Name ____ Mailbox ____________

ECE-320 Linear Control Systems

Winter 2013, Exam 1

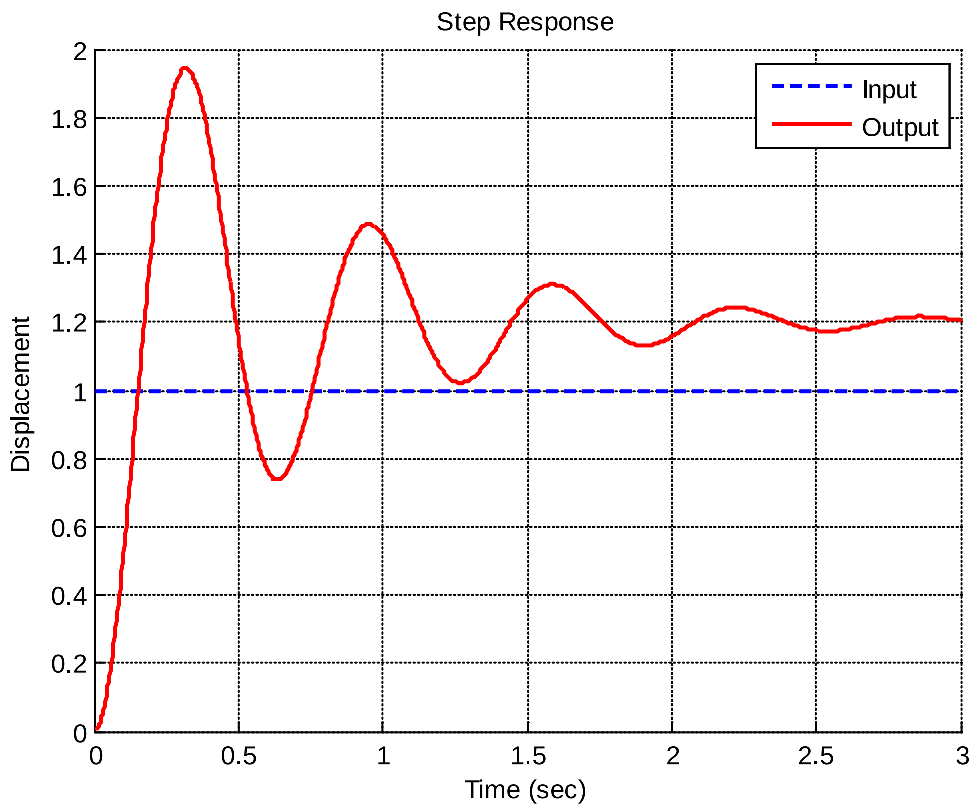
No calculators or computers allowed, you may leave your answers as fractions.

All problems are worth 3 points unless noted otherwise.

