

Name Solutions Mailbox _____

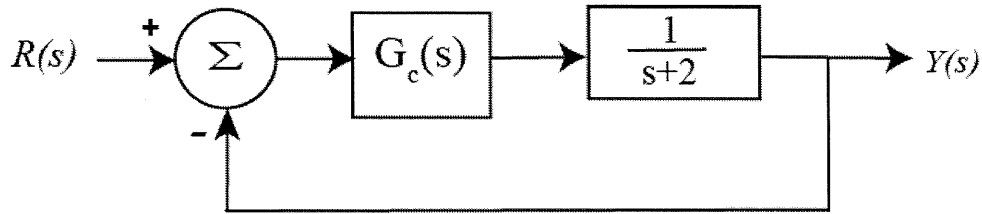
ECE-320 Linear Control Systems
Spring 2015, Exam 2

No calculators or computers allowed.

Problem 1 _____/15
Problem 2 _____/20
Problem 3 _____/15
Problems 4-13 _____/30
Problem 14 _____/20

Total _____/100

1) (15 points) Consider the following simple control system.



- Assume a proportional controller, $G_c(s) = k_p$, and determine an expression for the system sensitivity to the parameter k_p . You may leave your answer in terms of s , but it must be simplified as much as possible (i.e., it should be a ratio of polynomials and all numbers except for the letter s).
- Assume an integral controller, $G_c(s) = \frac{k_i}{s}$, and determine an expression for the system sensitivity to the parameter k_i . You may leave your answer in terms of s , but it must be simplified as much as possible (i.e., it should be a ratio of polynomials and all numbers except for the letter s).
- Assume the nominal values for k_p and k_i are 4 and 2, respectively. Determine simplified expressions for the magnitude of the sensitivities as a function of frequency. Your answers need to be written as the ratio of polynomials and cannot include any j 's

$$a) G_{0(s)} = \frac{k_p}{s+2+k_p} \quad S_{k_p}^{G_0} = 1 - \frac{k_p}{s+2+k_p} = \frac{s+2}{s+2+k_p}$$

$$b) G_{0(s)} = \frac{k_i}{s^2+2s+k_i} \quad S_{k_i}^{G_0} = 1 - \frac{k_i}{s^2+2s+k_i} = \frac{s^2+2s}{s^2+2s+k_i}$$

$$c) \left| \frac{j\omega+2}{j\omega+6} \right| = \sqrt{\frac{\omega^2+4}{\omega^2+36}}$$

$$\left| \frac{-\omega^2+2j\omega}{-\omega^2+2j\omega+2} \right| = \sqrt{\frac{\omega^4+4\omega^2}{(2-\omega^2)^2+4\omega^2}}$$

2) (20 points) For the state variable model

$$\dot{q} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} q + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = [0 \quad 1]q + [0]u$$

Determine the closed loop transfer function with state variable feedback, $u(t) = G_{pf}r(t) - Kq(t)$. Note that you need to write out the determinant, but you don't need to simplify it.

$$\tilde{A} = A - BK = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} k_1 & k_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1-k_1 & -k_2 \\ 2 & 1 \end{bmatrix}$$

$$sI - \tilde{A} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1-k_1 & -k_2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} s+k_1-1 & k_2 \\ -2 & s-1 \end{bmatrix}$$

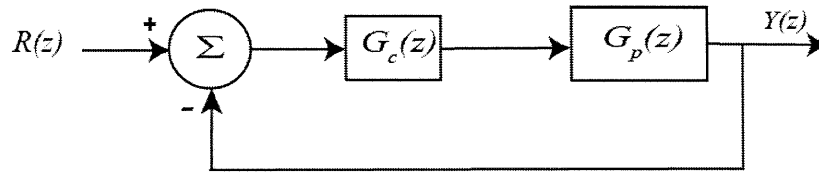
$$(sI - \tilde{A})^{-1} = \frac{1}{(s+k_1-1)(s-1) + 2k_2} \begin{bmatrix} s-1 & -k_2 \\ 2 & s+k_1-1 \end{bmatrix}$$

$$G_o(s) = C(sI - \tilde{A})^{-1} \tilde{B} = \frac{[0 \quad 1]}{\Delta(s)} \begin{bmatrix} s-1 & -k_2 \\ 2 & s+k_1-1 \end{bmatrix} \begin{bmatrix} G_{pf} \\ 0 \end{bmatrix}$$

$$= \frac{2G_{pf}}{(s+k_1-1)(s-1) + 2k_2} = G_o(s)$$

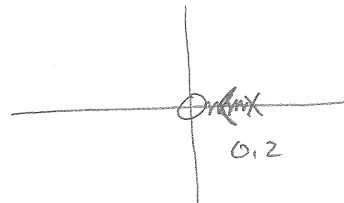
3) (15 points) For the following problem, assume the closed loop system below and assume

$$G_p(s) = \frac{z}{(z-0.2)}$$

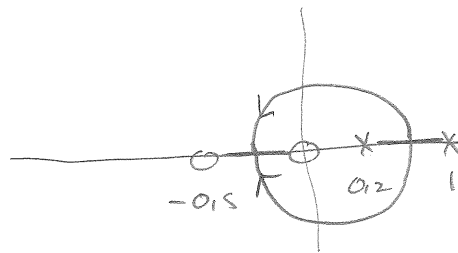


Hint: Drawing a root locus plot for a discrete-time system is identical to drawing the root locus plot for a continuous time system (treat z as you would s). The only difference is the interpretation.

