

ECE-320 Quiz #4

For problems 1-3 , consider a closed loop system with transfer function

$$G_0(s) = \frac{s+a}{s^2+bs+k}$$

1) The sensitivity to variations in k , $S_k^{G_0}(s)$, is

- a) $\frac{k}{s^2+bs+k}$ b) $\frac{-k}{s^2+bs+k}$ c) 1 d) $\frac{k}{s+a} - \frac{k}{s^2+bs+k}$ e) none of these

2) The sensitivity to variations in b , $S_b^{G_0}(s)$, is

- a) $\frac{-b}{s^2+bs+k}$ b) $\frac{-bs}{s^2+bs+k}$ c) 1 d) $\frac{b}{s+a} - \frac{bs}{s^2+bs+k}$ e) none of these

3) The sensitivity to variations in a , $S_a^{G_0}(s)$, is

- a) $\frac{a}{s^2+bs+k}$ b) $\frac{-a}{s^2+bs+k}$ c) 1 d) $\frac{a}{s+a}$ e) none of these

4) Assume we compute the sensitivity of a system with nominal value $a = 4$ to be

$$S_a^{G_0}(s) = \frac{1}{s+a}$$

For what frequencies will the sensitivity function be less than $\frac{1}{\sqrt{32}}$?

- a) $\omega < 4$ rad/sec b) $\omega > 4$ rad/sec c) $\omega > 16$ rad/sec d) $\omega < 16$ rad/sec e) none of these

5) Assume we compute the sensitivity of a system with nominal value $a = 3$

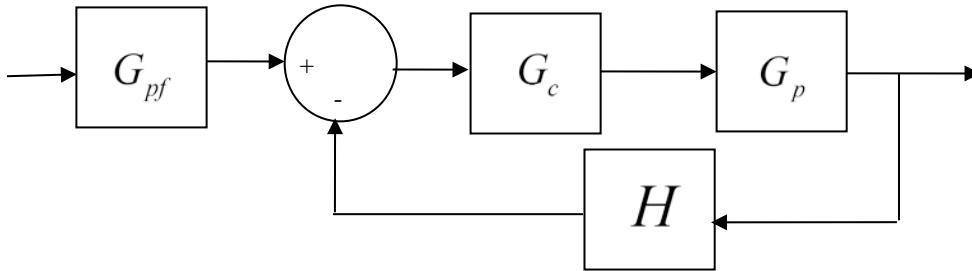
to be

$$S_a^{G_0}(s) = \frac{s+2}{s+1+a}$$

For what frequencies will the sensitivity function be greater than $\sqrt{\frac{10}{16}}$?

- a) $\omega < 4$ rad/sec b) $\omega > 4$ rad/sec c) $\omega > 16$ rad/sec d) $\omega < 16$ rad/sec e) none of these

Problems 6-9 refer to the following system



6) To reduce the sensitivity of the closed loop transfer function variations in the plant G_p , we should

- a) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ large
- b) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ small
- c) make G_{pf} large
- d) do nothing, we cannot change the sensitivity

7) To reduce the sensitivity of the closed loop transfer function to variations in the prefilter G_{pf} , we should

- a) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ large
- b) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ small
- c) make G_{pf} small
- d) do nothing, we cannot change the sensitivity

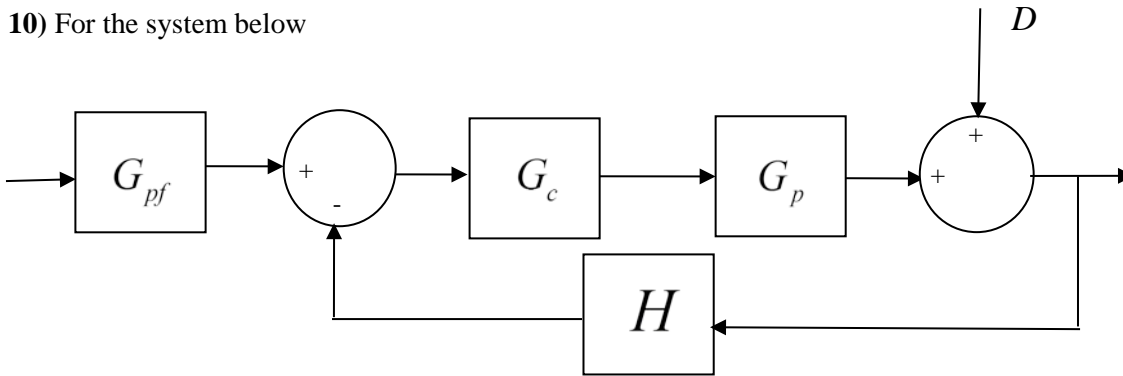
8) To reduce the sensitivity of the closed loop transfer function to variations in the controller G_c we should

