

Name Solutions Mailbox _____

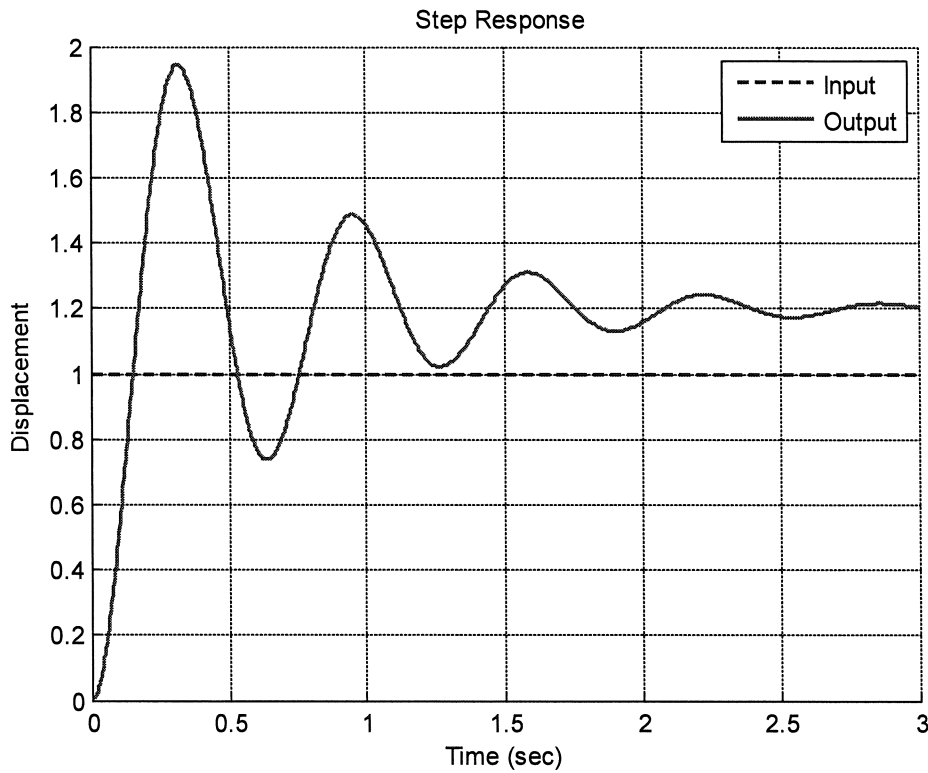
ECE-320 Linear Control Systems
Winter 2013, Exam 1

No calculators or computers allowed, you may leave your answers as fractions.

All problems are worth 3 points unless noted otherwise.