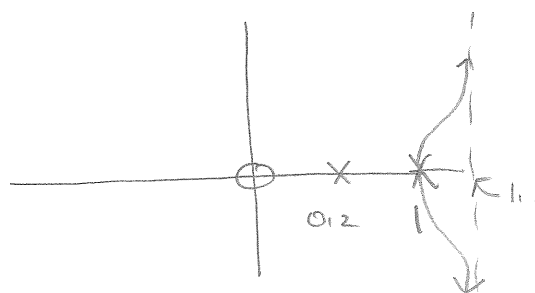
a) Assume we are using the proportional controller $G_c(z) = k$. Sketch the root locus, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary.



b) Assume we are using the proportional + integral controller $G_c(z) = \frac{k(z+0.5)}{(z-1)}$. Sketch the root locus, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary.



c) Assume we are using the double integral controller $G_c(z) = \frac{k}{(z-1)^2}$. Sketch the root locus, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary.



$$\sigma = \frac{1+1+0.2}{3-1} = \pm 90^\circ$$

$$\theta = \frac{[1+1+0.2]}{2} = 1.1$$

4) Which of the following transfer functions represents an (asymptotically) unstable systems? (circle all of them)

a) $G(z) = \frac{z-0.1}{z+0.1}$ b) $G(z) = \frac{z+0.1}{z-0.1}$ c) $G(z) = \frac{z}{(z-1.2+j3)(z-1.2-j3)}$ d) $G(z) = \frac{1}{z^2}$

5) Which of the following systems will have a smaller settling time?

a) $G(z) = \frac{z}{z-0.9}$ b) $G(z) = \frac{z}{z-0.7}$ c) $G(z) = \frac{z}{(z+0.1+j0.1)(z+0.1-j0.1)}$ d) $G(z) = \frac{z}{z+0.3}$

6) Is the following system **controllable**? $G(s) = \frac{8G_{pf}}{s^2 + 12k_1s + (k_1 + k_2 + 2)}$

- a) Yes b) No c) impossible to determine

7) Is the following system **controllable**? $G(s) = \frac{G_{pf}}{s^2 + (k_2 + k_1)s + (k_1 + k_2)}$

- a) Yes b) No c) impossible to determine

8) Consider a plant that is unstable but is a controllable system. Is it possible to use state variable feedback to make this system stable?

- a) Yes b) No

9) Is it possible for a system with state variable feedback to change the zeros of the plant (other than by pole-zero cancellation) ?

- a) Yes b) No

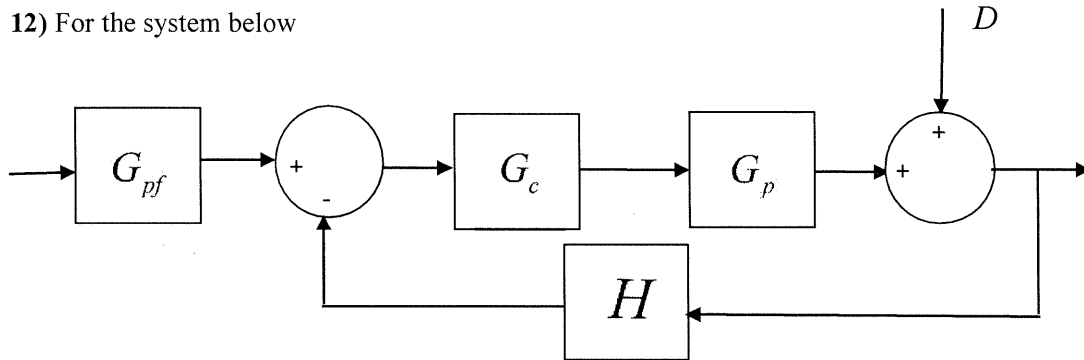
10) Is it possible for a system with state variable feedback to introduce zeros into the closed loop system?

- a) Yes b) No

11) If a plant has n poles, then a system with state variable feedback with no pole-zero cancellations will have

- a) more than n poles b) less than n poles **c) n poles** d) it is not possible to tell

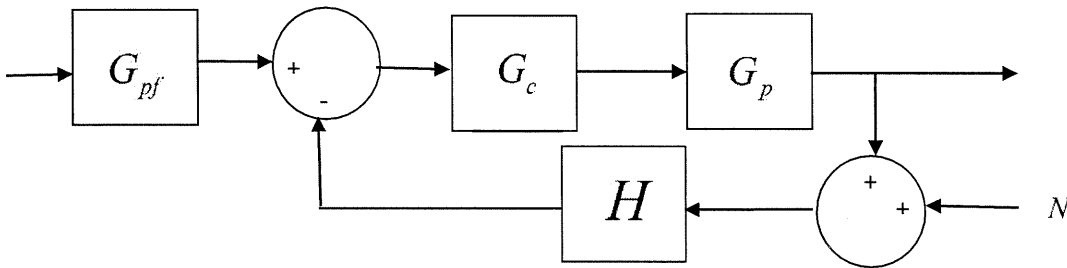
12) For the system below



to reduce the effects of the external disturbance D on the system output, we should

