Name Solutions Mailbox ____________

ECE-320 Linear Control Systems

Winter 2012, Exam 2

No calculators or computers allowed.

Problem 1	**_______/20**
Problem 2	**_______/20**
Problem 3	**_______/12**
Problem 4	**_______/12**
Problems 5-16	**_______/36**
Total	**_______/100**

Name ____ Mailbox ____________

1) **(20 points)** For impulse response $h(n) = \left(\frac{1}{2}\right)^{n+1} u(n+2)$ and input $x(n) = \left(\frac{1}{4}\right)^{n-1} u(n-2)$,

determine the system output by evaluating the convolution sum $y(n) = \sum_{k=-\infty}^{\infty} h(n-k)x(k)$

Note: you do not have to simplify your answer, but you must remove all sums and include a unit step function of some sort.

$n-k+2 \geq 0$

$n+2 \geq k$

$k-2 \geq 0$

$k \geq 2$

$n+2 \geq k \geq 2$

$n+2 \geq 2$ $\quad n \geq 0$

$$y(n) = \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^{n-k+1} u(n-k+2) \left(\tfrac{1}{4}\right)^{k-1} u(k-2)$$

$$= \sum_{k=2}^{k=n+2} \left(\tfrac{1}{2}\right)^{n+1} \left(\tfrac{1}{2}\right)^{-k} \left(\tfrac{1}{4}\right)^{k} \left(\tfrac{1}{4}\right)^{-1}$$

$$= \left(\tfrac{1}{2}\right)^{n+1} 4 \sum_{k=2}^{k=n+2} \left(\tfrac{1}{2}\right)^{k}$$

let $l = k-2$

$l+2 = k$

$$= \left(\tfrac{1}{2}\right)^{n+1} 4 \sum_{l=0}^{l=n} \left(\tfrac{1}{2}\right)^{l+2} = \left(\tfrac{1}{2}\right)^{n+1} 4 \left(\tfrac{1}{2}\right)^{2} \sum_{l=0}^{l=n} \left(\tfrac{1}{2}\right)^{l}$$

$$= \left(\tfrac{1}{2}\right)^{n+1} \left[\frac{1-\left(\tfrac{1}{2}\right)^{n+1}}{1-\tfrac{1}{2}}\right] = \left(\tfrac{1}{2}\right)^{n} \left[1-\left(\tfrac{1}{2}\right)^{n+1}\right]$$

$$\boxed{y(n) = \left(\tfrac{1}{2}\right)^{n} \left[1-\left(\tfrac{1}{2}\right)^{n+1}\right] u(n)}$$

Name ____ Mailbox ____________

2) (20 points) For impulse response $h(n) = \left(\frac{1}{2}\right)^{n+1} u(n-1)$ and input $x(n) = \left(\frac{1}{3}\right)^{n+1} u(n-2)$,

a) determine the z-transform of $h(n)$, $H(z)$

b) determine the z-transform of $x(n)$, $X(z)$

c) determine $y(n)$

Hint: Assume $Y(z) = z^{-2}G(z)$, determine $g(n)$ and then $y(n)$

a) $h(n) = \left(\frac{1}{2}\right)^{n+1} u(n-1) = \left(\frac{1}{2}\right)^{n-1}\left(\frac{1}{2}\right)^{2} u(n-1) \Rightarrow H(z) = \frac{1}{4} z^{-1} \frac{z}{z-\frac{1}{2}}$

$$\boxed{H(z) = \frac{\frac{1}{4}}{z-\frac{1}{2}}}$$

b) $x(n) = \left(\frac{1}{3}\right)^{n+1} u(n-2) = \left(\frac{1}{3}\right)^{n-2}\left(\frac{1}{3}\right)^{3} u(n-2) \Rightarrow X(z) = \frac{1}{27} z^{-2} \frac{z}{z-\frac{1}{3}}$

$$\boxed{X(z) = \frac{\frac{1}{27} z^{-1}}{z-\frac{1}{3}}}$$

c) $Y(z) = H(z)\,X(z) = \dfrac{\frac{1}{4}\frac{1}{27} z^{-1}}{(z-\frac{1}{2})(z-\frac{1}{3})}$ $\qquad G(z) = z^{2}Y(z) = \dfrac{\frac{1}{4}\frac{1}{27} z}{(z-\frac{1}{2})(z-\frac{1}{3})}$

$$\frac{G(z)}{z} = \frac{\frac{1}{4}\frac{1}{27}}{(z-\frac{1}{2})(z-\frac{1}{3})} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{3}}$$

$$A = \frac{\frac{1}{4}\cdot\frac{1}{27}}{\frac{1}{6}} = \frac{6}{4\cdot 27} = \frac{1}{18}$$

$$B = \frac{\frac{1}{4}\cdot\frac{1}{27}}{-\frac{1}{6}} = -\frac{6}{4\cdot 27} = \frac{-1}{18}$$

$g(n) = \frac{1}{18}\left(\frac{1}{2}\right)^{n} u(n) - \frac{1}{18}\left(\frac{1}{3}\right)^{n} u(n)$

