ECE-320: Linear Control Systems
Homework 3

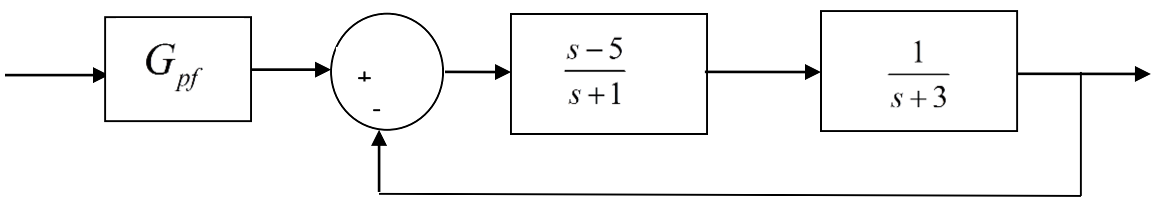
Due: Friday March 27 at the beginning of class.

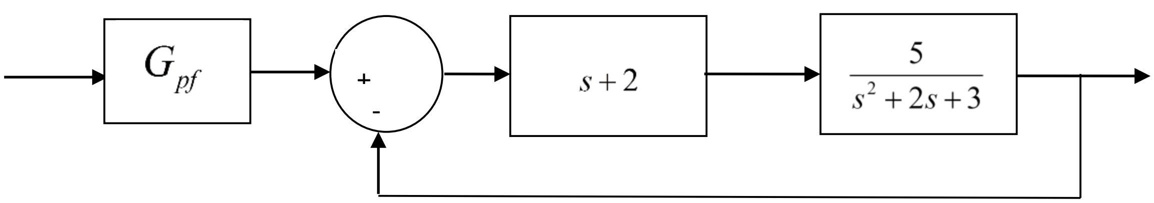
1) For the following systems

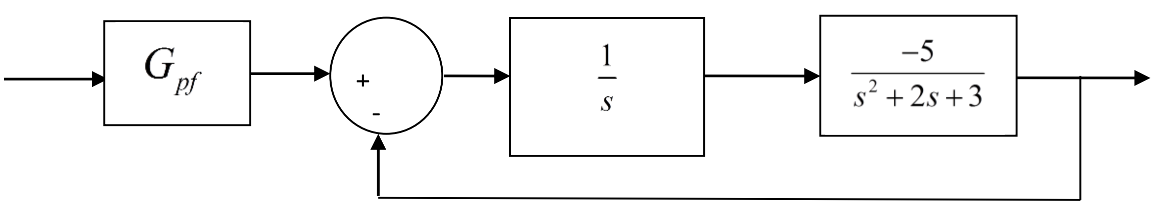
a) Determine the system type (0, 1, 2, ...)

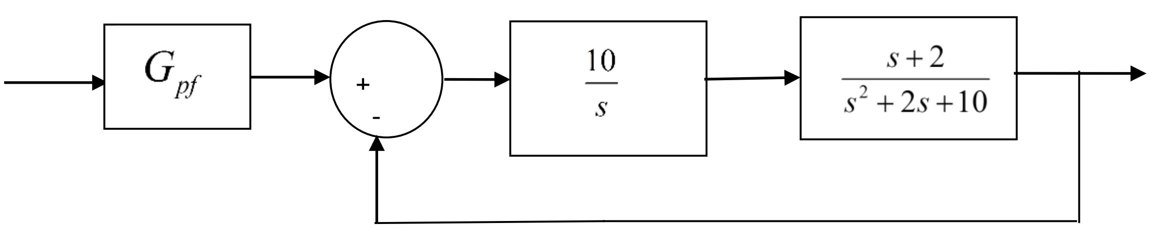
b) If the system is type 0 assume $G_{pf} = 1$ and determine the position error constant K_p and the steady state error for a unit step input. Then determine the value of G_{pf} to make this error zero. If the system is type 1, assume $G_{pf} = 1$ and determine the steady state error for a unit step, the velocity error constant K_v, and the steady state error for a unit ramp.

Ans. (steady state errors) $-\frac{3}{2}, \frac{3}{13}, -\frac{3}{5}, \frac{1}{2}$; (prefilers) $\frac{2}{5}, \frac{13}{10}$

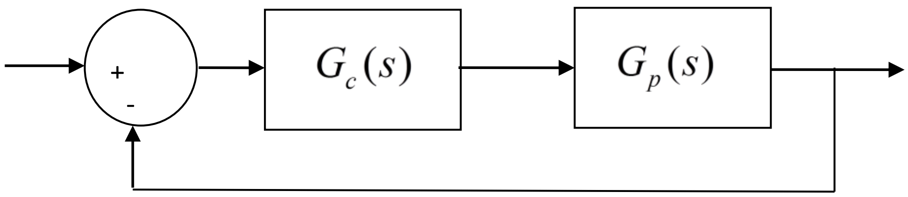








2) For the following problem, assume we are using the following control system



where the plant is given by

$$G_p(s) = \frac{1}{s^2 + 4s + 29} = \frac{1}{(s+2-5j)(s+2+5j)}$$

For the following controllers, sketch the root locus with arrows showing the direction of travel as k increases. If there are any poles going to zeros at infinity, you need to compute the centroid of the asymptotes (σ_c) and the angles of the asymptotes.

