ECE-320: Linear Control Systems
Homework 2

Due: Tuesday December 13 *at the beginning of class*

Reading: Chapters 4, 5, 6, 8, 9

1) For systems with the following transfer functions:

$$H_a(s) = \frac{1}{s+2}$$

$$H_b(s) = \frac{s+6}{(s+2)(s+3)}$$

a) Determine the unit step and unit ramp response for each system using Laplace transforms. Your answer should be time domain functions $y_a(t)$ and $y_b(t)$.

b) From these time domain functions, determine the steady state errors for a unit step and unit ramp input.

c) Using the equations derived in class (and in the notes), determine the steady state errors for a unit step and a unit ramp input directly from the transfer functions.

The following Matlab code can be used to estimate the step and ramp response for 5 seconds for transfer function $H_b(s)$.

```
H = tf([1 6],[1 5 6]);                 % enter the transfer function
t = [0:0.01:5];                        % t goes from 0 to 5 by increments of 0.01
ustep = ones(1,length(t));             % the step input is all ones, u(t) = 1;
uramp = t;                             % the ramp input is has the input u(t) = t;
ystep = lsim(H,ustep,t);               % find the step response
yramp = lsim(H,uramp,t);               % find the ramp response
figure;                                % make a new figure
orient tall                            % or orient landscape, use more of the page
subplot(2,1,1);                        % put two graphs on one piece of paper
plot(t,ustep,'.-',t,ystep,'-');        % plot input/output with different line types
grid;                                  % put a grid on the graph
legend('Step Input','Step Response',4);   % put a legend on the graph
subplot(2,1,2);                        % second of two graphs on one piece of paper
plot(t,uramp,'.-',t,yramp,'-');        % plot input/output with different line types
grid;                                  % put a grid on the graph
legend('Ramp Input','Ramp Response',4);   % put a legend on the graph
```

d) Plot the step and ramp response for both systems (a and b) and indicate the steady state errors on the graph. Draw on the graph to show you know what the steady state errors are.

Ans. Steady state errors for a unit step input: 0.5,0; for a unit ramp input : infinity and 0.666

**2)** For the following systems

a) Determine the system type (0, 1, 2, ...)

b) If the system is type 0 assume $G_{pf} = 1$ and determine the position error constant K_p and the steady state error for a unit step input. Then determine the value of G_{pf} to make this error zero. If the system is type 1, assume $G_{pf} = 1$ and determine the steady state error for a unit step, the velocity error constant K_v, and the steady state error for a unit ramp. Is there any constant value of G_{pf} that can change the steady state error for a ramp?

