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ECE-320 Linear Control Systems

Spring 2015, Exam 1

No calculators or computers allowed, you may leave your answers as fractions.

All problems are worth 2 points unless noted otherwise.

Total ______/100

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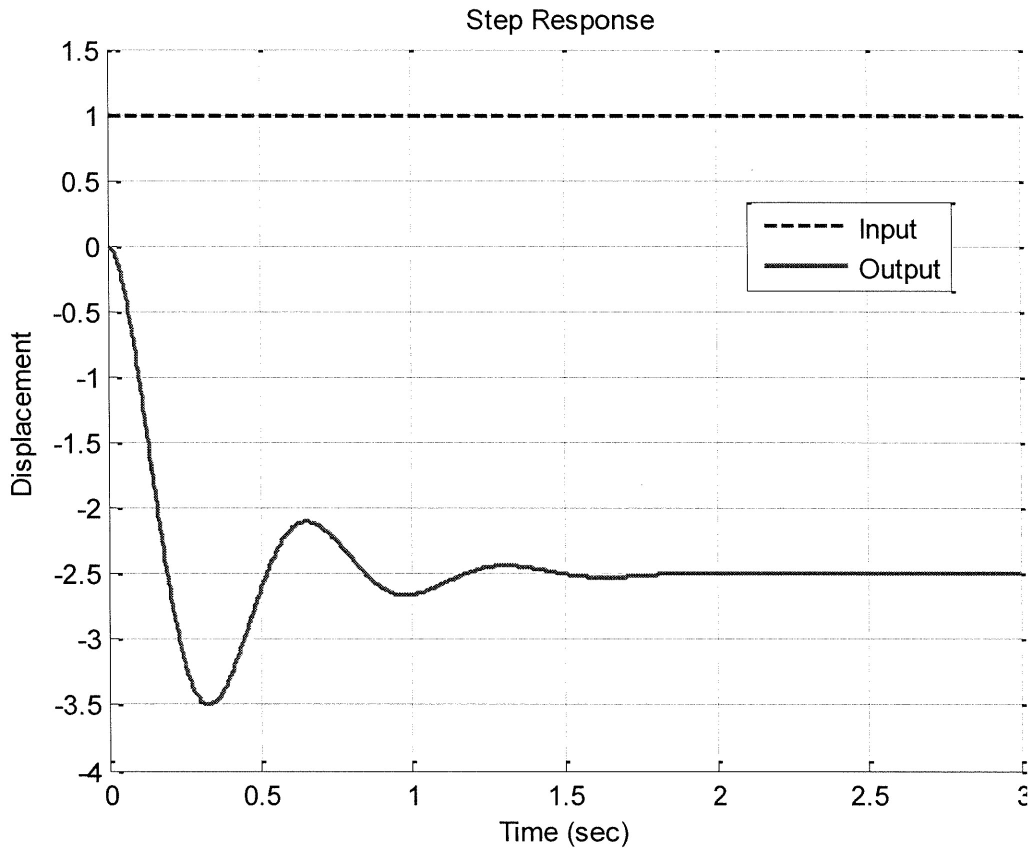
Problems 1-3 refer to the **unit step response** of a system, shown below