Total _____ **/100**

Problems 1-3 refer to the unit step response of a system, shown below

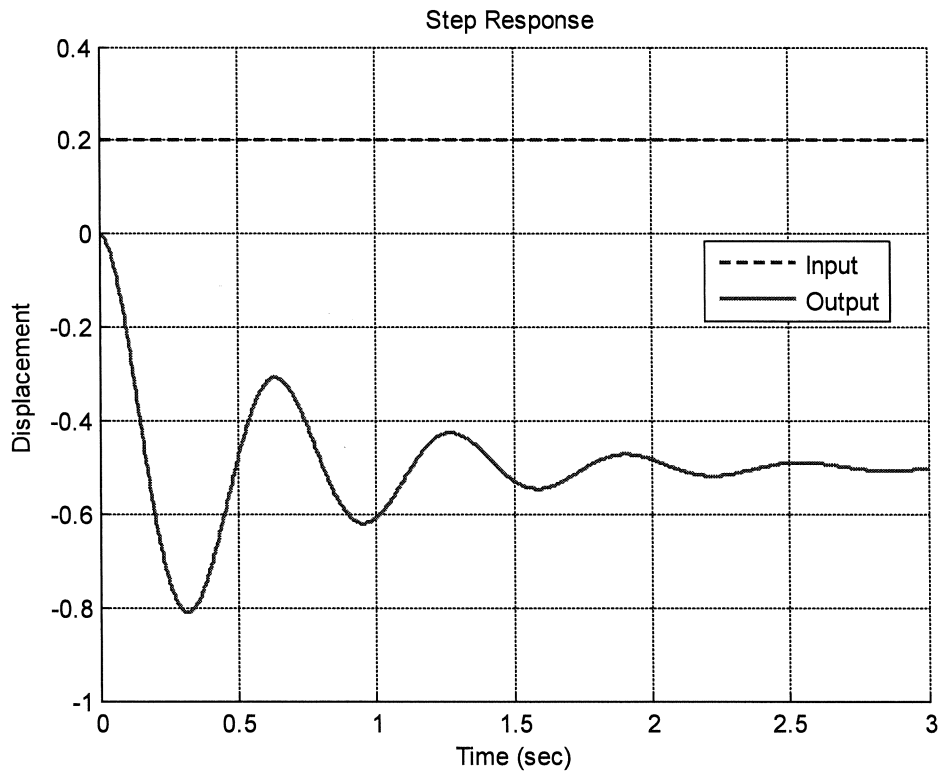


1) Estimate the steady state error $1 - 1.2 = -0.2 = e_{ss}$

2) Estimate the percent overshoot $\frac{1.9 - 1.2}{1.2} \times 100\% = \frac{0.7}{1.2} \times 100\% = PO$

