8}\sum_{l=0}^{n}\left(\frac{1}{2}\right)^{l}$$

$$\boxed{y(n)=\left(\frac{1}{2}\right)^{n-1}\frac{1}{8}\left[\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right]u(n)}$$

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2) (25 points) For impulse response $h(n) = \left(\frac{1}{2}\right)^{n-1} u(n)$ and input $x(n) = \left(\frac{1}{3}\right)^{n+1} u(n-1)$,

a) determine the z-transform of $h(n)$, $H(z)$

b) determine the z-transform of $x(n)$, $X(z)$

c) determine $y(n)$

Hint: Assume $Y(z) = z^{-2}G(z)$, determine $g(n)$ and then $y(n)$

$$H(z) = Z\left\{\left(\tfrac{1}{2}\right)^n u(n) \left(\tfrac{1}{2}\right)^{-1}\right\} = 2\,\frac{z}{z-\frac{1}{2}}$$

$$X(z) = Z\left\{\left(\tfrac{1}{3}\right)^{n-1} u(n-1) \left(\tfrac{1}{3}\right)^{2}\right\} = \tfrac{1}{9} z^{-1} \frac{z}{z-\frac{1}{3}} = \frac{\frac{1}{9}}{z-\frac{1}{3}}$$

$$Y(z) = H(z)X(z) = \frac{\frac{2}{9} z}{(z-\frac{1}{2})(z-\frac{1}{3})}$$

$$\frac{Y(z)}{z} = \frac{\frac{2}{9}}{(z-\frac{1}{2})(z-\frac{1}{3})} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{3}}$$

$$A = \frac{\frac{2}{9}}{\frac{1}{6}} = \frac{12}{9} = \frac{4}{3}$$

$$B = \frac{\frac{2}{9}}{-\frac{1}{6}} = \frac{-12}{9} = \frac{-4}{3}$$

$$Y(z) = \frac{4}{3}\frac{z}{z-\frac{1}{2}} - \frac{4}{3}\frac{z}{z-\frac{1}{3}}$$

$$\boxed{y(n) = \frac{4}{3}\left(\tfrac{1}{2}\right)^n u(n) - \frac{4}{3}\left(\tfrac{1}{3}\right)^n u(n)}$$

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Problem 3-9, 3 points each

3) Is the following system ***controllable***? $G(s) = \dfrac{G_{pf}}{(s - k_1 k_2)^2}$

a) Yes b) No c) impossible to determine

4) Is the following system ***controllable***? $G(s) = \dfrac{8G_{pf}}{s^2 + 12s + (k_1 + k_2 + 20)}$

a) Yes b) No c) impossible to determine

5) Is the following system **controllable**? $G(s) = \dfrac{G_{pf}}{s^2 + (k_2 + k_1 - 1)s + (k_2 + 2)}$

a) Yes b) No c) impossible to determine

6) Consider a plant that is unstable but is a controllable system. Is it possible to use state variable feedback to make this system stable?

a) Yes b) No

7) Is it possible for a system with state variable feedback to change the zeros of the plant (other than by pole-zero cancellation) ?

a) Yes b) No

8) Is it possible for a system with state variable feedback to introduce zeros into the closed loop system?

a) Yes b) No

9) If a plant has n poles, then a system with state variable feedback with no pole-zero cancellations will have

a) more than n poles b) less than n poles c) n poles d) it is not possible to tell

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10) (29 points) ***(sisotool problem)***

Consider the plant

$$G_p(s) = \frac{50}{s^2 + 10s + 100}$$

Design a PID controller using *sisotool* with ***complex conjugate zeros*** so that

$$\begin{aligned} T_s &\leq 2.0\,\mathrm{sec} \\ P.O. &\leq 10\% \end{aligned}$$

In addition, your controller must be designed so that

$$\begin{aligned} k_p &\leq 1.0 \\ k_i &\leq 5 \\ k_d &\leq 0.05 \end{aligned}$$

Write your final values for k_p, k_i, k_d, and the transfer function of the controller in the space below.