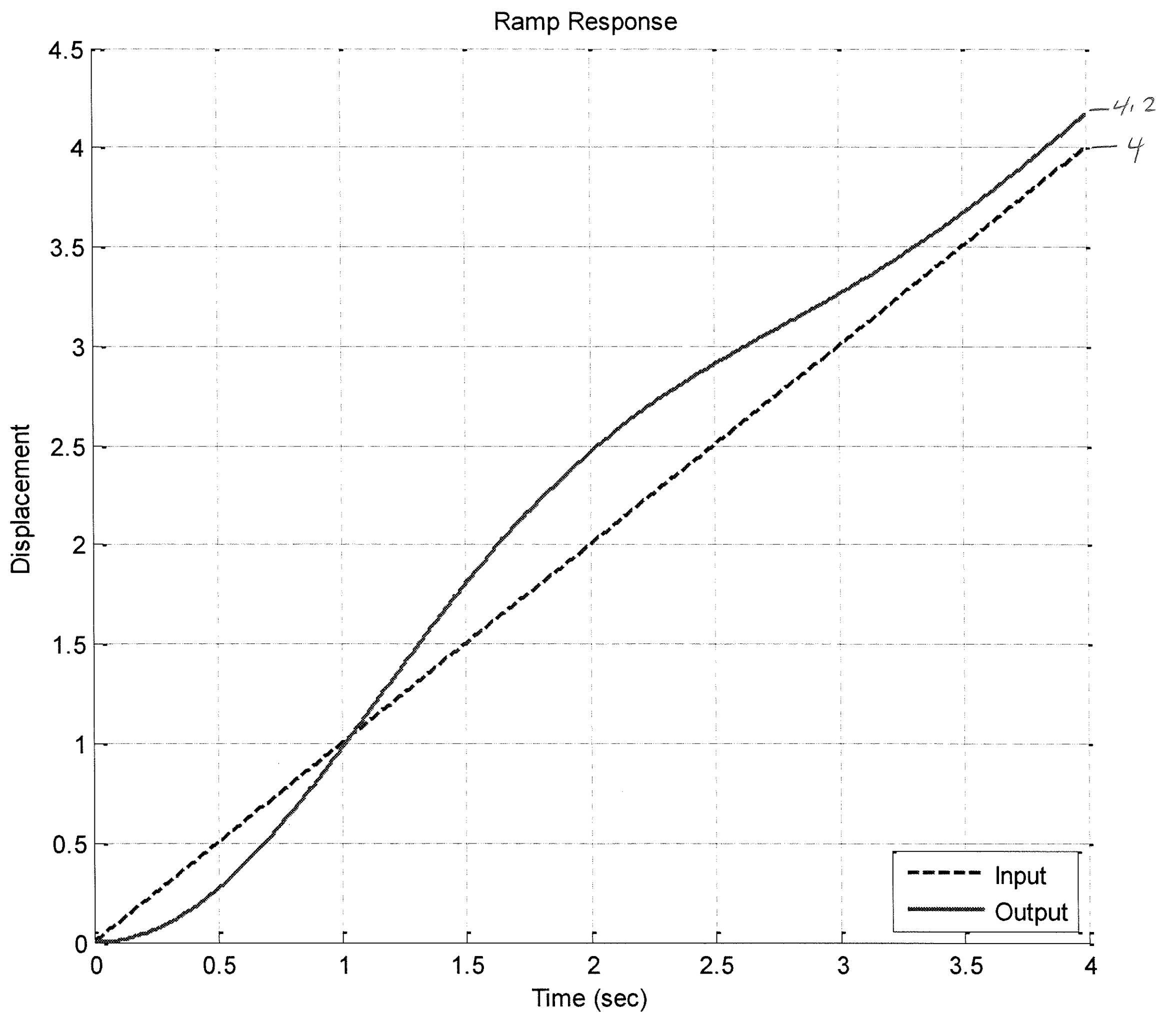
1) Estimate the **steady state error** $1-(-2.5) = \boxed{3.5}$

2) Estimate the **percent overshoot** $\frac{-3.5-(-2.5)}{-2.5} \times 100\% = \frac{-1}{-2.5} \times 100\% = \boxed{40\%}$

3) Estimate the **static gain** $K(1) = -2.5$ $\boxed{K = -2.5}$

Name ______________________________ Mailbox __________

4) Estimate the steady state error for the ramp response of the system shown below:



$e_{ss} = 4 - 4.2 \approx \boxed{-0.2}$

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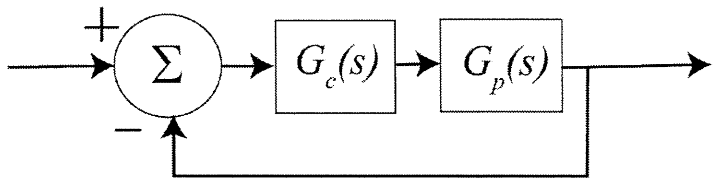
5) (10 points) For this problem assume the following unity feedback system



with $G_p(s) = \dfrac{2}{s(s+1)(s+2)}$ and $G_c(s) = 2$

a) Determine the position error constant K_p

$\boxed{K_p = \infty}$

b) Estimate the steady state error for a unit step using the position error constant.