- a) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ large** b) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ small
 c) make G_{pf} large d) do nothing, we cannot change the sensitivity

13) For the system below

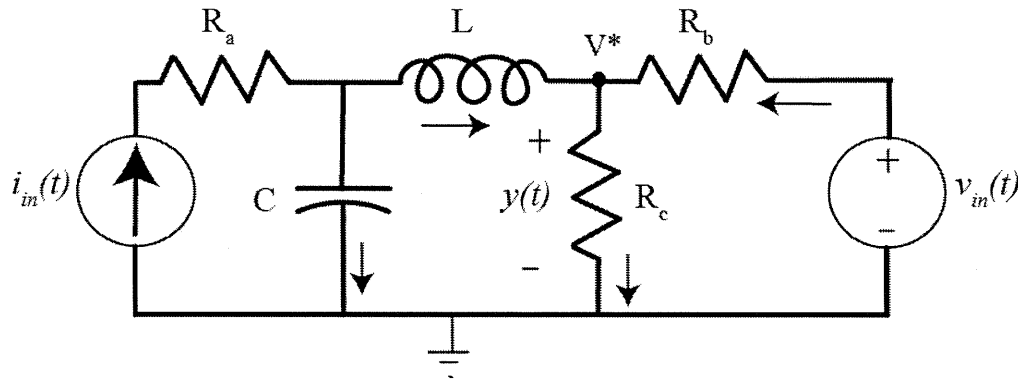


to reduce the effects of sensor noise N on the closed loop system, we should

- a) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ large **b) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ small**
 c) make $|H(j\omega)|$ large d) do nothing, we cannot change the sensitivity

14) (20 points) For the following circuit, the state variables are the current through the inductor and the voltage across the capacitor. Determine a state variable model for this system. Specifically, you need to identify the A, B, C, and D matrices/vectors/scalars. You surely recall the useful relationships

$v(t) = L \frac{di(t)}{dt}$, $i(t) = C \frac{dv(t)}{dt}$. For my sanity, and to get full credit, you should assume the currents flow in the direction shown by the arrows. Also, you may find it easier to use the node voltage v^* and then figure out what it is in terms of state variable. Also note that there are two inputs in this system.



$$i_{in}(t) = C \frac{dv_c(t)}{dt} + i_L(t)$$

$$v_c(t) - L \frac{di_L(t)}{dt} = v^*(t)$$

$$\frac{dv_c}{dt} = -\frac{1}{C} i_L(t) + \frac{1}{C} i_{in}(t)$$

$$i_L(t) + \frac{v_{in}(t) - v^*(t)}{R_b} = \frac{v^*(t)}{R_c}$$

$$i_L(t) + \frac{v_{in}(t)}{R_b} = v^*(t) \left[\frac{1}{R_b} + \frac{1}{R_c} \right] = v^*(t) \left[\frac{R_c + R_b}{R_b R_c} \right]$$

$$v^*(t) = \frac{R_b R_c}{R_b + R_c} i_L(t) + \frac{R_c}{R_b + R_c} v_{in}(t)$$

$$v_c(t) - L \frac{di_L(t)}{dt} = \frac{R_b R_c}{R_b + R_c} i_L(t) + \frac{R_c}{R_b + R_c} v_{in}(t)$$

$$\frac{di_L(t)}{dt} = \frac{-R_b R_c}{L(R_b + R_c)} i_L(t) - \frac{R_c}{L(R_b + R_c)} v_{in}(t) + \frac{1}{L} v_c(t)$$

$$\frac{d}{dt} \begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{R_b R_c}{L(R_b + R_c)} \end{bmatrix}}_A \begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{R_c}{L(R_b + R_c)} & 0 \end{bmatrix}}_B \begin{bmatrix} v_{in}(t) \\ i_{in}(t) \end{bmatrix}$$

Name _____

Mailbox _____

$$i_L(t) + \frac{v_{in}(t) - y(t)}{R_b} = \frac{y(t)}{R_c}$$

$$i_L(t) + \frac{v_{in}(t)}{R_b} = y(t) \left[\frac{1}{R_c} + \frac{1}{R_b} \right] = y(t) \left[\frac{R_b + R_c}{R_b R_c} \right]$$

$$y(t) = \frac{R_b R_c}{R_b + R_c} i_L(t) + \frac{R_c}{R_b + R_c} v_{in}(t)$$

$$y(t) = \underbrace{\begin{bmatrix} 0 & \frac{R_b R_c}{R_b + R_c} \end{bmatrix}}_C \begin{bmatrix} v_{in}(t) \\ i_L(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{R_c}{R_b + R_c} & 0 \end{bmatrix}}_D \begin{bmatrix} v_{in}(t) \\ i_{in}(t) \end{bmatrix}$$