ECE-320: Linear Control Systems
Homework 1

Due: Friday March 13, 2015 *at the beginning of class*

1) For the following transfer functions, determine both the **impulse response** and the **unit step response**.

$$H(s) = \frac{s}{(s+1)(s+2)^2} \quad H(s) = \frac{1}{(2s+1)(3s+2)}$$

$$H(s) = \frac{2}{s^2+8s+25} \quad H(s) = \frac{s+2}{s^2+2s+4}$$

Scrambled Answers:

$$h(t) = \frac{2}{3}e^{-4t}\sin(3t)u(t),\ h(t) = -e^{-t}u(t) + e^{-2t}u(t) + 2te^{-2t}u(t),\ h(t) = e^{-t/2}u(t) - e^{-2t/3}u(t),$$

$$h(t) = e^{-t}cos(\sqrt{3}t)u(t) + \frac{1}{\sqrt{3}}e^{-t}\sin(\sqrt{3}t)u(t),\ y(t) = \frac{1}{2}u(t) - 2e^{-t/2}u(t) + \frac{3}{2}e^{-2t/3}u(t),$$

$$y(t) = \frac{1}{2}u(t) + \frac{1}{2\sqrt{3}}e^{-t}\sin(\sqrt{3}t)u(t) - \frac{1}{2}e^{-t}\cos(\sqrt{3}t)u(t),\ y(t) = e^{-t}u(t) - e^{-2t}u(t) - te^{-2t}u(t),$$

$$y(t) = \frac{2}{25}u(t) - \frac{8}{75}e^{-4t}\sin(3t)u(t) - \frac{2}{25}e^{-4t}cos(3t)u(t)$$

2) For the following transfer functions

$$H(s) = \frac{2}{s^2+2s+2} \quad H(s) = \frac{3}{s^2+4s+6} \quad H(s) = \frac{5}{s^2+6s+10}$$

$$H(s) = \frac{4}{s^2-4s+7} \quad H(s) = \frac{1}{s^2+4}$$

By computing the inverse Laplace transform show that the step responses are given by

$$y(t) = \left[1 - e^{-t}\cos(t) - e^{-t}\sin(t)\right]u(t) \quad y(t) = \left[\frac{1}{2} - \frac{1}{\sqrt{2}}e^{-2t}\sin(\sqrt{2}t) - \frac{1}{2}e^{-2t}\cos(\sqrt{2}t)\right]u(t)$$

$$y(t) = \left[\frac{1}{2} - \frac{3}{2}e^{-3t}\sin(t) - \frac{1}{2}e^{-3t}\cos(t)\right]u(t) \quad y(t) = \left[\frac{4}{7} + \frac{8\sqrt{3}}{21}e^{2t}\sin(\sqrt{3}t) - \frac{4}{7}e^{2t}\cos(\sqrt{3}t)\right]u(t)$$

$$y(t) = \left[\frac{1}{4} - \frac{1}{4}\cos(2t)\right]u(t)$$

3) (Mason's Rule) For the block diagram shown below, determine a corresponding signal flow diagram and show that the closed loop transfer function is

$$H_{system} = \frac{G_1 G_2 G_3 + G_4 (1 - G_1 G_2 H_1 + G_2 H_1 + G_2 G_3 H_2)}{1 - G_1 G_2 H_1 + G_2 H_1 + G_2 G_3 H_2}$$