3) Estimate the static gain $K(1) = 1.2$ $K = 1.2$

Problems 4-6 refer to the unit step response of a system, shown below



4) Estimate the steady state error

$$0.2 - (-0.5) = 0.7 = e_{ss}$$

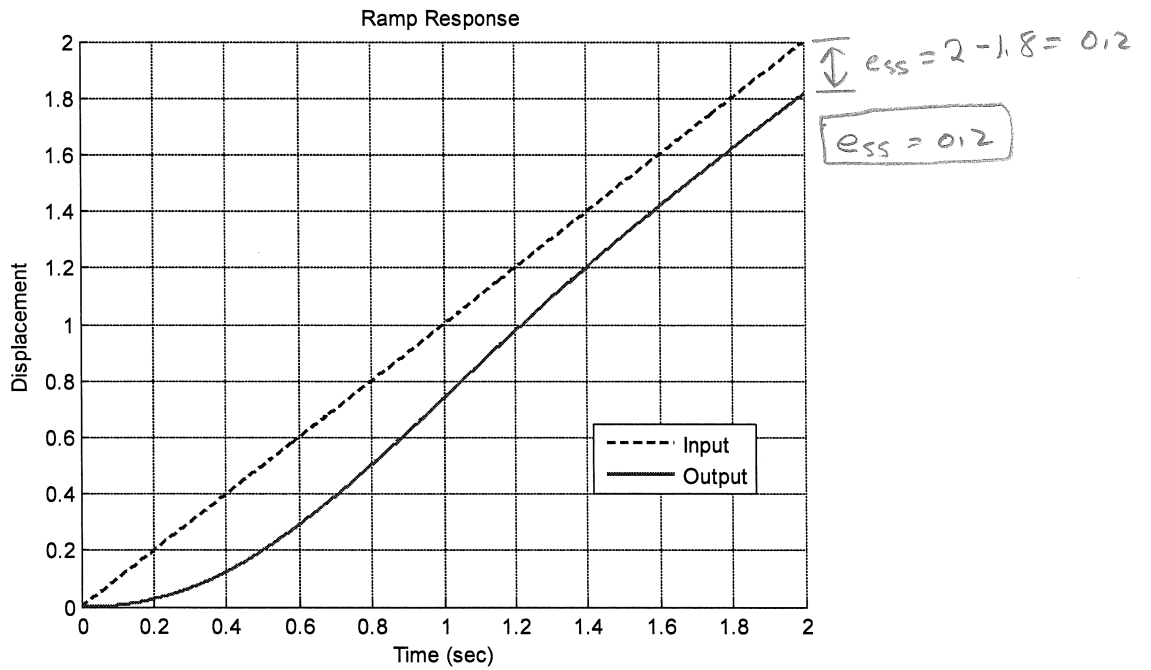
5) Estimate the percent overshoot

$$\frac{-0.8 - (-0.5)}{-0.5} \times 100\% = \frac{3}{5} \times 100\% = 60\%$$

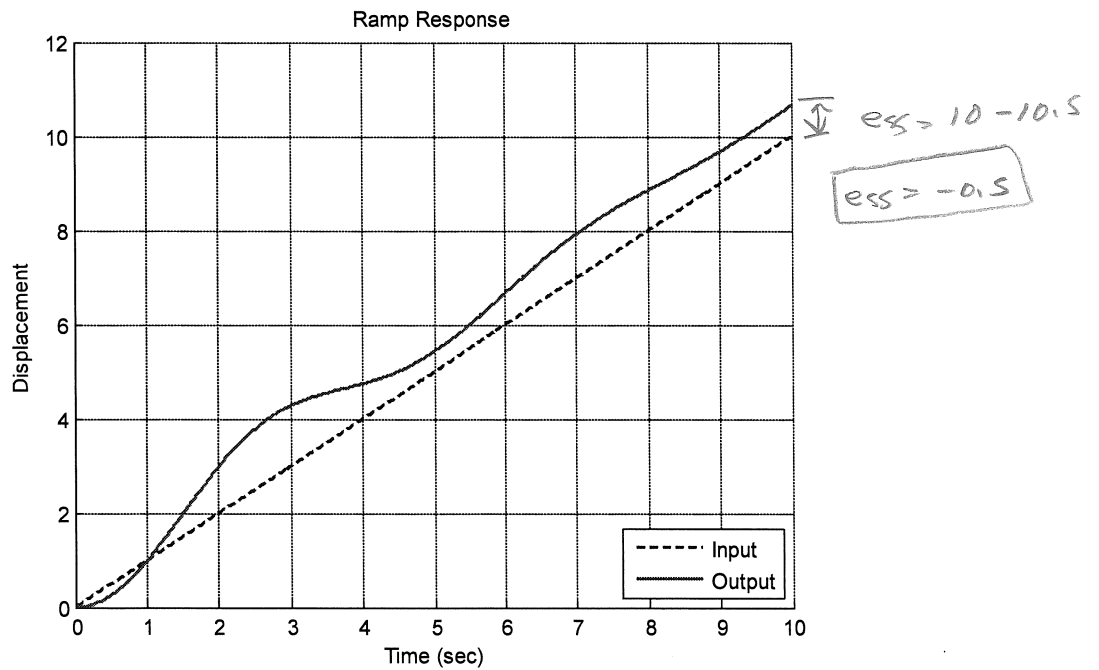
6) Estimate the static gain

$$K(0.2) = -0.5 \quad K = -2.5$$

7) Estimate the steady state error for the ramp response of the system shown below:



8) Estimate the steady state error for the ramp response of the system shown below:



9) The **unit step response** of a system is given by $y(t) = -u(t) - t^4 e^{-t} u(t) + e^{-2t} u(t)$

Estimate the **steady state error** for this system

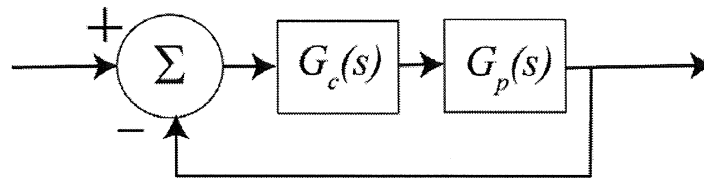
$$e_{ss} = \lim_{t \rightarrow \infty} [u(t) - y(t)] = \boxed{2 = e_{ss}}$$

10) The **unit ramp response** of a system is given by $y(t) = -2u(t) + tu(t) + e^{-t} u(t)$

Estimate the **steady state error** for this system

$$e_{ss} = \lim_{t \rightarrow \infty} [tu(t) - y(t)] = \boxed{2 = e_{ss}}$$

11) (10 points) For this problem assume the following unity feedback system



with $G_p(s) = \frac{3}{(s+2)(s+4)}$ and $G_c(s) = 2(s+3)$

a) Determine the position error constant K_p

$$K_p = G_p(0)G_c(0) = \frac{3}{8} \cdot 6 = \frac{18}{8} = \boxed{\frac{9}{4} = K_p}$$

b) Estimate the steady state error for a unit step using the position error constant.

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\frac{9}{4}} = \boxed{\frac{4}{13} = e_{ss}}$$

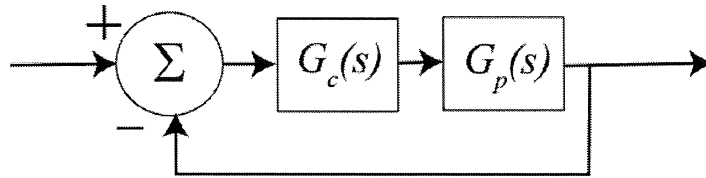
c) Determine the velocity error constant K_v

$$K_v = \lim_{s \rightarrow 0} s G_p(s) G_c(s) = \boxed{0 = K_v}$$

d) Estimate the steady state error for a unit ramp using the velocity error constant.

$$e_{ss} = \frac{1}{K_v} = \boxed{\infty = e_{ss}}$$

12) (10 points) For this problem assume the following unity feedback system



with $G_p(s) = \frac{3}{(s+2)(s+4)}$ and $G_c(s) = \frac{3(s+5)}{s}$

a) Determine the position error constant K_p $K_p = G_c(0)G_p(0) = \infty = K_p$

b) Estimate the steady state error for a unit step using the position error constant.

