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ECE-320 Linear Control Systems

Winter 2012, Exam 1

No calculators or computers allowed, except for Problem 6 when you should use Matlab's sisotool.

You must simplify your answers as much as possible, or points will be deducted.

Problem 1 _______/24

Problem 2 _______/12

Problem 3 _______/8

Problem 4 _______/8

Problem 5 _______/24

Problem 6 _______/24

Total _______/100

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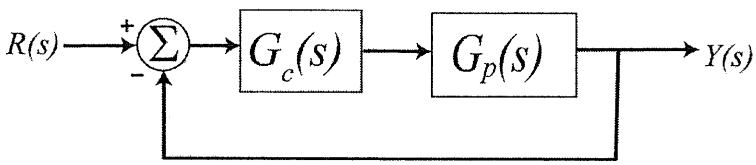
1) (24 points) Consider the following simple feedback control block diagram. The plant is $G_p(s) = \frac{3}{s+5}$. The input is a unit step.



a) Determine the settling time, steady state error for a unit step input, and the bandwidth of the plant alone (assuming there is no feedback)

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the steady state error for a unit step is 1/4, and the corresponding settling time for the system.

d) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the settling time is 4/11 seconds, and the corresponding steady state error.

e) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the bandwidth is 17 rad/sec.

(a) $\boxed{T_s = \frac{4}{5} \text{ sec}}$ $\boxed{e_{ss} = 1 - 3/5 = 2/5}$ $\boxed{BW = 5 \text{ rad/sec}}$

(b) $\boxed{G_0(s) = \frac{3k_p}{s+5+3k_p}}$

(c) $e_{ss} = \frac{5}{5+3k_p} = \frac{1}{4}$ $\boxed{k_p = 5}$ $T_s = \frac{4}{5+15} = \frac{4}{20} = \boxed{\frac{1}{5} = T_s}$

(d) $T_s = \frac{4}{5+3k_p} = \frac{4}{11}$ $\boxed{k_p = 2}$ $\boxed{e_{ss} = 5/11}$

(e) $BW = 5 + 3k_p = 17$ $\boxed{k_p = 4}$

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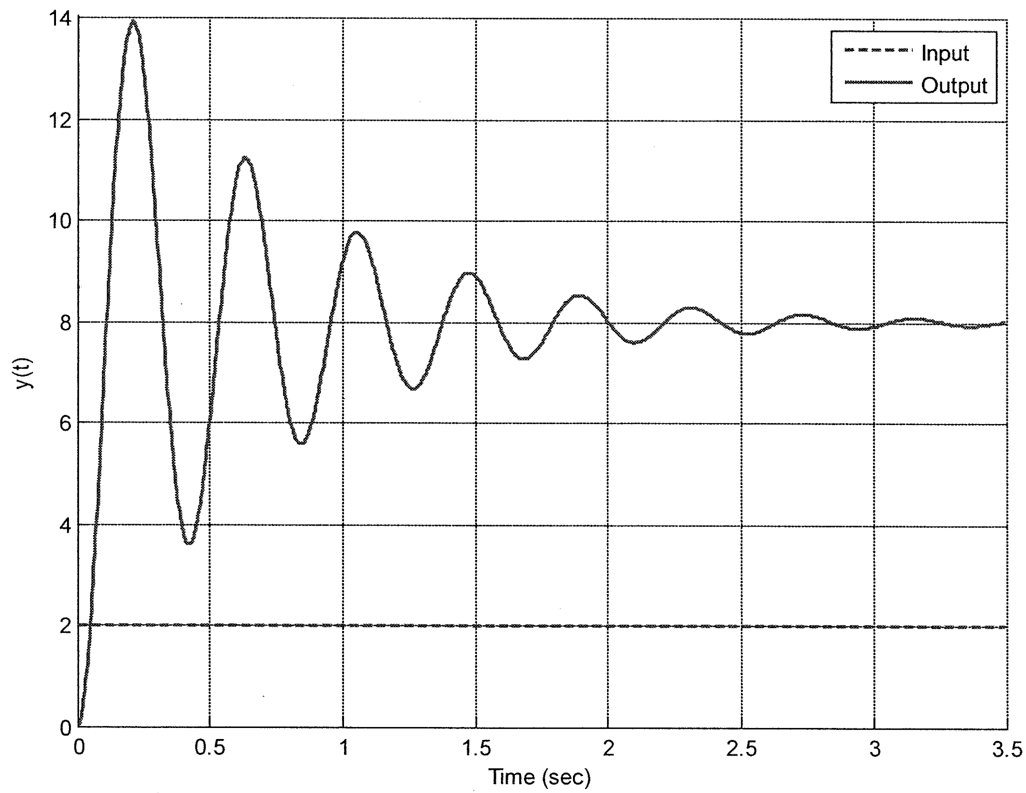
2) (12 points) For the following questions, refer to the following graph showing the input and output of a second order system. For this system the input is a step of amplitude 2. (You can leave your answers as fractions.)



