ECE-320: Linear Control Systems
Homework 4

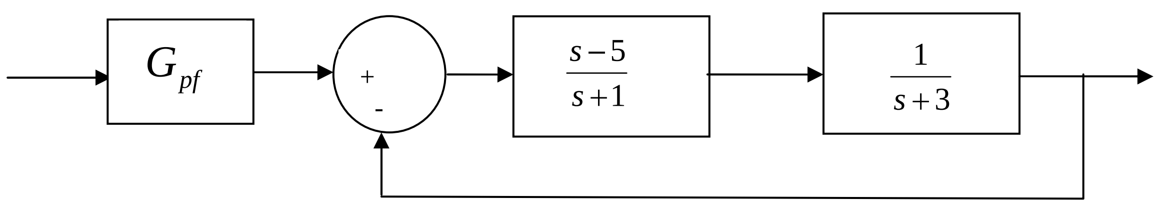
Due: Thursday January 7 at the beginning of class

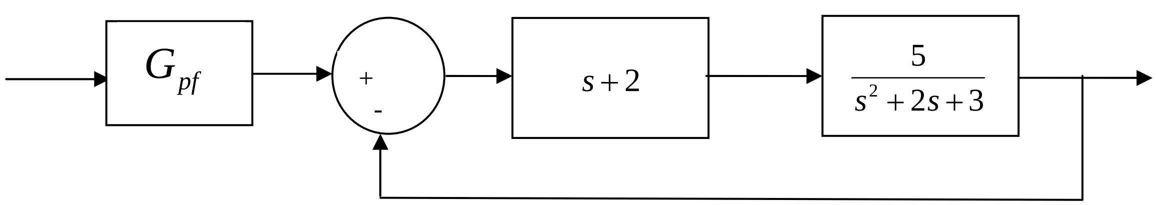
1) For the following systems

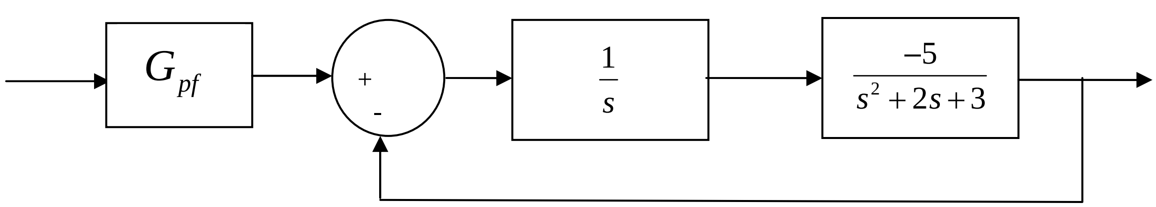
a) Determine the system type (0, 1, 2, ...)

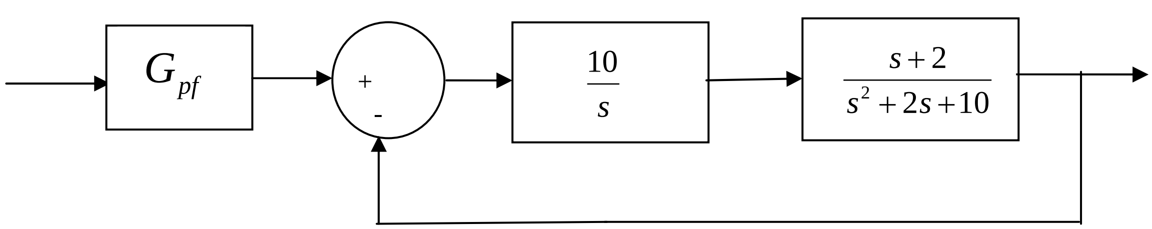
b) If the system is type 0 assume $G_{pf} = 1$ and determine the position error constant K_p and the steady state error for a unit step input. Then determine the value of G_{pf} to make this error zero. If the system is type 1, assume $G_{pf} = 1$ and determine the steady state error for a unit step, the velocity error constant K_v, and the steady state error for a unit ramp. Is there any constant value of G_{pf} that can change the velocity error?

Ans. (steady state errors) $-\frac{3}{2}$, $\frac{3}{13}$, $-\frac{3}{5}$, $\frac{1}{2}$; (prefilers) $\frac{2}{5}$, $\frac{13}{10}$, G_{pf} has no effect

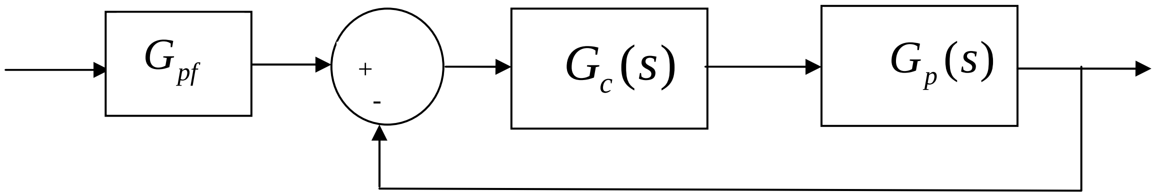








2) Consider the following control system.