Total _______/100

Name ____ Mailbox ____________

Problems 1-3 refer to the **unit step response** of a system, shown below



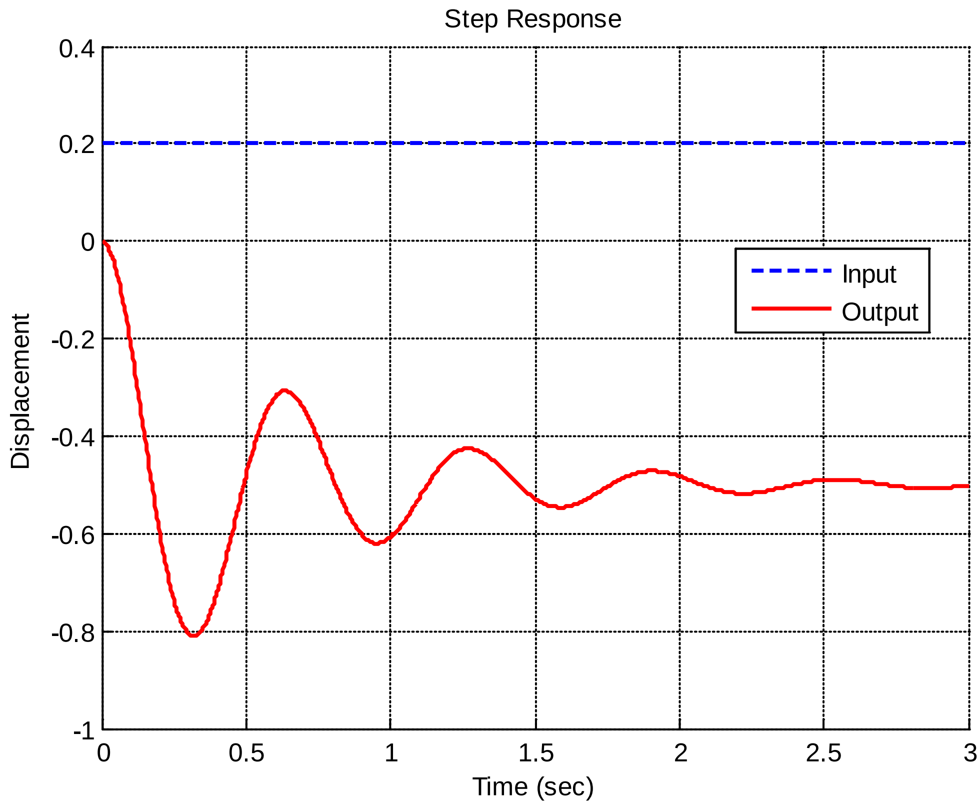
1) Estimate the **steady state error**

2) Estimate the **percent overshoot**

3) Estimate the **static gain**

Name ____ Mailbox ____________

Problems 4-6 refer to the **unit step response** of a system, shown below



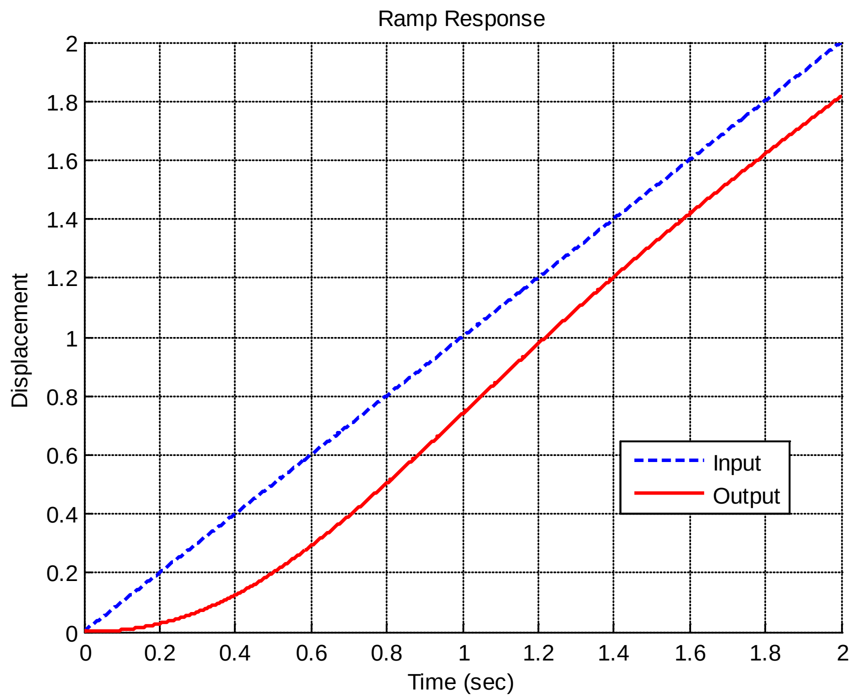
4) Estimate the **steady state error**

5) Estimate the **percent overshoot**

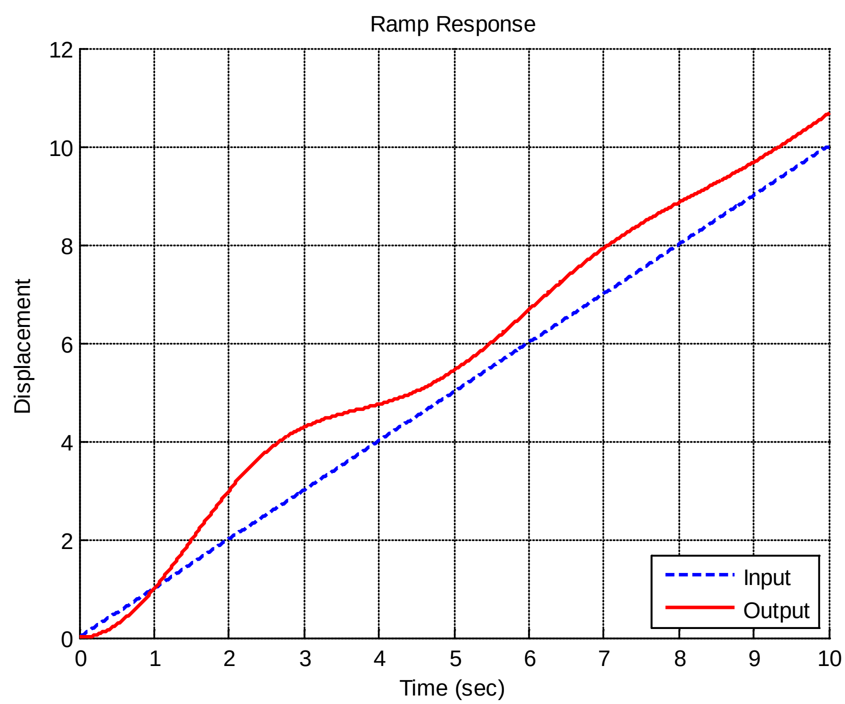
6) Estimate the **static gain**

Name ____ Mailbox ____________

7) Estimate the steady state error for the ramp response of the system shown below:



8) Estimate the steady state error for the ramp response of the system shown below:



Name ____ Mailbox ____________