$y(n) = g(n-2) = \boxed{\frac{1}{18}\left[\left(\frac{1}{2}\right)^{n-2} - \left(\frac{1}{3}\right)^{n-2}\right] u(n-2) = y(n)}$

Name ____ Mailbox ____________

3) (12 points) Consider the continuous-time plant with transfer function

$$G_p(s) = \frac{1}{s}$$

We want to determine the discrete-time equivalent to this plant, $G_p(z)$, by assuming a zero order hold is placed before the continuous-time plant to convert the discrete-time control signal to a continuous time control signal.

$$G_p(z) = (1 - z^{-1}) \mathcal{Z}\left\{\frac{1}{s^2}\right\} \qquad \mathcal{Z}\left\{\frac{1}{s^2}\right\} = \frac{Tz}{(z-1)^2}$$

$$G_p(z) = (1 - z^{-1}) \frac{Tz}{(z-1)^2} = \left(\frac{z-1}{z}\right) \frac{Tz}{(z-1)^2} = \boxed{\frac{T}{z-1} = G_p(z)}$$

Name ______________________________ Mailbox __________

4) (12 points) In this problem we derive discrete-time equivalents to continuous-time controllers. *You must show work or you will receive no points (you cannot just write down the answers!)*

a) Consider the continuous-time integral controller, $G_c(s) = k_i\left(\frac{1}{s}\right)$. By determining the discrete-time equivalent to the function $\frac{1}{s}$, find the equivalent discrete-time integral controller.

$$Z\left\{\frac{1}{s}\right\} = \frac{z}{z-1} = \frac{1}{1-z^{-1}} \qquad \boxed{G_c(z) = \frac{k_i}{1-z^{-1}}}$$

b) Consider the continuous-time derivative controller, $G_c(s) = k_d s = \frac{U(s)}{E(s)}$. Using the continuous-time approximation $u(t) = k_d\left[\frac{e(t)-e(t-T)}{T}\right]$, take z-transforms to find the equivalent discrete-time derivative controller.

$$u(nT) = K_d\left[\frac{e(nT) - e((n-1)T)}{T}\right] \qquad U(z) = \frac{K_d}{T}\left[E(z) - z^{-1}E(z)\right]$$

$$\frac{U(z)}{E(z)} = \boxed{G_c(z) = \frac{k_d}{T}(1-z^{-1})}$$

c) Assuming we using *sisotool* to determine the PI controller $G_c(z) = \frac{7(z-\frac{5}{7})}{z-1} = k_p + \frac{k_i}{1-z^{-1}}$. Determine k_p and k_i.

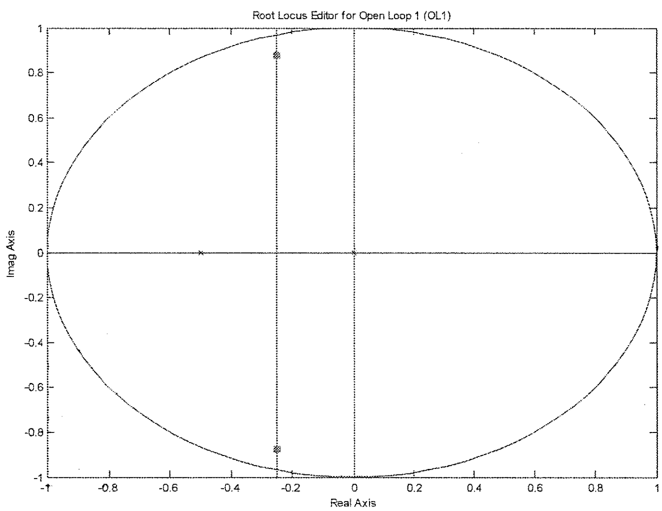
$$\frac{7(z-5/7)}{z-1} = \frac{7z-5}{z-1} = K_p + \frac{K_i}{1-z^{-1}} = K_p + \frac{K_i z}{z-1}$$

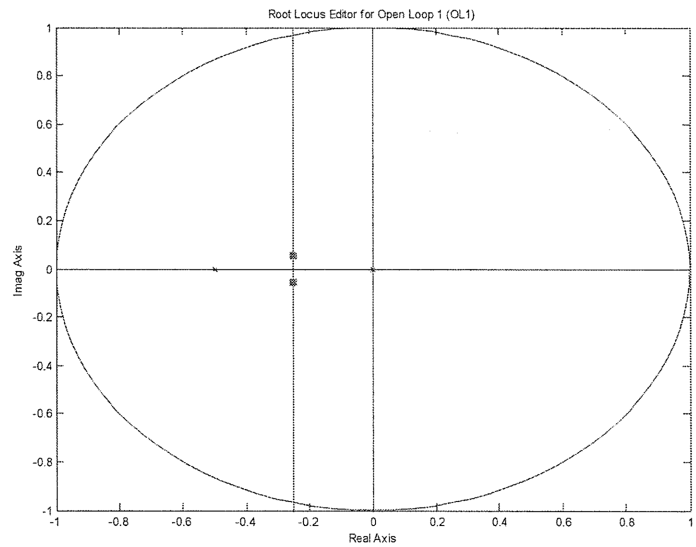
$$= \frac{K_p z - K_p + K_i z}{z-1}$$

$$7z - 5 = (K_p + K_i)z - K_p \qquad \boxed{K_p = 5 \quad K_i = 2}$$

Name ____ Mailbox ____________

Problems 5 and 6 refer to the following two root locus plot for a discrete-time system