- a) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ large
- b) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ small
- c) make $|H(j\omega)|$ large
- d) do nothing, we cannot change the sensitivity

9) To reduce the sensitivity of the closed loop transfer function to variations in the sensor H , we should

- a) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ large
- b) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ small
- c) make G_{pf} large
- d) do nothing, we cannot change the sensitivity

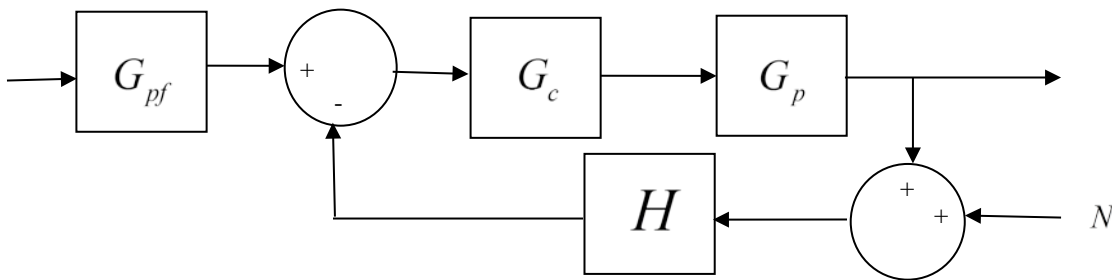
10) For the system below



to reduce the effects of the external disturbance D on the system output, we should

- a) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ large
- b) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ small
- c) make G_{pf} large
- d) do nothing, we cannot change the sensitivity

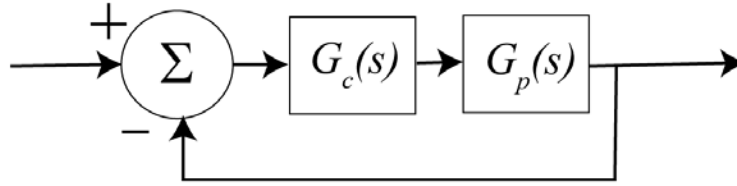
11) For the system below



to reduce the effects of sensor noise N on the closed loop system, we should

- a) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ large
- b) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ small
- c) make $|H(j\omega)|$ large
- d) do nothing, we cannot change the sensitivity

For the problems 12-16, assume the closed loop system below and assume $G_p(s) = \frac{3}{(s+2)(s+3)}$



For each of the following problems sketch the root locus, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary.

12) Assume the proportional controller $G_c(s) = k_p$

13) Assume the integral controller $G_c(s) = \frac{k_i}{s}$

14) Assume the PI controller $G_c(s) = \frac{k(s+5)}{s}$

15) Assume the PD controller $G_c(s) = k(s+6)$

16) Assume the PID controller $G_c(s) = \frac{k(s+6+2j)(s+6-2j)}{s}$

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For the problems 17 - 19, assume a , b , c , d , e , and f are real-valued numbers, and write an expression for the magnitude of the following:

17) $Z = \frac{a + j\omega b}{c - j\omega d}$

18) $Z = \frac{a + b - j\omega c}{d + j\omega}$

19) $Z = \frac{a + j + j\omega c + j\omega d}{1 - j\omega e + f}$

Root Locus Construction**Once each pole has been paired with a zero, we are done**1. *Loci Branches*

$$\underset{k=0}{\text{poles}} \rightarrow \underset{k=\infty}{\text{zeros}}$$

Continuous curves, which comprise the locus, start at each of the n poles of $G(s)$ for which $k = 0$. As k approaches ∞ , the branches of the locus approach the m zeros of $G(s)$. Locus branches for excess poles extend to infinity.

The root locus is **symmetric** about the real axis.

2. *Real Axis Segments*

The root locus includes all points along the real axis to the left of an odd number of poles plus zeros of $G(s)$.

3. *Asymptotic Angles*

As $k \rightarrow \infty$, the branches of the locus become asymptotic to straight lines with angles

$$\theta = \frac{180^\circ + i360^\circ}{n - m}, i = 0, \pm 1, \pm 2, \dots$$

until all $(n - m)$ angles not differing by multiples of 360° are obtained. n is the number of poles of $G(s)$ and m is the number of zeros of $G(s)$.

4. *Centroid of the Asymptotes*

The starting point on the real axis from which the asymptotic lines radiate is given by

$$\sigma_c = \frac{\sum_i p_i - \sum_j z_j}{n - m}$$

where p_i is the i^{th} pole of $G(s)$, z_j is the j^{th} zero of $G(s)$, n is the number of poles of $G(s)$ and m is the number of zeros of $G(s)$. This point is termed the *centroid of the asymptotes*.

5. *Leaving/Entering the Real Axis*

When two branches of the root locus leave or enter the real axis, they usually do so at angles of $\pm 90^\circ$.