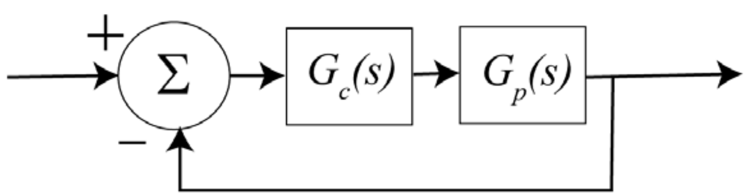
9) The **unit step response** of a system is given by $y(t) = -u(t) - t^4 e^{-t} u(t) + e^{-2t} u(t)$

Estimate the **steady state error** for this system

10) The **unit ramp response** of a system is given by $y(t) = -2u(t) + tu(t) + e^{-t} u(t)$.

Estimate the **steady state error** for this system

11) (10 points) For this problem assume the following unity feedback system



with $G_p(s) = \dfrac{3}{(s+2)(s+4)}$ and $G_c(s) = 2(s+3)$

a) Determine the position error constant K_p

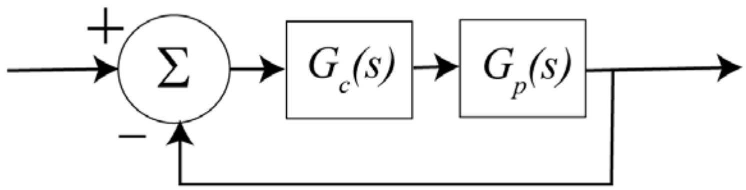
b) Estimate the steady state error for a unit step using the position error constant.

c) Determine the velocity error constant K_v

d) Estimate the steady state error for a unit ramp using the velocity error constant.