Ans. (steady state errors) $-\dfrac{3}{2}, \dfrac{3}{13}, -\dfrac{3}{5}, \dfrac{1}{2};$ (prefilers) $\dfrac{2}{5}, \dfrac{13}{10}, G_{pf}$ has no effect
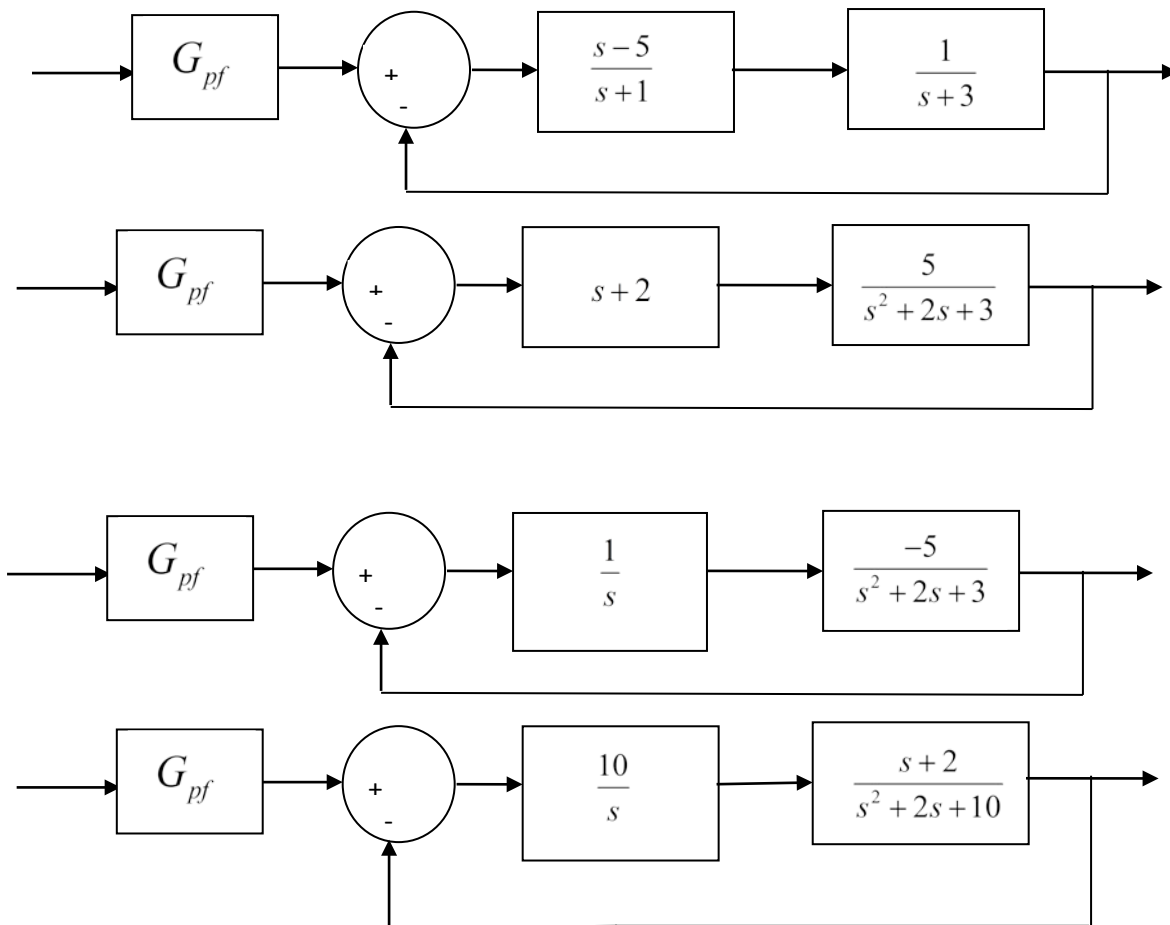
3) Consider the following control system.
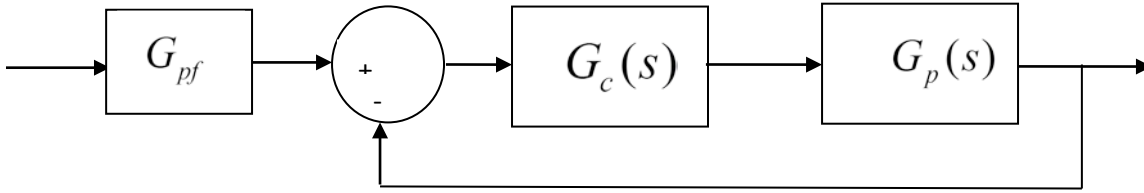
We can compute the position error constant K_p as $K_p = G_c(0)G_p(0)$

a) Determine an expression for the closed loop transfer function (from input to output) $G_o(s)$ in terms of G_{pf}, $G_c(s)$, and $G_p(s)$.

b) For a steady state error of zero for a step input we want $G_o(0) = 1$. Use this information to show that we can determine the prefilter gain to be
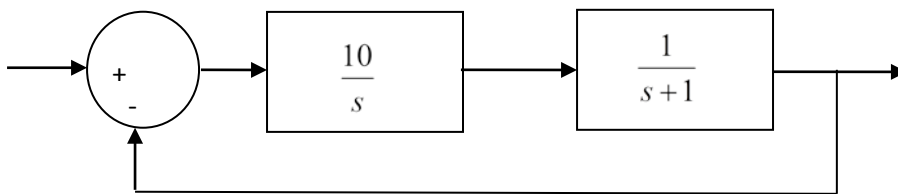
$$G_{pf} = 1 + \frac{1}{K_p}$$

c) We can find the steady state error for a unit step input as $e_{ss} = \frac{1}{1+K_p}$. Using this, show that we can

determine the prefilter gain to be

$$G_{pf} = \frac{1}{1-e_{ss}}$$

Note that if $K_p = \infty$ (or equivalently $e_{ss} = 0$), we get $G_{pf} = 1$)