5) For which system is the settling time likely to be smallest?

a) The system on the left b) the system on the right c) the settling time will be the same

poles close to origin

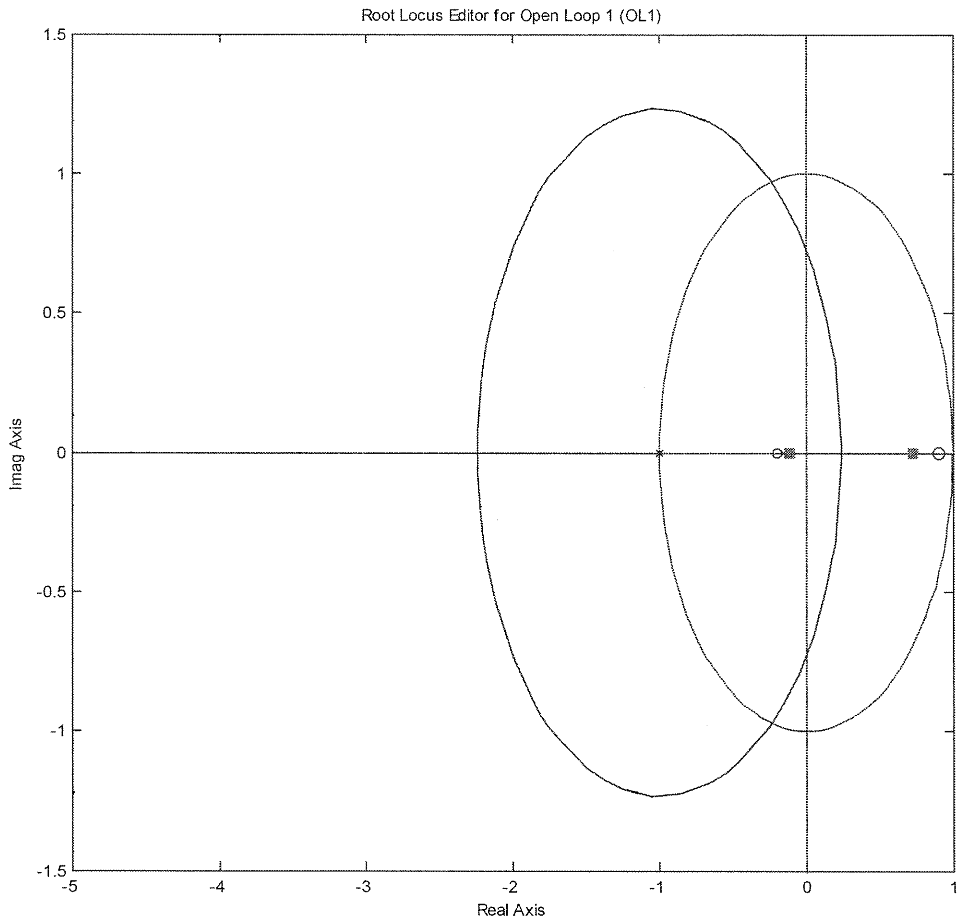
6) Is this a type 1 system?

a) yes b) no c) not enough information

no poles at $z = 1$

Name ______________________________ Mailbox __________

Problems 7-9 refer to the following root locus plot for a discrete-time system



7) With the closed loop pole locations shown in the figure, is the closed loop system stable?

a) yes b) no c) not enough information *all poles inside unit circle*

8) Is there any value of k for which the closed loop system is stable?

a) yes b) no c) not enough information

9) Is this a type one system?

a) yes b) no c) not enough information

no poles at z=1

Name ______________________________________ Mailbox ____________

10) Is the following system ***controllable***? $G(s) = \dfrac{G_{pf}}{(s - k_1 k_2)^2}$

a) Yes b) No c) impossible to determine

11) Is the following system ***controllable***? $G(s) = \dfrac{8G_{pf}}{s^2 + 12s + (k_1 + k_2 + 20)}$

a) Yes b) No c) impossible to determine

12) Is the following system **controllable**? $G(s) = \dfrac{G_{pf}}{s^2 + (k_2 + k_1 - 1)s + (k_2 + 2)}$

a) Yes b) No c) impossible to determine

13) Consider a plant that is unstable but is a controllable system. Is it possible to use state variable feedback to make this system stable?

a) Yes b) No

14) Is it possible for a system with state variable feedback to change the zeros of the plant (other than by pole-zero cancellation) ?

a) Yes b) No

15) Is it possible for a system with state variable feedback to introduce zeros into the closed loop system?

a) Yes b) No

16) If a plant has *n* poles, then a system with state variable feedback with no pole-zero cancellations will have

a) more than *n* poles b) less than *n* poles c) n poles d) it is not possible to tell