Name Solutions Mailbox ____________

ECE-320 Linear Control Systems

Spring 2014, Exam 2

No calculators or computers allowed.

Problem 1 ________	**/15**
Problem 2 ________	**/25**
Problem 3 ________	**/20**
Problem 4 ________	**/20**
Problem 5 ________	**/20**
Total ________	**/100**

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1) (15 Points) Consider a system with closed loop transfer function $G_o(s) = \frac{k_p s}{s+1+k_p s}$. The *nominal* values for the parameter k_p is 2.

a) Determine an expression for the sensitivity of the closed loop system to variations in k_p. Your final answer should be written as numbers and the complex variable s.

b) Determine expressions for the *magnitude* of the sensitivity functions in terms of frequency, ω

a) $$S_{K_p}^{G_o} = \frac{K_p}{N}\frac{\partial N}{\partial K_p} - \frac{K_p}{D}\frac{\partial D}{\partial K_p}$$

$$= \frac{K_p}{K_p s}(s) - \frac{K_p}{s+1+K_p}(s) = 1 - \frac{K_p s}{s+1+K_p s} = \frac{s+1}{s+1+K_p s}$$

$$\boxed{S_{K_p}^{G_o} = \frac{s+1}{s+1+K_p s} = \frac{s+1}{3s+1}}$$

b) $$\boxed{\left|S_{K_p}^{G_o}(j\omega)\right| = \sqrt{\frac{\omega^2+1}{9\omega^2+1}}}$$

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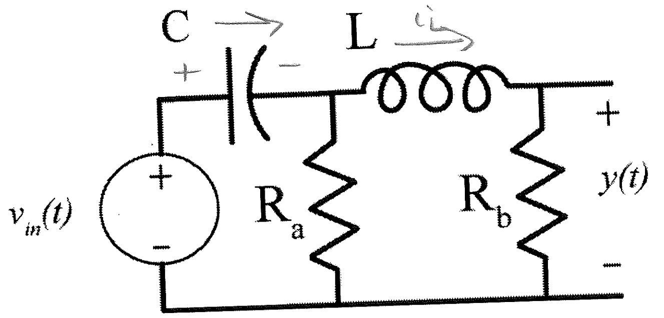
2) (25 Points) For the following circuit, the state variables are the current through the inductor and the voltage across the capacitor. Determine a state variable model for this system. Specifically, you need to identify the A, B, C, and D matrices/vectors/scalars. You surely recall the useful relationships

$v(t) = L\frac{di(t)}{dt}, i(t) = C\frac{dv(t)}{dt}$



$$C\frac{dV_c}{dt} = i_L + \frac{(V_{in} - V_c)}{R_a}$$

$$V_{in} - V_c - L\frac{di_L}{dt} = i_L R_b$$

$$\frac{dV_c}{dt} = \frac{-1}{R_a C} V_c + \frac{1}{C} i_L + \frac{V_{in}}{R_a C}$$

$$\frac{di_L}{dt} = \frac{-1}{L} V_c - \frac{R_b}{L} i_L + \frac{1}{L} V_{in}$$

$$\boxed{\frac{d}{dt}\begin{bmatrix} V_c \\ i_L \end{bmatrix} = \overbrace{\begin{bmatrix} \frac{-1}{R_a C} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_b}{L} \end{bmatrix}}^{A} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \overbrace{\begin{bmatrix} \frac{1}{R_a C} \\ \frac{1}{L} \end{bmatrix}}^{B} V_{in}}$$

$$y = i_b R_b$$

$$\boxed{y = \underbrace{\begin{bmatrix} 0 & R_b \end{bmatrix}}_{C} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} V_{in}}$$

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3) (20 Points) For the state variable model

$$\dot{q} = \begin{bmatrix} 2 & 0 \\ 2 & 3 \end{bmatrix} q + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} q + [0] u$$

Determine the closed loop transfer function with ***state variable feedback***, $u(t) = G_{pf} r(t) - Kq(t)$

$$\tilde{A} = A - BK = \begin{bmatrix} 2 & 0 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 2 - K_1 & 3 - K_2 \end{bmatrix}$$

$$sI - \tilde{A} = \begin{bmatrix} s-2 & 0 \\ K_1 - 2 & s + K_2 - 3 \end{bmatrix}$$

$$(sI - \tilde{A})^{-1} = \frac{1}{(s-2)(s+K_2-3)} \begin{bmatrix} s + K_2 - 3 & 0 \\ 2 - K_1 & s - 2 \end{bmatrix}$$

$$\tilde{B} = B G_{pf} = \begin{bmatrix} 0 \\ G_{pf} \end{bmatrix}$$

$$G_0(s) = C(sI - \tilde{A})^{-1} \tilde{B} = \frac{\begin{bmatrix} 0 & 1 \end{bmatrix}}{(s-2)(s+K_2-3)} \begin{bmatrix} s + K_2 - 3 & 0 \\ 2 - K_1 & s - 2 \end{bmatrix} \begin{bmatrix} 0 \\ G_{pf} \end{bmatrix}$$

$$\boxed{G_0(s) = \frac{G_{pf}}{s + K_2 - 3}}$$

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4) (20 points) For impulse response $h(n)=\left(\frac{1}{2}\right)^{n+2}u(n-1)$ and input $x(n)=\left(\frac{1}{4}\right)^{n-3}u(n-2)$, use z-transforms of the input and impulse response to determine the system output $y(n)$