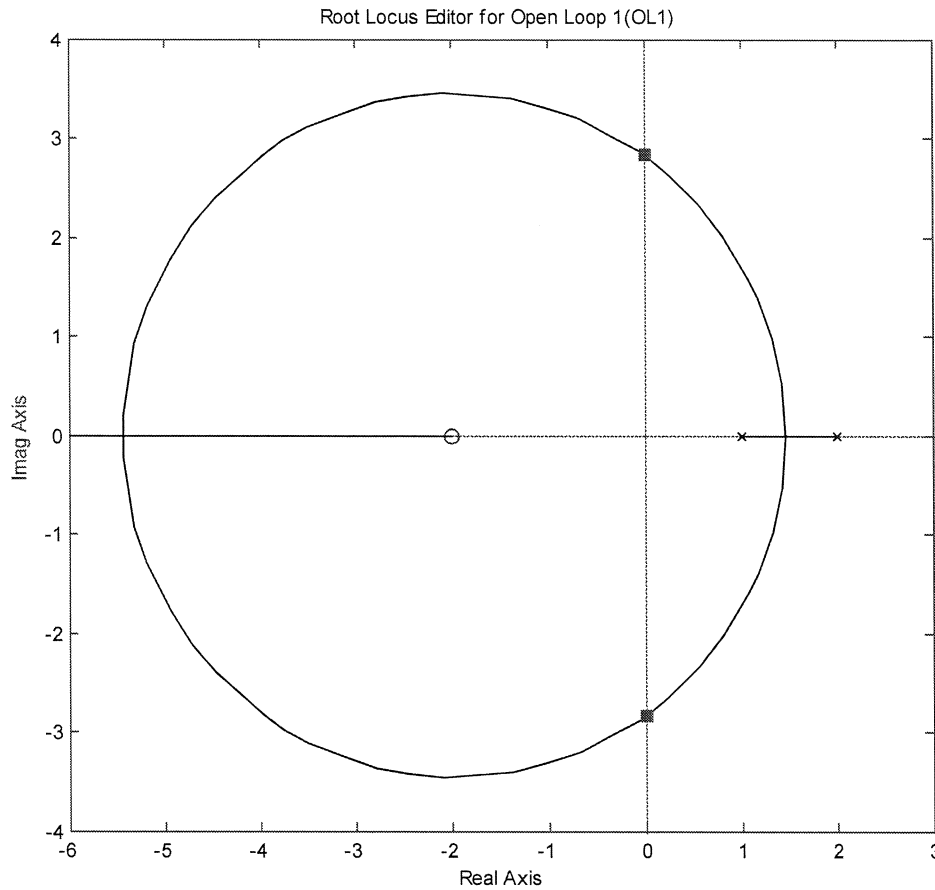
$e_{ss} = \frac{1}{1+K_p} = 0 = e_{ss}$

c) Determine the velocity error constant K_v $K_v = \lim_{s \rightarrow 0} s G_c(s) G_p(s) = \frac{3}{8} \cdot 15 = \frac{45}{8} = K_v$

d) Estimate the steady state error for a unit ramp using the velocity error constant.

$e_{ss} = \frac{1}{K_v} = \frac{8}{45} = e_{ss}$

Problems 13-17 refer to the following root locus plot (from sisotool)



13) Is it possible for -1 to be a closed loop pole for this system? (Yes or No)

14) When the gain is approximately 3 the closed loop poles are as shown in the figure. If we want the system to be stable what conditions do we need to place on the gain k?

$$K > 3$$

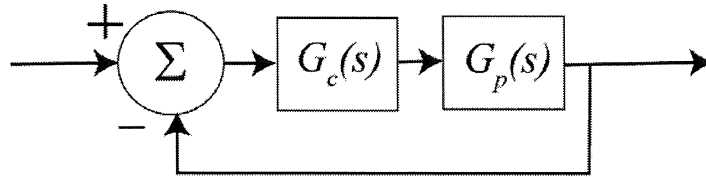
15) For this system, is it possible to get a response that is underdamped? (Yes or No)