4) Consider the following control system:

a) If the input to the system is $r(t) = 8u(t)$, what is the steady state output?

b) If the input to the system is $r(t) = 8\sin(3t)u(t)$, what is the output in steady state?
What is the time lag between the input signal and the output signal?
Hint: you can write $\omega t - \theta = \omega(t - t_d)$ if θ is measured in radians.
Answers: $y(t) = 8, y(t) = 8\sqrt{10}\sin(3t - 71.57°), t_d = 0.416\sec$

5) For the following problem, assume we are using the following control system
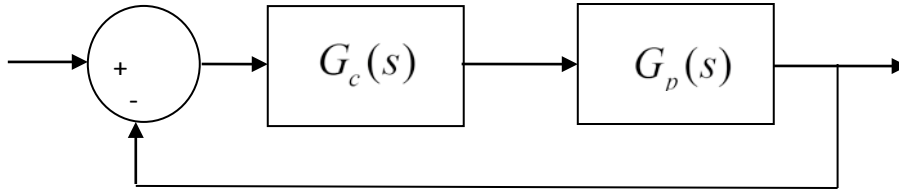
where the plant is given by

$$G_p(s) = \frac{1}{s^2 + 4s + 29} = \frac{1}{(s+2-5j)(s+2+5j)}$$

For the following controllers, sketch the root locus with arrows showing the direction of travel as k increases. If there are any poles going to zeros at infinity, you need to compute the centroid of the asymptotes (σ_c) and the angles of the asymptotes.

You may (and should) check your answers with Matlab (use the **rlocus** command), but you need to do this by hand.

a) $G_c(s) = k$ (proportional (P) controller)

b) $G_c(s) = \dfrac{k}{s}$ (an integral (I) controller)

c) $G_c(s) = \dfrac{k(s+z)}{s}$ (a proportional + integral (PI) controller) Write the centroid σ_c as a function of z. For what values of z will the two asymptotes be in the right half plane? *(For plotting purposes, assume z is equal to 2.)*

d) $G_c(s) = k(s+z)$ (a proportional+derivative (PD) controller)

e) $G_c(s) = \dfrac{k(s+z_1)(s+z_2)}{s}$ (a proportional+integral+derivative (PID) controller) Sketch this for the case where both zeros are real and then when both zeros are complex conjugates.

f) $G_c(s) = \dfrac{k(s+z)}{(s+p)}$ (a lead controller, $p > z$) Write an expression for σ_c as a function of the distance between the pole and the zero, $l = p - z$. What happens to the asymptotes as l gets larger? *(For plotting purposes, assume p is 5 and z is 1.)*

6) For the following problem, assume we are using the following control system
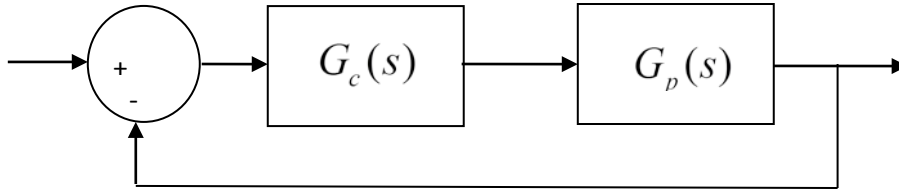
where the plant is given by

$$G_p(s) = \frac{1}{s+3}$$

Want to determine if it is possible to meet the following constraints

- $T_s \le 1\sec$
- Steady state error for a unit step input < 0.1

with P, I, PD, and PID (real zeros and complex conjugate zeros) controllers. For each of these controllers, you need to

- Sketch the root locus with arrows showing the direction of travel as k increases. If there are any poles going to zeros at infinity, you need to compute the centroid of the asymptotes (σ_c) and the angles of the asymptotes.

- State whether it is possible, *based only on the closed loop pole locations*, to meet the constraints. If it is necessary to put constraints on k, p, or z to meet the constraints, you must specify them. *(At this point you can only put conditions on k to meet the steady state error constraints.)*

- Determine if it is possible for the output to oscillate (nonzero ω_d)

You may (and should) check your answers with Matlab (use the **rlocus** command), but you need to do this by hand.