You may (and should) check your answers with Matlab (use the **rlocus** command), but you need to do this by hand.

a) $G_c(s) = k$ (proportional (P) controller)

b) $G_c(s) = \frac{k}{s}$ (an integral (I) controller)

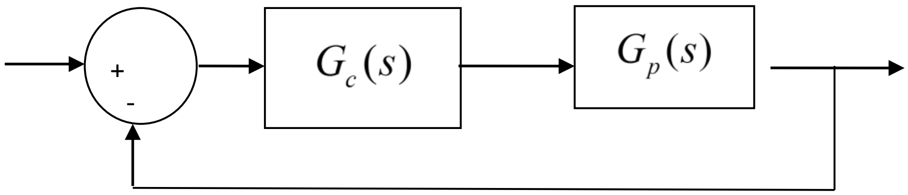
c) $G_c(s) = \frac{k(s+z)}{s}$ (a proportional + integral (PI) controller) Write the centroid σ_c as a function of z. For what values of z will the two asymptotes be in the right half plane? *(For plotting purposes, assume z is equal to 2.)*

d) $G_c(s) = k(s+z)$ (a proportional+derivative (PD) controller) *(For plotting purposes, assume z is equal to 2.)*

e) $G_c(s) = \frac{k(s+z_1)(s+z_2)}{s}$ (a proportional+integral+derivative (PID) controller) Sketch this for the case where both zeros are real and then when both zeros are complex conjugates.

f) $G_c(s) = \frac{k(s+z)}{(s+p)}$ (a lead controller, $p > z$) Write an expression for σ_c as a function of the distance between the pole and the zero, $l = p - z$. What happens to the asymptotes as l gets larger? *(For plotting purposes, assume p is 5 and z is 1.)*

3) For the following problem, assume we are using the following control system



where the plant is given by

$$G_p(s) = \frac{1}{s+3}$$

For the following controllers, sketch the root locus with arrows showing the direction of travel as k increases. If there are any poles going to zeros at infinity, you need to compute the centroid of the asymptotes (σ_c) and the angles of the asymptotes.

You may (and should) check your answers with Matlab (use the **rlocus** command), but you need to do this by hand.

a) $G_c(s) = k$ (proportional (P) controller)

b) $G_c(s) = \frac{k}{s}$ (an integral (I) controller)

c) $G_c(s) = \frac{k(s+z)}{s}$ (a proportional + integral (PI) controller) *Sketch this for the case when z is equal to 2 and then assume z is equal to 4; there will be two plots.*

d) $G_c(s) = k(s+z)$ (a proportional+derivative (PD) controller) *Sketch this for the case where z is equal to 2 and then assume z = 4; there will be two plots.*

e) $G_c(s) = \frac{k(s+z_1)(s+z_2)}{s}$ (a proportional+integral+derivative (PID) controller) *Sketch this for the case where there are zeros at* $-4 \pm 4j$ *and when they are at -6 and -8; there will be two plots.*

4) (*sisotool problem*) For the plant modeled by the transfer function

$$G_1(s) = \frac{6000}{s^2 + 4s + 400}$$

You are to design a PI controller, a PID controller with **complex conjugate zeros**, and a PID controller with **real zeros** that meet the following specifications

$$\begin{aligned} &PO \le 10\% \\ &T_s \le 2.5\,\text{sec} \\ &k_p \le 0.5 \\ &k_i \le 5 \\ &k_d \le 0.01 \end{aligned}$$

In *sisotool*, in the LTI viewer, if you right click on the graph and select **Characteristics** you can let *sisotool* find the settling time. You should copy your step response and root locus plots to a word document, as well as including your values of the controller coefficients.

5) (*sisotool problem*) For the plant modeled by the transfer function

$$G_2(s) = \frac{6250}{s^2 + 0.5s + 625}$$

You are to design a PI controller, a PID controller with **complex conjugate zeros**, and a PID controller with **real zeros** that meet the following specifications

$$\begin{aligned} &PO \le 10\% \\ &PI\ T_s \le 15.0\ sec, PID\ T_s \le 0.5\ sec \\ &k_p \le 0.5 \\ &k_i \le 5 \\ &k_d \le 0.01 \end{aligned}$$

In *sisotool*, in the LTI viewer, if you right click on the graph and select **Characteristics** you can let *sisotool* find the settling time. You should copy your step response and root locus plots to a word document, as well as including your values of the controller coefficients.