$\boxed{e_{ss} = 0}$

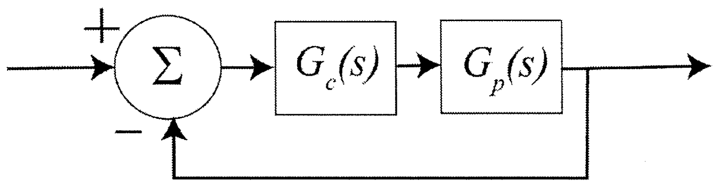
c) Determine the velocity error constant K_v $\boxed{K_v = 2}$

d) Estimate the steady state error for a unit ramp using the velocity error constant.

$\boxed{e_{ss} = \frac{1}{2}}$

Name ____ Mailbox ____________

6) (10 points) For this problem assume the following unity feedback system



with $G_p(s) = \dfrac{3}{(s+2)(s+4)}$ and $G_c(s) = \dfrac{5(s+1)}{s}$

a) Determine the position error constant K_p

$\boxed{K_p = \infty}$

b) Estimate the steady state error for a unit step using the position error constant.

$\boxed{e_{ss} = 0}$

c) Determine the velocity error constant K_v

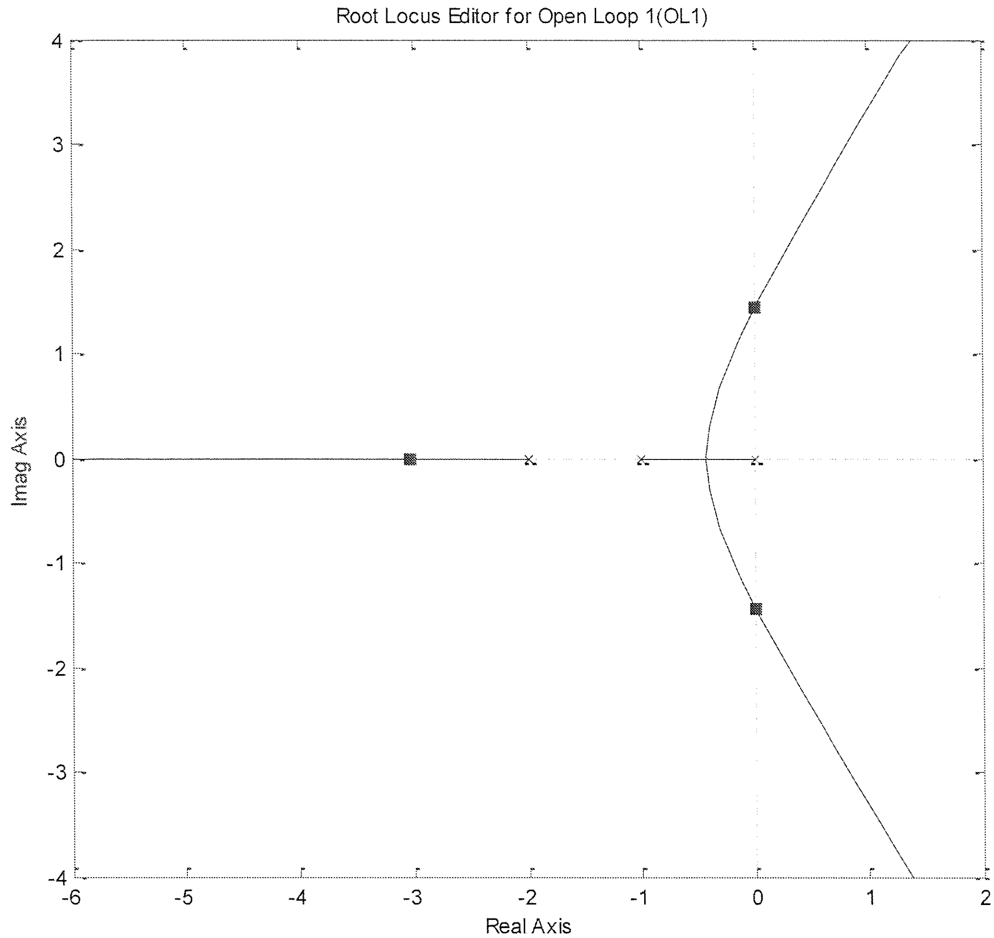
$\boxed{K_v = \frac{15}{8}}$

d) Estimate the steady state error for a unit ramp using the velocity error constant.

