**ECE-320**: Linear Control Systems
Homework 7

Due: **<u>Tuesday</u>** January 29 at the beginning of class

**<u>Exam 2</u>**, Thursday January 31

**1)** For the $z$-transform

$$X(z) = \frac{3}{z-2}$$

**a)** Show that, by multiplying and dividing by $z$ and then using partial fractions, the corresponding discrete-time sequence is

$$x(n) = -\frac{3}{2}\delta(n) + \frac{3}{2}2^k u(n)$$

**b)** By starting with the $z$-transform

$$G(z) = \frac{3z}{z-2}$$

where $X(z) = z^{-1}G(z)$, determine $g(n)$ and use the delay property to show that

$$x(n) = 3 \times 2^{n-1} u(n-1)$$

**2)** For impulse response $h(n) = \left(\frac{1}{3}\right)^{n-2} u(n-1)$ and input $x(n) = \left(\frac{1}{2}\right)^{n} u(n-1)$, use z-transforms of the input and impulse response to show the output is

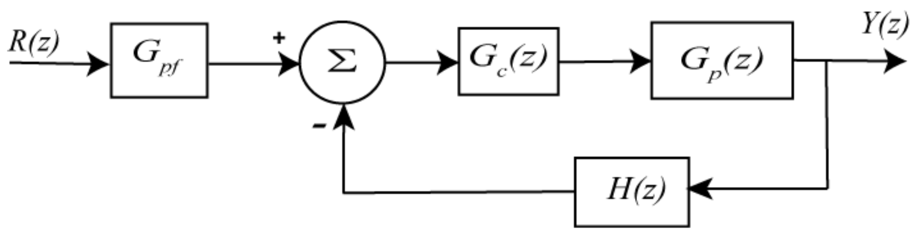
$$y(n) = 9\left[\left(\frac{1}{2}\right)^{n-1} - \left(\frac{1}{3}\right)^{n-1}\right]u(n-2) = 9\left[\left(\frac{1}{2}\right)^{n-1} - \left(\frac{1}{3}\right)^{n-1}\right]u(n-1)$$

*Hint:* Assume $Y(z) = z^{-1}G(z)$, determine $g(n)$ and then $y(n)$

**3)** For impulse response $h(n) = \left(\frac{1}{2}\right)^{n-3} u(n-1)$ and input $x(n) = \left(\frac{1}{4}\right)^{n+1} u(n-2)$, use $z$-transforms of the input and impulse response to show the system output is $y(n) = \left[\left(\frac{1}{2}\right)^{n} - \left(\frac{1}{4}\right)^{n-1}\right]u(n-3)$

*Hint:* Assume $Y(z) = z^{-2}G(z)$,

**4)** For the following system, assuming the closed loop systems are stable, determine the prefilter gain $G_{pf}$ that will result in zero steady state error for a unit step input. Are any of these systems type one systems?




**a)** $G_p(z) = \frac{0.2}{z^2 + 0.1z + 0.2}, \quad G_c(z) = \frac{z}{z-1}, \quad H(z) = 1$

**b)** $G_p(z) = \frac{0.2}{z^2 + 0.1z + 0.2}, \quad G_c(z) = \frac{1}{z}, \quad H(z) = 1$

**c)** $G_p(z) = \frac{1}{z^2 + 0.4z + 0.04}, \quad G_c(z) = \frac{0.2}{z+0.2}, \quad H(z) = \frac{1}{z+0.2}$

Answers: 7.5, 9.47, one is type one (so the prefilter has value 1)

**5)** Consider the continuous-time plant with transfer function

$$G_p(s) = \frac{1}{(s+1)(s+2)}$$

We want to determine the discrete-time equivalent to this plant, $G_p(z)$, by assuming a zero order hold is placed before the continuous-time plant to convert the discrete-time control signal to a continuous time control signal.

Show that if we assume a sampling interval of $T$, the equivalent discrete-time plant is

$$G_p(z) = \frac{z(0.5 - e^{-T} + 0.5e^{-2T}) + (0.5e^{-T} - e^{-2T} + 0.5e^{-3T})}{(z - e^{-T})(z - e^{-2T})}$$

Note that we have poles were we expect them to be, but we have introduced a zero in going from the continuous time system to the discrete-time system.