a) What is the static gain of the system? $K \cdot 2 = 8$ $\boxed{K = 4}$

b) What is the percent overshoot? $PO = \frac{14-8}{8} \times 100\% = \frac{6}{8} \times 100\% = \boxed{75\%}$

c) What is the steady state error? $e_{ss} = 2 - 8 = \boxed{-6}$

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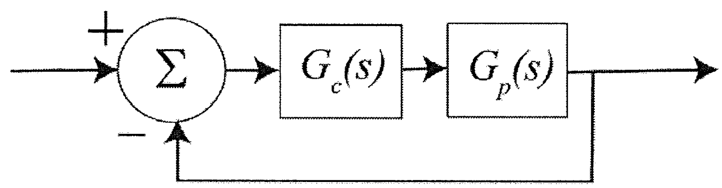
3) **(8 points)** For the following systems, assume $G_c(s) = \frac{1}{s+2}$ and $G_p(s) = \frac{1}{s+5}$



a) Determine the position error constant K_p $\boxed{K_p = \frac{1}{10}} = \lim_{s \to 0} G_c(s) G_p(s)$

b) Determine the steady state error for a unit step input.

$e_{ss} = \frac{1}{1+K_p} = \boxed{\frac{10}{11} = e_{ss}}$

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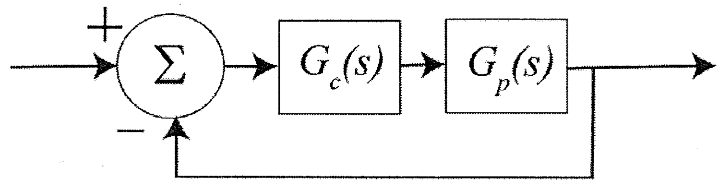
4) (8 points) For the following systems, assume $G_c(s) = \frac{3}{s}$ and $G_p(s) = \frac{1}{s+4}$



a) Determine the velocity error constant K_v.

$\boxed{K_v = \frac{3}{4}} = \lim_{s \to 0} s\, G_c(s)\, G_p(s)$

b) Determine the steady state error for a unit ramp input.

$e_{ss} = \frac{1}{K_v} = \boxed{\frac{4}{3}}$

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5) (24 points) For a system with the transfer function $H(s)=\dfrac{1}{(s+1)(s+2)^2}$

a) Determine the **impulse response** $h(t)$

$$H(s) = \frac{1}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$A = 1 \quad C = -1$

$\times s$, let $s \to \infty$ $\quad 0 = A + B \quad B = -A = -1$

$$\boxed{h(t) = \left(e^{-t} - e^{-2t} - te^{-2t}\right)u(t)}$$

b) Determine the **unit step response.**

$$Y(s) = \frac{1}{s(s+1)(s+2)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} + \frac{D}{(s+2)^2}$$

$A = \frac{1}{4}$

$B = -1$

$D = \frac{1}{2}$

$\times s$, let $s \to \infty$

$0 = A + B + C$

$C = -A - B = -\frac{1}{4} + 1 = 3/4$

$$\boxed{y(t) = \left(\frac{1}{4} - e^{-t} + \frac{3}{4}e^{-2t} + \frac{1}{2}te^{-2t}\right)u(t)}$$

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6) (24 points) ***(sisotool problem)***

Consider the plant

$$G_p(s) = \frac{100}{s^2 + 2s + 20}$$

Design a PID controller using *sisotool* with ***complex zeros*** so that

$$\begin{aligned} T_s &\leq 1.0\,\text{sec} \\ P.O. &\leq 10\% \end{aligned}$$

In addition, your controller must be designed so that

$$\begin{aligned} k_p &\leq 0.5 \\ k_i &\leq 5 \\ k_d &\leq 0.1 \end{aligned}$$

Write your final values for k_p, k_i, k_d, and the transfer function of the controller in the space below.

$k_p =$

$k_i =$

$k_d =$

$G_c(s) =$