$\boxed{e_{ss} = \frac{8}{15}}$

Name ______________________________________ Mailbox ____________

Problems 7-9 refer to the following root locus plot (from sisotool)



7) Is it possible for -3 to be a closed loop pole for this system ? (Yes or No)

8) When the gain is approximately 3.2 the closed loop poles are as shown in the figure. If we want the system to be stable what conditions do we need to place on the gain k?

$\boxed{0 < K < 3.2}$

9) Is this a type one system? (Yes or No)

1 pole at origin

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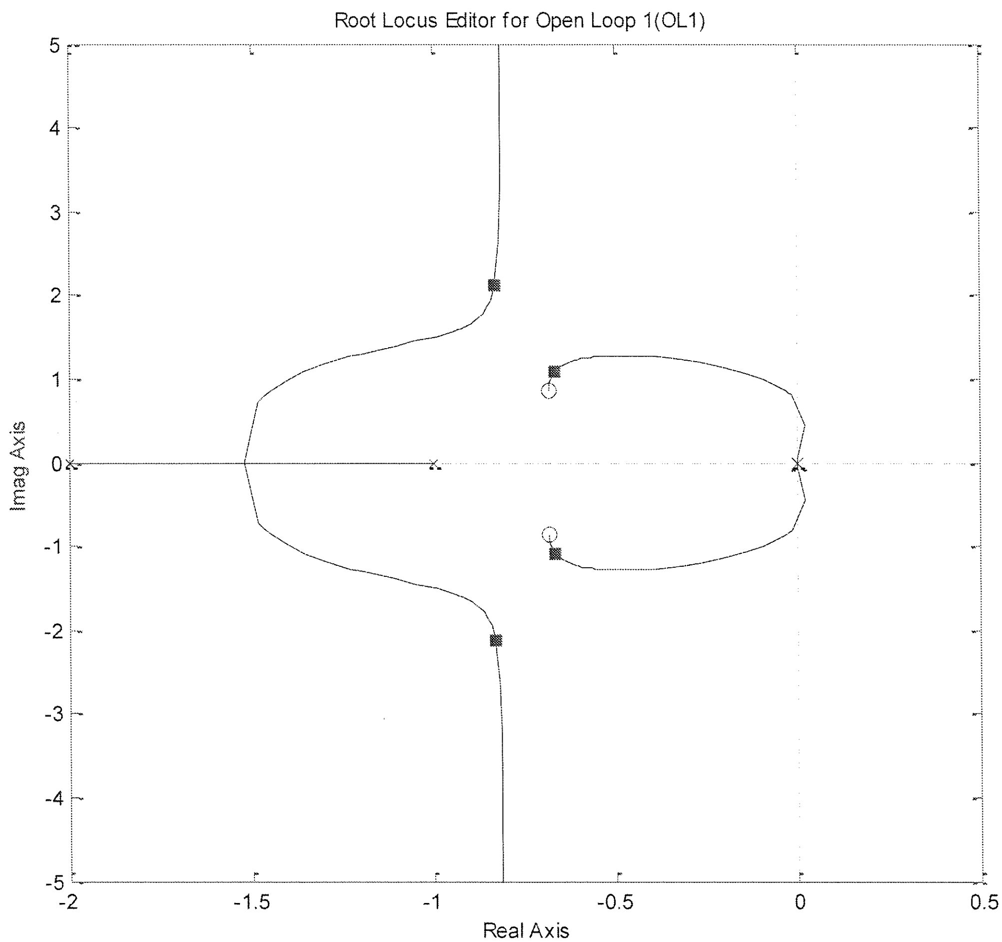
Problems 10 -12 refer to the following root locus plot (from sisotool)



10) When $k = 3.5$ the poles are as they are shown in the figure. Estimate the closed loop poles.