We can compute the position error constant K_p as $K_p = G_c(0)G_p(0)$

a) Determine an expression for the closed loop transfer function (from input to output) $G_o(s)$ in terms of G_{pf} , $G_c(s)$, and $G_p(s)$.

b) For a steady state error of zero for a step input we want $G_o(0) = 1$. Use this information to show that we can determine the prefilter gain to be

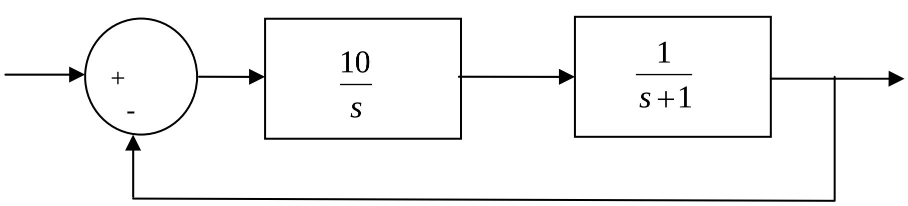
$$G_{pf} = 1 + \frac{1}{K_p}$$

c) We can find the steady state error for a unit step input as $e_{ss} = \frac{1}{1+K_p}$. Using this, show that we can determine the prefilter gain to be

$$G_{pf} = \frac{1}{1-e_{ss}}$$

Note that if $K_p = \infty$ *(or equivalently* $e_{ss} = 0$*), we get* $G_{pf} = 1$)

3) Consider the following control system:



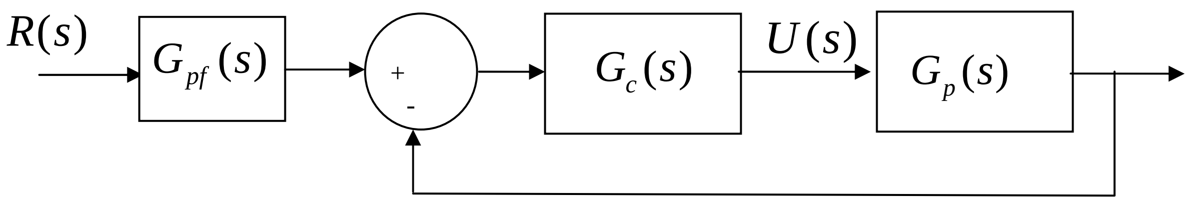
a) If the input to the system is $r(t) = 8u(t)$, what is the steady state output?

b) If the input to the system is $r(t) = 8\sin(3t)u(t)$, what is the output in steady state? What is the time lag between the input signal and the output signal?
Hint: you can write $\omega t - \theta = \omega(t - t_d)$ *if* θ *is measured in radians.*

Answers: $y(t) = 8, y(t) = 8\sqrt{10}\sin(3t - 71.57^\circ), t_d = 0.416\,\text{sec}$

4) One of the things that will be coming up in lab more and more is the limitation of the amplitude of the control signal, or the *control effort*. This is also a problem for most practical systems. In this problem we will do some simple analysis to better understand why Matlab's sisotool won't give us a good estimate of the control effort for some types of systems, and why dynamic prefilters can often really help us out here.

a) For the system below,



show that U(s) and R(s) are related by

$$U(s) = \frac{G_c(s)G_{pf}(s)}{1+G_p(s)G_c(s)}R(s)$$

b) For many types of controllers, the maximum value of the control signal is just after the step is applied, at $t = 0^+$. Although most of the time we are concerned with steady state values and use the final value Theorem in the *s*-plane, in this case we want to use the initial value Theorem, which can be written as

$$\lim_{t\to 0^+} u(t) = \lim_{s\to\infty} sU(s)$$

If the system input is a step of amplitude *A*, show that

$$u(0^+) = \lim_{s\to\infty} \frac{AG_c(s)G_{pf}(s)}{1+G_p(s)G_c(s)}$$

This result shows very clearly that the initial control signal is directly proportional to the amplitude of the input signal, which is pretty intuitive.

c) Now let's assume

$$G_c(s) = \frac{N_c(s)}{D_c(s)} \quad G_p(s) = \frac{N_p(s)}{D_p(s)} \quad G_{pf}(s) = \frac{N_{pf}(s)}{D_{pf}(s)}$$

If we want to look at the initial value for a unit step, we need to look at

$$u(0^+) = \lim_{s\to\infty} \frac{sG_c(s)G_{pf}(s)}{1+G_c(s)G_p(s)}\frac{1}{s} = \lim_{s\to\infty}\frac{G_c(s)G_{pf}(s)}{1+G_c(s)G_p(s)}$$

Let's also then define

$$\tilde{U}(s) = \frac{G_c(s)G_{pf}(s)}{1+G_c(s)G_p(s)}$$

so that

$$u(0^+) = \lim_{s\to\infty}\tilde{U}(s)$$

Show that

$$\tilde{U}(s) = \frac{N_{pf}(s)}{\left(\frac{D_c(s)}{N_c(s)}\right)D_{pf}(s) + \left(\frac{N_p(s)}{D_p(s)}\right)D_{pf}(s)}$$

and

$$\deg\tilde{U} = \deg N_{pf} - \max\left[\deg D_c - \deg N_c + \deg D_{pf}, \deg N_p - \deg D_p + \deg D_{pf}\right]$$

where $\deg Y$ is the degree of polynomial Y.

d) Since we are going to take the limit as $s \to \infty$, we need the degree of $\tilde{U}(s)$ to be less than or equal to zero for a step input to have a finite $u(0^+)$. Why?

For our 1 dof systems in lab, we have $\deg N_p = 0$ *and* $\deg D_p = 2$. *Use this for the remainder of this problem*

e) If the prefilter is a constant, show that in order to have a finite $u(0^+)$ we must have

$$\deg D_c \geq \deg N_c$$

f) If the numerator of the prefilter is a constant, then in order to have a finite $u(0^+)$ we must have

$$\deg D_c - \deg N_c + \deg D_{pf} \geq 0 \text{ or } -2 + \deg D_{pf} \geq 0$$

g) For P, I, D, PI, PD, PID, and lead controllers, determine if $u(0^+)$ is finite if the prefilter is a constant.

Note: Although it may appear that the control effort is sometimes infinite, in practice this is not true since our motor cannot produce an infinite signal. This large initial control signal is referred to as a *set- point kick*. There are different ways to implement a PID controller to avoid this, and we will cover two of them in Lab 7.