Name ____ Mailbox ____________

12) (10 points) For this problem assume the following unity feedback system



with $G_p(s) = \frac{3}{(s+2)(s+4)}$ and $G_c(s) = \frac{3(s+5)}{s}$

a) Determine the position error constant K_p

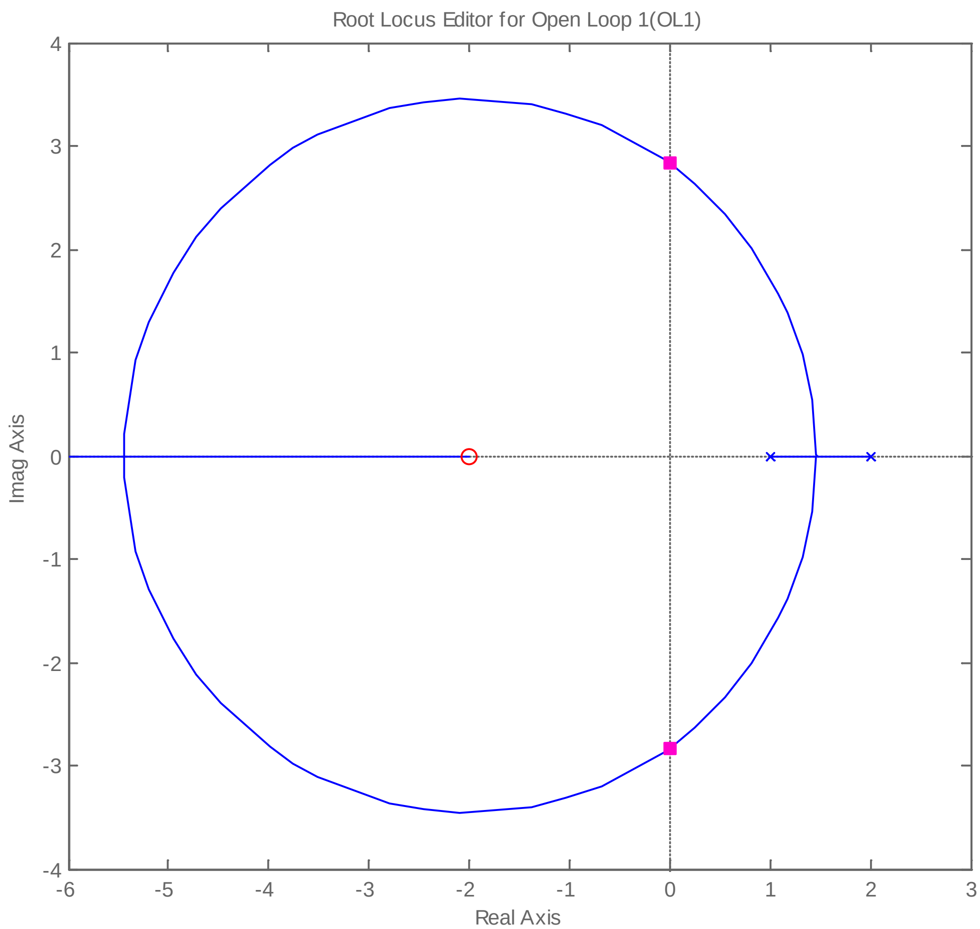
b) Estimate the steady state error for a unit step using the position error constant.

c) Determine the velocity error constant K_v

d) Estimate the steady state error for a unit ramp using the velocity error constant.

Name ____ Mailbox ____________

Problems 13-17 refer to the following root locus plot (from sisotool)



13) Is it possible for -1 to be a closed loop pole for this system ? (Yes or No)

14) When the gain is approximately 3 the closed loop poles are as shown in the figure. If we want the system to be stable what conditions do we need to place on the gain k?

15) For this system, is it possible to get a response that is ***underdamped***? (Yes or No)

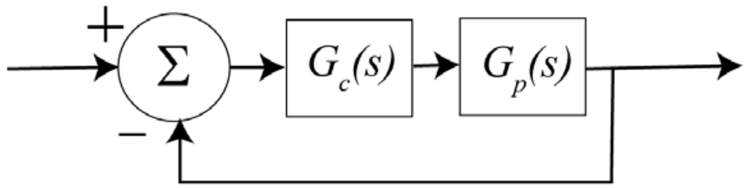
16) For this system, is it possible to get a response that is ***overdamped***? (Yes or No)

17) Is this a type one system? (Yes or No)