$\boxed{-0.6 \pm j1, \ -0.8 \pm j2}$

11) Estimate the settling time as the gain $k \to \infty$

$\frac{4}{0.8}, \frac{4}{0.6}$ $\boxed{T_s \approx \frac{4}{0.6}}$

12) Is this a type one system? (Yes or No)

2 poles at origin

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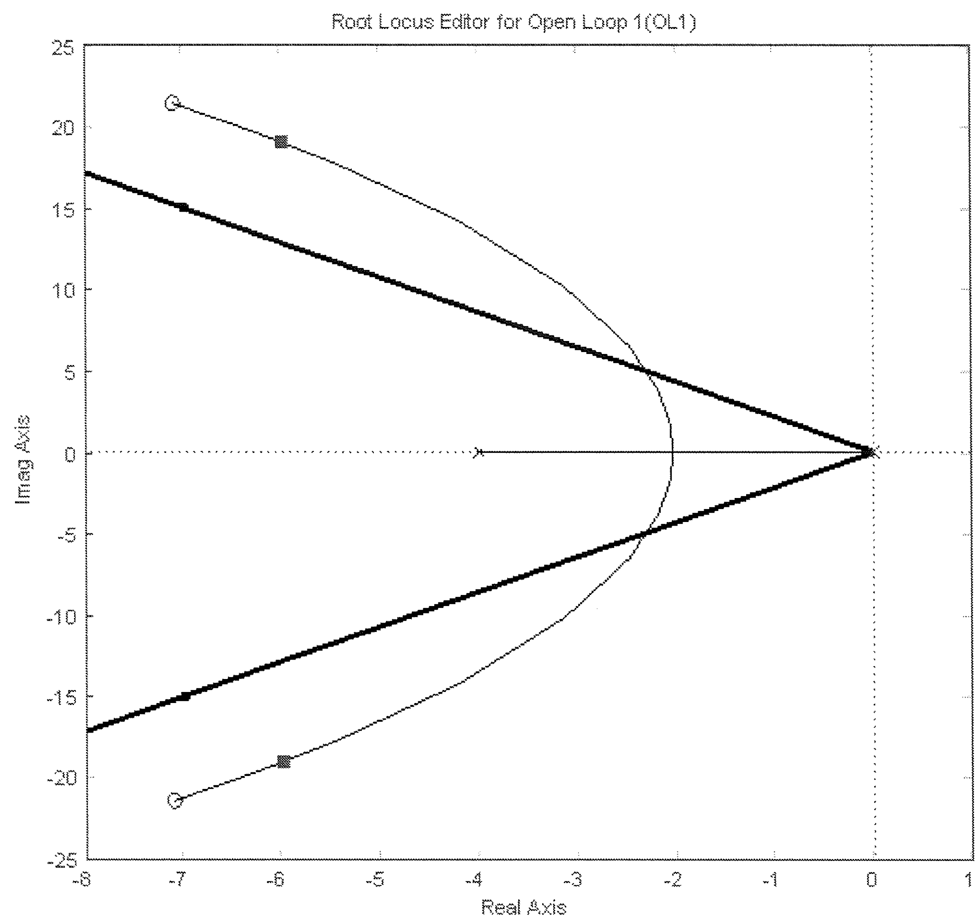
13) (6 points) For the following two controllers, determine k_p, k_i, and k_d

$$G_c(s) = \frac{2(s+4)}{s}$$ $K_p = 2 \quad K_i = 8 \quad K_d = 0$

$$G_c(s) = 4(s+2)$$ $K_p = 8 \quad K_i = 0 \quad K_d = 4$

14) (5 points) Consider a system with plant $G_p(s) = \frac{1}{s+4}$ and controller $G_c(s) = \frac{3.6(s^2 + 14.2s + 507)}{s}$.
This information was entered into *sisotool*, as well as the constraint that the percent overshoot should be less than 20%. The corresponding root locus plot is shown below. With this information, are you guaranteed that the unit step response of the system with this root locus plot will have percent overshoot greater than or equal to 20%? Explain why you answered the way you did.