Hint: Assume $Y(z)=z^{-2}G(z)$

$$H(z) = \mathcal{Z}\left\{\left(\tfrac{1}{2}\right)^{n-1}\left(\tfrac{1}{2}\right)^{3}u(n-1)\right\} = \frac{1}{8}z^{-1}\frac{z}{z-\frac{1}{2}} = \boxed{\frac{1/8}{z-1/2} = H(z)}$$

$$X(z) = \mathcal{Z}\left\{\left(\tfrac{1}{4}\right)^{n-2}\left(\tfrac{1}{4}\right)^{-1}u(n-2)\right\} = 4z^{-2}\frac{z}{z-\frac{1}{4}} = \boxed{\frac{4z^{-1}}{z-1/4} = X(z)}$$

$$Y(z) = \frac{\frac{1}{2}z^{-1}}{(z-\frac{1}{2})(z-\frac{1}{4})} \qquad G(z) = \frac{\frac{1}{2}z}{(z-\frac{1}{2})(z-\frac{1}{4})} \qquad Y(z) = z^{-2}G(z)$$

$$\frac{G(z)}{z} = \frac{\frac{1}{2}}{(z-\frac{1}{2})(z-\frac{1}{4})} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{4}}$$

$$A = \frac{1/2}{1/4} = 2$$

$$B = \frac{1/2}{-1/4} = -2$$

$$G(z) = 2\frac{z}{z-\frac{1}{2}} - 2\frac{z}{z-\frac{1}{4}}$$

$$g(n) = \left[2\left(\tfrac{1}{2}\right)^{n} - 2\left(\tfrac{1}{4}\right)^{n}\right]u(n)$$

$$y(n) = g(n-2) = \boxed{2\left[\left(\tfrac{1}{2}\right)^{n-2} - \left(\tfrac{1}{4}\right)^{n-2}\right]u(n-2) = y(n)}$$

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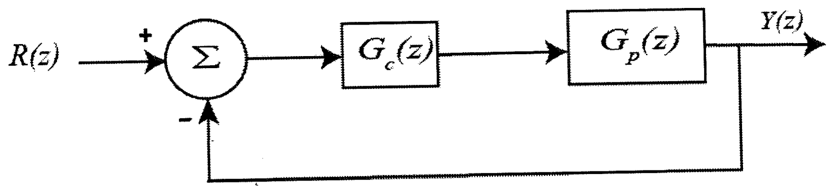
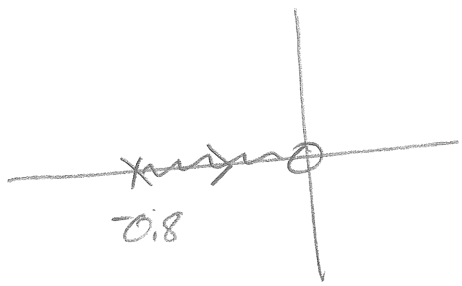
5) (20 points) For the following problem, assume the closed loop system below and assume

$$G_p(s) = \frac{z}{(z+0.8)}$$



Hint: Drawing a root locus plot for a discrete-time system is identical to drawing the root locus plot for a continuous time system (treat z as you would s). The only difference is the interpretation.

a) Assume we are using the controller $G_c(z) = k$. Sketch the root locus, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary.



In order for the system to be stable, we should require which of the following conditions (assume k_o is a suitable chosen positive number) $k < k_o$ $\quad k > k_o$ $\quad k > 0$

Assuming the system is stable, will the closed loop system respond faster or slower as k is increased?

faster as pole moves closer to origin

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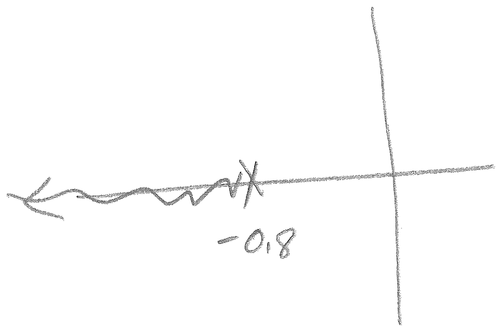
b) Assume we are using the controller $G_c(z) = kz^{-1}$. Sketch the root locus, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary.



In order for the system to be stable, we should require which of the following conditions (assume k_o is a suitable chosen positive number) $k < k_o$ $k > k_o$ $k > 0$

Assuming the system is stable, will the closed loop system respond faster or slower as k is increased?

slower