Name ____ Mailbox ____________

18) (19 points) For this problem assume the closed loop system below and assume

$$G_p(s) = \frac{3}{(s+2+2j)(s+2-2j)}$$



For each of the following problems sketch the root locus, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary.

a) Assume the proportional controller $G_c(s) = k_p$

b) Assume the integral controller $G_c(s) = \frac{k_i}{s}$

c) Assume the PI controller $G_c(s) = \frac{k(s+5)}{s}$

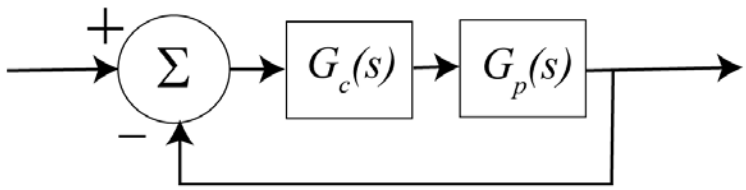
d) Assume the PD controller $G_c(s) = k(s+6)$

e) Assume the PID controller $G_c(s) = \frac{k(s+6+2j)(s+6-2j)}{s}$

Name ______________________________________ Mailbox ____________

Name ____ Mailbox ____________

19) (10 points) For this problem assume the following unity feedback system



with $G_p(s) = \dfrac{1}{(s+3)}$ and $G_c(s) = k(s+2)$

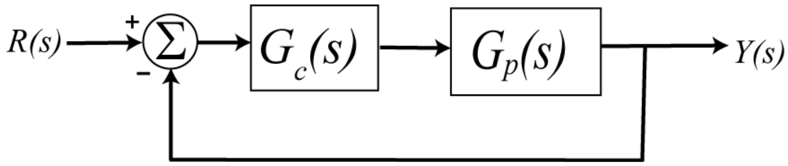
a) Determine an expression for the closed loop transfer function $G_o(s)$

b) Determine an expression for the sensitivity of the closed loop transfer function $G_o(s)$ to variations in the parameter k. Your answer should be a function of s and should be a ratio of two polynomials.

c) Determine an expression for the magnitude of the sensitivity function as a function of ω. Your final answer must be simplified as much as possible and cannot contain any j's

Name ____ Mailbox ____________

20) (6 points) Consider the following simple feedback control block diagram. The plant is $G_p(s) = \dfrac{2}{s+4}$. The input is a unit step.



a) Determine the settling time of the plant alone (assuming there is no feedback)

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the settling time is 0.5 seconds

Name ______________________________________ Mailbox ____________

Name ____ Mailbox ____________

Root Locus Construction

Once each pole has been paired with a zero, we are done

1. *Loci Branches*

$$\underset{k=0}{poles} \rightarrow \underset{k=\infty}{zeros}$$

Continuous curves, which comprise the locus, start at each of the n poles of $G(s)$ for which $k=0$. As k approaches ∞, the branches of the locus approach the m zeros of $G(s)$. Locus branches for excess poles extend to infinity.

The root locus is s**<u>ymmetric</u>** about the real axis.

2. *Real Axis Segments*

The root locus includes all points along the real axis to the left of an odd number of poles plus zeros of $G(s)$.

3. *Asymptotic Angles*

As $k \rightarrow \infty$, the branches of the locus become asymptotic to straight lines with angles

$$\theta = \frac{180^o + i360^o}{n-m}, i = 0, \pm 1, \pm 2, ..$$

until all ($n-m$) angles not differing by multiples of 360^o are obtained. n is the number of poles of $G(s)$ and m is the number of zeros of $G(s)$.

4. *Centroid of the Asymptotes*

The starting point on the real axis from which the asymptotic lines radiate is given by

$$\sigma_c = \frac{\sum_i p_i - \sum_j z_j}{n-m}$$

where p_i is the i^{th} pole of $G(s)$, z_j is the j^{th} zero of $G(s)$, n is the number of poles of $G(s)$ and m is the number of zeros of $G(s)$. This point is termed the *centroid of the asymptotes*.

5. *Leaving/Entering the Real Axis*

When two branches of the root locus leave or enter the real axis, they usually do so at angles of $\pm 90^o$.