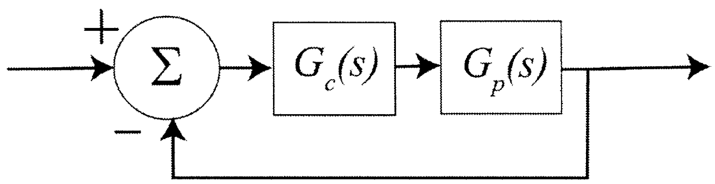
No, This is not an ideal 2nd order system!

$$G_p G_c = \frac{3.6(s^2+14.2s+507)}{s(s+4)} \neq \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \quad 0<\zeta<1$$

Name ______________________________ Mailbox __________

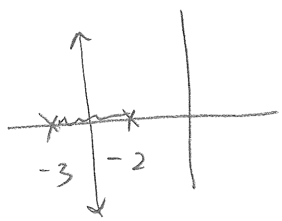
15) (10 points) For this problem assume the closed loop system below and assume

$$G_p(s) = \frac{3}{(s+2)(s+3)}$$



For each of the following problems you should sketch the root locus to answer the following questions. (You ***will not*** be graded on your root locus sketches, just your answers.)

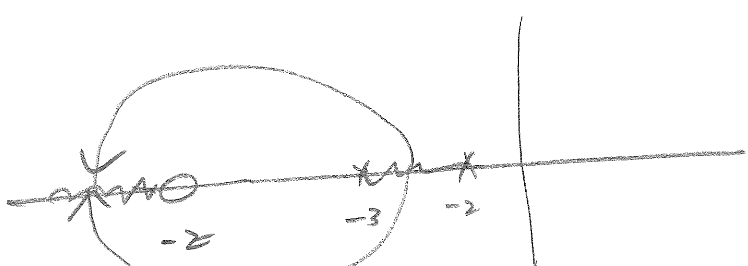
a) Assuming a proportional controller $G_c(s) = k_p$, what is the settling time as $k_p \to \infty$?



$\sigma_c = \frac{[(-3)+(-2)]}{2} = \frac{-5}{2}$ $\theta = \frac{180+i360}{2} = \pm 90^\circ$

$T_s = \frac{4}{5/2} = \boxed{\frac{8}{5} = T_s}$

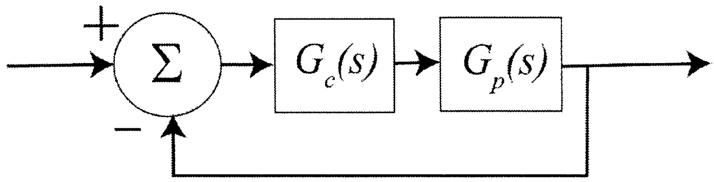
b) Assuming a proportional + derivative controller $G_c(s) = k(s+z)$, what is the value of z so that the settling time $T_s = \frac{1}{2}$ as $k \to \infty$



$T_s = \frac{4}{z} = \frac{1}{2}$ $\boxed{z = 8}$

Name ______________________________ Mailbox __________

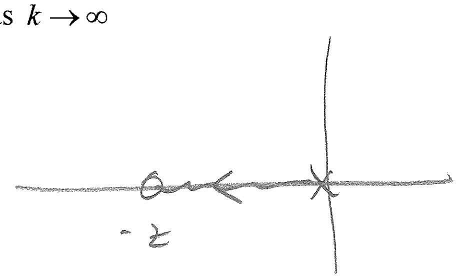
16) (10 points) For this problem assume the following unity feedback system



with $G_p(s) = \frac{1}{s}$

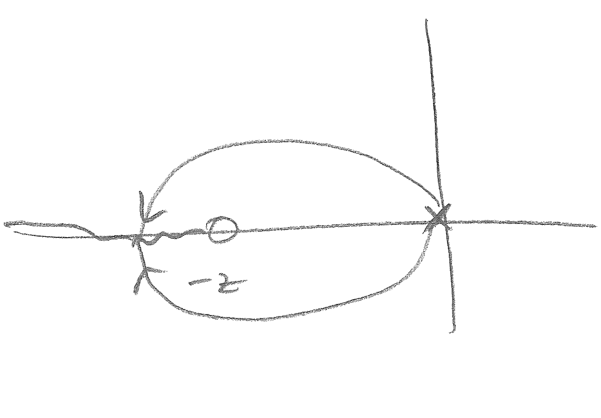
For each of the following problems you should sketch the root locus to answer the following questions. (You will not be graded on your root locus sketches, just your answers.)

