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ECE-320,

Quiz #4

For your ease, assume the form of convolution $y(n)=\sum_{k=-\infty}^{k=\infty} x(k)h(n-k)$ in all of the following problems.

1) The finite summation $S_N=\sum_{k=0}^{N} a^k$ is equal to

a) $\frac{1-a^N}{1-a}$ b) $\frac{1-a^{N-1}}{1-a}$ c) $\frac{1-a^{N+1}}{1-a}$ d) $\frac{1+a^{N+1}}{1-a}$ e) none of these

2) The finite summation $S=\sum_{k=-1}^{N+2} a^k$ is equal to

a) $a^{-1}\frac{1-a^{N+3}}{1-a}$ b) $a^1\left(\frac{1-a^{N+4}}{1-a}\right)$ c) $a^{-1}\left(\frac{1-a^{N+4}}{1-a}\right)$ d) $a^{-1}\left(\frac{1-a^{N-4}}{1-a}\right)$ e) none of these

3) For a discrete time system, $\delta(0)$ is equal to

a) 0 b) 1 c) ∞ d) it is not defined

4) If an LTI system with impulse response $h(n)=4^{n-1}u(n-1)$ has input $x(n)=\delta(n)$, the output of the system is

a) $y(n)=4^{n-1}u(n-1)\delta(n)$ b) $y(n)=4^{n-1}u(n)$ c) $y(n)=4^{n-1}u(n-1)$ d) none of these

5) If an LTI system with impulse response $h(n)=3^{n+1}u(n)$ has input $x(n)=3\delta(n-1)$, the output of the system is

a) $y(n)=3^{n+1}u(n-1)$ b) $y(n)=3^{n}u(n-1)$ c) $y(n)=3^{n}u(n)$ d) none of these

6) If an LTI system with impulse response $h(n)=2^{n-1}u(n-1)$ has input $x(n)=2\delta(n-1)$, the output of the system is

a) $y(n)=2^{n-2}u(n-2)$ b) $y(n)=2^{n}u(n-2)$ c) $y(n)=2^{n-1}u(n-2)$ d) none of these

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7) If an LTI system with impulse response $h(n) = 3\delta(n-1)$ has input $x(n) = 2\delta(n-1)$, the output of the system is

a) $y(n) = 3\times 2u(n-2)$ b) $y(n) = 3\times 2\delta(n-1)$ c) $y(n) = 3\times 2\delta(n-2)$ d) none of these

8) If an LTI system with impulse response $h(n) = 3^n u(n)$ has input $x(n) = u(n)$, the output of the system is

a) $y(n) = 3^n u(n)$ b) $y(n) = 3^{n+1}u(n)$ c) $y(n) = \dfrac{1-3^{n+1}}{1-3}u(n)$ d) $y(n) = \dfrac{1-3^{n-1}}{1-3}u(n)$ e) none of these

9) If an LTI system with impulse response $h(n) = 3^n u(n)$ has input $x(n) = 2^n u(n)$, the output of the system is

a) $y(n) = 3^n 2^n u(n)$ b) $y(n) = 3^n \dfrac{1-\left(\frac{2}{3}\right)^{n+1}}{1-\frac{2}{3}}u(n)$ c) $y(n) = 2^n \dfrac{1-\left(\frac{3}{2}\right)^{n+1}}{1-\frac{3}{2}}u(n)$

d) $y(n) = \left[\dfrac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right]\left[\dfrac{1-\left(\frac{1}{3}\right)^{n+1}}{1-\frac{1}{3}}\right]u(n)$ e) none of these

10) The sum $S = \sum_{k=0}^{\infty} a^k$ will converge provided

a) $|a| > 1$ b) $|a| < 1$

11) If the sum $S = \sum_{k=0}^{\infty} a^k$ converges, it is equal to

a) $\dfrac{1}{1+a}$ b) $\dfrac{1}{1-a}$ c) $\dfrac{a}{1-a}$ d) $\dfrac{a}{1+a}$ e) none of these

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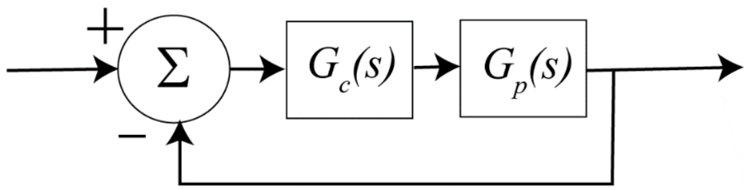
For problems 12-14 assume the closed loop system below and assume $G_p(s) = \dfrac{3}{(s+1)(s+2)}$



For the following problem you should sketch the root locus to answer the following questions. (You ***will not*** be graded on your root locus sketches, just your answers.)

12) Assuming a proportional controller $G_c(s) = k_p$, what is the settling time as $k_p \to \infty$?

13) Assuming a proportional + derivative controller $G_c(s) = k(s+z)$, what is the value of z so that the settling time $T_s = \dfrac{1}{2}$ as $k \to \infty$

14) Assuming an I controller $G_c(s) = \dfrac{k}{s}$, can the system become unstable for any value of k?

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Root Locus Construction

Once each pole has been paired with a zero, we are done

1. *Loci Branches*

$$\underset{k=0}{poles} \rightarrow \underset{k=\infty}{zeros}$$

Continuous curves, which comprise the locus, start at each of the *n* poles of $G(s)$ for which $k = 0$. As *k* approaches ∞, the branches of the locus approach the *m* zeros of $G(s)$. Locus branches for excess poles extend to infinity.

The root locus is s**<u>ymmetric</u>** about the real axis.

2. *Real Axis Segments*

The root locus includes all points along the real axis to the left of an odd number of poles plus zeros of $G(s)$.

3. *Asymptotic Angles*

As $k \rightarrow \infty$, the branches of the locus become asymptotic to straight lines with angles

$$\theta = \frac{180^o + i360^o}{n-m}, \; i = 0, \pm 1, \pm 2, ..$$

until all $(n-m)$ angles not differing by multiples of 360^o are obtained. *n* is the number of poles of $G(s)$ and *m* is the number of zeros of $G(s)$.

4. *Centroid of the Asymptotes*

The starting point on the real axis from which the asymptotic lines radiate is given by

$$\sigma_c = \frac{\sum_i p_i - \sum_j z_j}{n-m}$$

where p_i is the i^{th} pole of $G(s)$, z_j is the j^{th} zero of $G(s)$, *n* is the number of poles of $G(s)$ and *m* is the number of zeros of $G(s)$. This point is termed the *centroid of the asymptotes*.

5. *Leaving/Entering the Real Axis*

When two branches of the root locus leave or enter the real axis, they usually do so at angles of $\pm 90^o$.