16) For this system, is it possible to get a response that is overdamped? (Yes or No)

17) Is this a type one system? (Yes or No)

18) (19 points) For this problem assume the closed loop system below and assume

$$G_p(s) = \frac{3}{(s+2+2j)(s+2-2j)}$$



For each of the following problems sketch the root locus, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary.

a) Assume the proportional controller $G_c(s) = k_p$

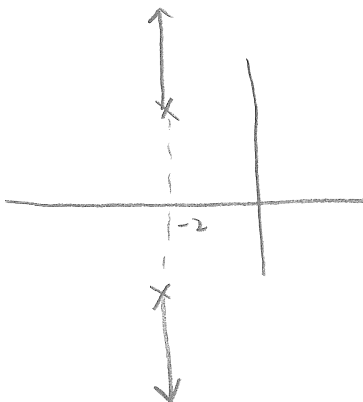
b) Assume the integral controller $G_c(s) = \frac{k_i}{s}$

c) Assume the PI controller $G_c(s) = \frac{k(s+5)}{s}$

d) Assume the PD controller $G_c(s) = k(s+6)$

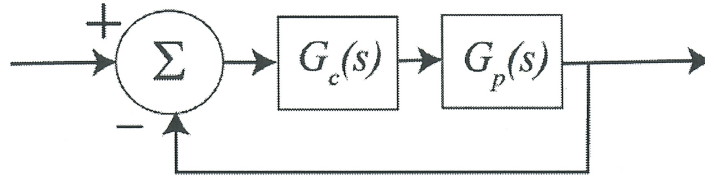
e) Assume the PID controller $G_c(s) = \frac{k(s+6+2j)(s+6-2j)}{s}$

a) $\sigma_c = \frac{[(-2-2j) + (-2+2j)]}{2} = -2$ $\theta = \frac{180 + i360}{2} = \pm 90^\circ$



18) (19 points) For this problem assume the closed loop system below and assume

$$G_p(s) = \frac{3}{(s+2+2j)(s+2-2j)}$$



For each of the following problems sketch the root locus, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary.

a) Assume the proportional controller $G_c(s) = k_p$

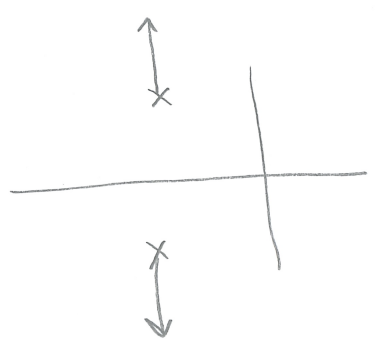
b) Assume the integral controller $G_c(s) = \frac{k_i}{s}$

c) Assume the PI controller $G_c(s) = \frac{k(s+5)}{s}$

d) Assume the PD controller $G_c(s) = k(s+6)$

e) Assume the PID controller $G_c(s) = \frac{k(s+6+2j)(s+6-2j)}{s}$

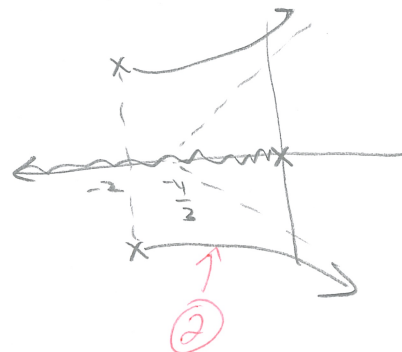
(a)
(3)



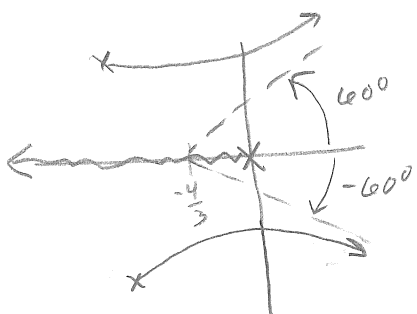
(b)
(5)

$$\sigma = \frac{[(-2-2j) + (-2+2j) + 0]}{3} = \frac{-4}{3}$$

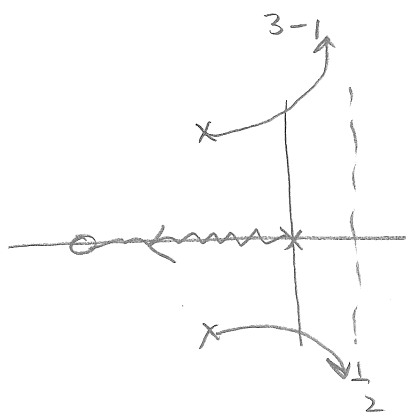
$$\theta = \frac{180 + i360}{3} = \pm 60^\circ, 180^\circ$$