a) Assuming a proportional + derivative controller $G_c(s) = k(s+z)$, what is the value of z so that the settling time $T_s = \frac{1}{2}$ as $k \to \infty$



$T_s = \frac{4}{z} = \frac{1}{2}$ $\boxed{z = 8}$

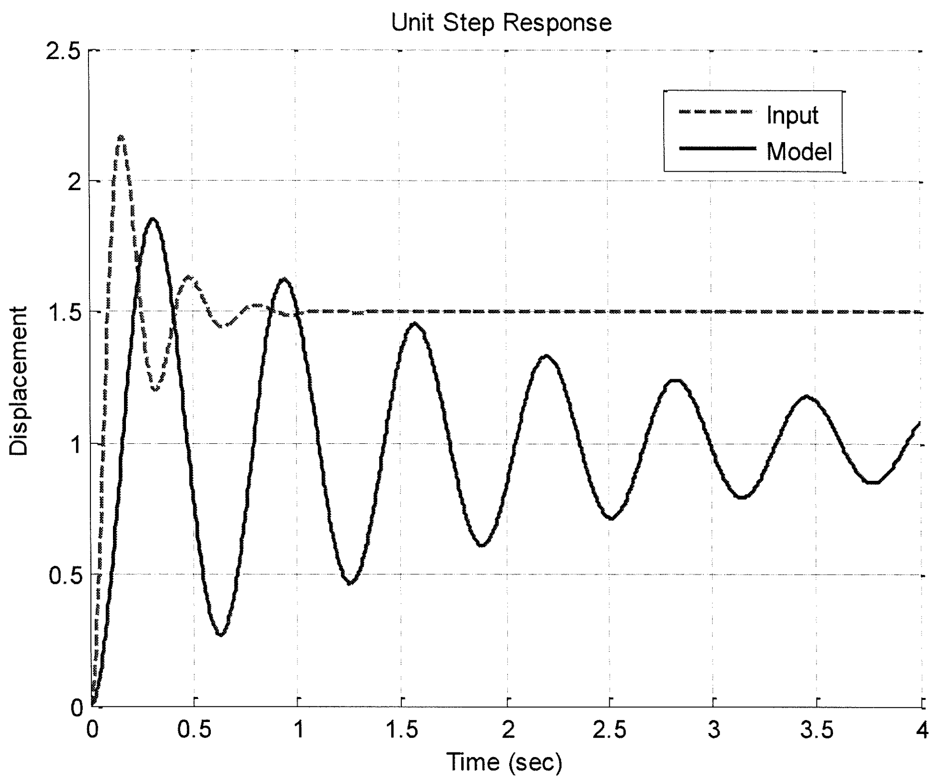
b) Assuming a the PI controller $G_c(s) = \frac{k(s+z)}{s}$, what is the value of z so that the settling time $T_s = 0.25$ as $k \to \infty$



$T_s = \frac{4}{z} = \frac{1}{4}$ $\boxed{z = 16}$

Name ______________________________________ Mailbox ____________

For problems 17-19 consider the following graph, which shows the unit step response of a real system and the unit step response of a model. We want to make the model match the real system as well as we can.



