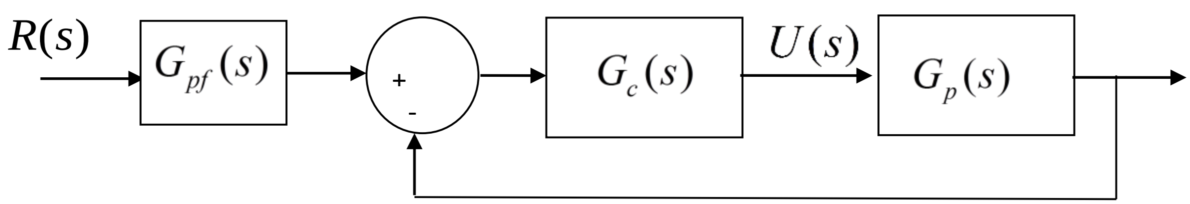
ECE-320: Linear Control Systems
Homework 9

Due: Friday May 15 at the beginning of class

1) One of the things that we need to worry about in practical systems is the limitation of the amplitude of the control signal, or the *control effort*. This is also a problem for most practical systems. In this problem we will do some simple analysis to better understand why Matlab's sisotool won't give us a good estimate of the control effort for some types of systems.

a) For the system below,



show that U(s) and R(s) are related by

$$U(s) = \frac{G_c(s)G_{pf}(s)}{1+G_p(s)G_c(s)}R(s)$$

b) For many types of controllers, the maximum value of the control signal is just after the step is applied, at $t = 0^+$. Although most of the time we are concerned with steady state values and use the final value Theorem in the *s*-plane, in this case we want to use the initial value Theorem, which can be written as

$$\lim_{t \to 0^+} u(t) = \lim_{s \to \infty} sU(s)$$

If the system input is a step of amplitude *A*, show that

$$u(0^+) = \lim_{s \to \infty} \frac{AG_c(s)G_{pf}(s)}{1+G_p(s)G_c(s)}$$

This result shows very clearly that the initial control signal is directly proportional to the amplitude of the input signal, which is pretty intuitive.

c) Now let's assume

$$G_c(s) = \frac{N_c(s)}{D_c(s)} \quad G_p(s) = \frac{N_p(s)}{D_p(s)} \quad G_{pf}(s) = \frac{N_{pf}(s)}{D_{pf}(s)}$$

If we want to look at the initial value for a unit step, we need to look at

$$u(0^+) = \lim_{s\to\infty} \frac{sG_c(s)G_{pf}(s)}{1+G_c(s)G_p(s)}\frac{1}{s} = \lim_{s\to\infty} \frac{G_c(s)G_{pf}(s)}{1+G_c(s)G_p(s)}$$

Let's also then define

$$\tilde{U}(s) = \frac{G_c(s)G_{pf}(s)}{1+G_c(s)G_p(s)}$$

so that

$$u(0^+) = \lim_{s\to\infty} \tilde{U}(s)$$

Show that

$$\tilde{U}(s) = \frac{N_{pf}(s)}{\left(\frac{D_c(s)}{N_c(s)}\right)D_{pf}(s) + \left(\frac{N_p(s)}{D_p(s)}\right)D_{pf}(s)}$$

and

$$\deg \tilde{U} = \deg N_{pf} - \max\left[\deg D_c - \deg N_c + \deg D_{pf}, \deg N_p - \deg D_p + \deg D_{pf}\right]$$

where $\deg Y$ is the degree of polynomial *Y*.

d) Since we are going to take the limit as $s \to \infty$, we need the degree of $\tilde{U}(s)$ to be less than or equal to zero for a step input to have a finite $u(0^+)$. Why?

For the remainder of this problem assume we have a 1 dof system such as an ideal second order system, so we have $\deg N_p = 0$ *and* $\deg D_p = 2$.

e) If the prefilter is a constant, show that in order to have a finite $u(0^+)$ we must have

$$\deg D_c \geq \deg N_c$$

f) If the numerator of the prefilter is a constant, then in order to have a finite $u(0^+)$ we must have

$$\deg D_c - \deg N_c + \deg D_{pf} \geq 0 \text{ or } -2 + \deg D_{pf} \geq 0$$

g) For P, I, D, PI, PD, PID, and lead controllers, determine if $u(0^+)$ is finite if the prefilter is a constant.

Note: Although it may appear that the control effort is sometimes infinite, in practice this is not true since our motor cannot produce an infinite signal. This large initial control signal is referred to as a *set- point kick*. There are different ways to implement a PID controller to avoid this.