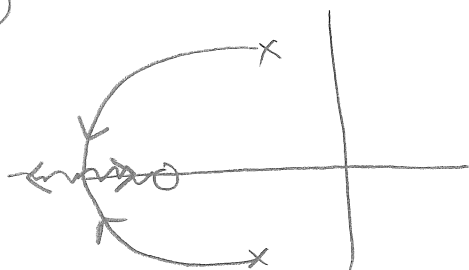
(b) $\sigma_c = \frac{[(-2-2j) + (-2+2j) + 0]}{3} = -\frac{4}{3}$ $\theta = \frac{180 + i360}{3} = \pm 60, 180$



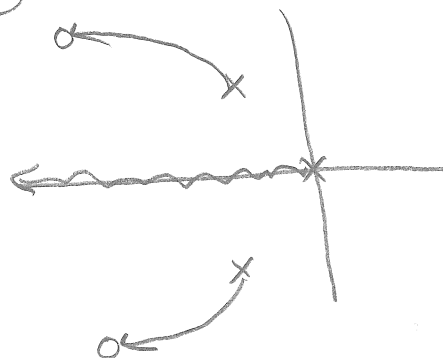
(c) $\sigma_c = \frac{[(-2-2j) + (-2+2j) + 0] - [-5]}{3-1} = \frac{1}{2}$ $\theta = \frac{180 + i360}{3-1} = \pm 90^\circ$



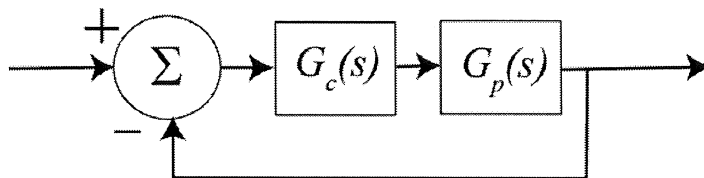
(d)



(e)



19) (10 points) For this problem assume the following unity feedback system



with $G_p(s) = \frac{1}{(s+3)}$ and $G_c(s) = k(s+2)$

a) Determine an expression for the closed loop transfer function $G_o(s)$

$$G_o = \frac{K(s+2)}{(s+3) + K(s+2)}$$

b) Determine an expression for the sensitivity of the closed loop transfer function $G_o(s)$ to variations in the parameter k . Your answer should be a function of s and should be a ratio of two polynomials.

$$\int_K^{G_o} = \frac{K(s+2)}{K(s+2)} - \frac{K(s+2)}{(s+3) + K(s+2)} = \frac{s+3}{(s+3) + K(s+2)} = \int_K^{G_o}$$

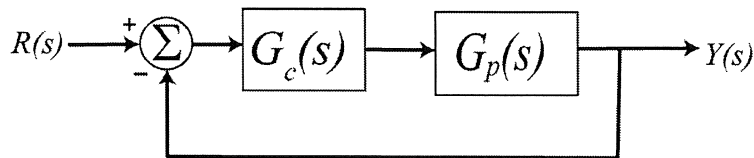
c) Determine an expression for the magnitude of the sensitivity function as a function of ω . Your final answer must be simplified as much as possible and cannot contain any j 's

$$\int_K^{G_o}(j\omega) = \frac{j\omega + 3}{j\omega + 3 + j\omega K + 2K}$$

$$\left| \int_K^{G_o}(j\omega) \right| = \sqrt{\frac{\omega^2 + 9}{\omega^2(1+K)^2 + (3+2K)^2}}$$

20) (6 points) Consider the following simple feedback control block diagram. The plant is

$G_p(s) = \frac{2}{s+4}$. The input is a unit step.



a) Determine the settling time of the plant alone (assuming there is no feedback)

$$T_S = \frac{4}{4} = 1 = T_S$$

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

$$G_0(s) = \frac{2k_p}{s+4+2k_p}$$

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the settling time is 0.5 seconds

$$T_S = \frac{4}{4+2k_p} = \frac{1}{2}$$

$$8 = 4 + 2k_p \quad k_p = 2$$