17) To make the model match the real system, what should we do to the static gain of the model?

a) reduce it b) increase it c) leave it alone

18) Assuming we have matched the static gain correctly, what should we do to the damping ratio of the model to make the model better fit the real system?

a) reduce it b) increase it c) leave it alone

or (close)

19) What should we do with the natural frequency of the model to make the model better fit the real system?

a)reduce it b) increase it c) leave it alone

Name ______________________________ Mailbox __________

20) (13 points) Determine ***both*** the *impulse response* and the *unit step response* of systems with transfer functions

a) $H(s)=\dfrac{2s}{(s+1)^2+2^2}$, b) $H(s)=\dfrac{1}{(s+2)^2}$

a) $H(s)=\dfrac{2s}{(s+1)^2+2^2}=\dfrac{2(s+1-1)}{(s+1)^2+2^2}=2\dfrac{(s+1)}{(s+1)^2+2^2}-\dfrac{2}{(s+1)^2+2^2}$

$$\boxed{h(t)=\left[2e^{-t}\cos(2t)-e^{-t}\sin(2t)\right]u(t)}$$

$Y(s)=H(s)\frac{1}{s}=\dfrac{2}{(s+1)^2+2^2}$ $\boxed{y(t)=e^{-t}\sin(2t)u(t)}$

b) $H(s)=\dfrac{1}{(s+2)^2}$ $\boxed{h(t)=te^{-2t}u(t)}$

$Y(s)=H(s)\frac{1}{s}=\dfrac{1}{s(s+2)^2}=\dfrac{A}{s}+\dfrac{B}{s+2}+\dfrac{C}{(s+2)^2}$ $A=\frac{1}{4}$ $C=-\frac{1}{2}$

$0=A+B$ $B=-\frac{1}{4}$

$$\boxed{y(t)=\left[\frac{1}{4}-\frac{1}{4}e^{-2t}-\frac{1}{2}te^{-2t}\right]u(t)}$$

Name ____ Mailbox ____________

21) (10 points) For impulse response $h(n)=\left(\frac{1}{2}\right)^{n+1}u(n-1)$ and input $x(n)=\left(\frac{1}{4}\right)^{n-2}u(n+2)$, determine the system output by evaluating the convolution sum $y(n)=\sum_{k=-\infty}^{\infty}h(n-k)x(k)$

Note: you do not have to simplify your answer, but you must remove all sums and include a unit step function of some sort.

$$y(n) = \sum_{k=\infty}^{\infty} \left(\tfrac{1}{2}\right)^{n-k+1} u(n-k-1) \left(\tfrac{1}{4}\right)^{k-2} u(k+2)$$

$n-k-1 \geq 0$ $\quad n-1 \geq k$

$k+2 \geq 0$ $\quad k \geq -2$

$n-1 \geq -2$

$n \geq -1$

$n+1 \geq 0$

$$y(n) = \sum_{k=-2}^{n-1} \left(\tfrac{1}{2}\right)^{n+1}\left(\tfrac{1}{2}\right)^{-k}\left(\tfrac{1}{4}\right)^{-2}\left(\tfrac{1}{4}\right)^{k}$$

$$= \left(\tfrac{1}{2}\right)^{n+1} 16 \sum_{k=-2}^{n-1} \left(\tfrac{1}{2}\right)^{k}$$

let $l = k+2$ $\quad l-2 = k$

$$= \left(\tfrac{1}{2}\right)^{n+1} 16 \sum_{l=0}^{n+1} \left(\tfrac{1}{2}\right)^{l-2} = \left(\tfrac{1}{2}\right)^{n+1} 16 \left(\tfrac{1}{2}\right)^{-2} \sum_{l=0}^{n+1} \left(\tfrac{1}{2}\right)^{l}$$

$$= \left(\tfrac{1}{2}\right)^{n+1} 16 \left(\tfrac{1}{2}\right)^{-2} \left[\frac{1-\left(\frac{1}{2}\right)^{n+2}}{1-\frac{1}{2}}\right] u(n+1)$$

$$= \left(\tfrac{1}{2}\right)^{n+1}\left(\tfrac{1}{2}\right)^{-2}\left(\tfrac{1}{2}\right)^{-1} 16\left[1-\left(\tfrac{1}{2}\right)^{n+2}\right] u(n+1)$$

$$\boxed{y(n) = \left(\tfrac{1}{2}\right)^{n-2} 16 \left[1-\left(\tfrac{1}{2}\right)^{n+2}\right] u(n+1)}$$