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ECE-320 Linear Control Systems

Spring 2014, Exam 1

No calculators or computers allowed, you may leave your answers as fractions.

All problems are worth 3 points unless noted otherwise.

Total ________/100

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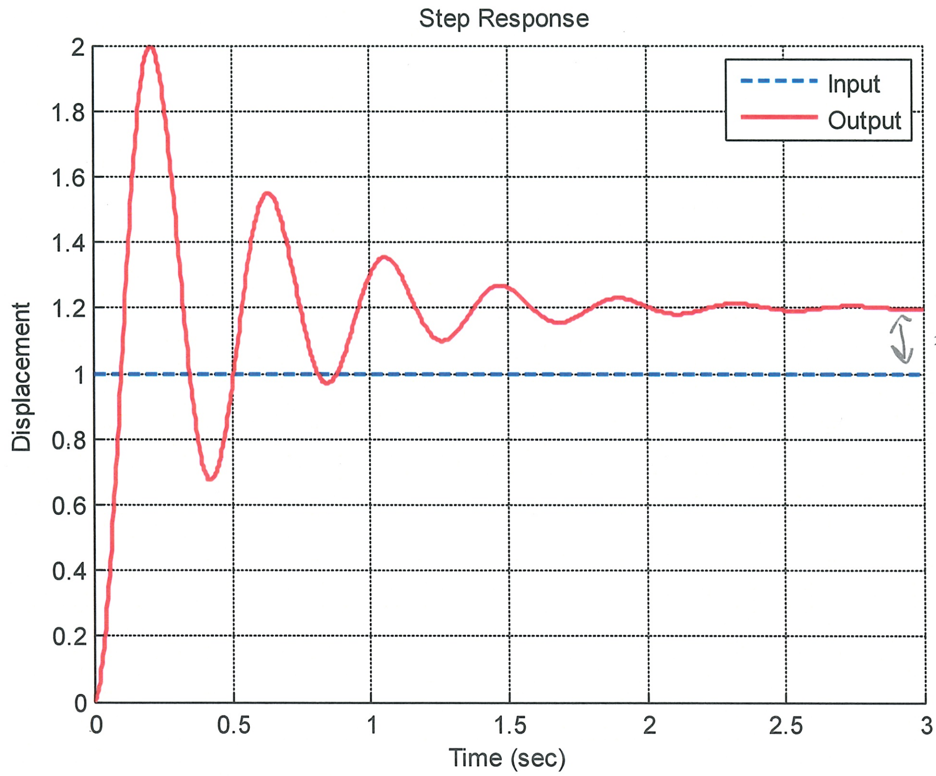
Problems 1-3 refer to the **unit step response** of a system, shown below



$= e_{ss}$

1) Estimate the **steady state error** $e_{ss} = \text{input} - \text{output} = 1 - 1.2 = \boxed{-0.2 = e_{ss}}$

2) Estimate the **percent overshoot** $PO = \frac{a}{b} \times 100\% = \frac{2-1.2}{1.2} \times 100\% = \frac{0.8}{1.2} \times 100\% = \boxed{67\%}$

3) Estimate the **static gain** $K(\text{input}) = (\text{output})$

$K(1) = 1.2 \quad \boxed{K = 1.2}$

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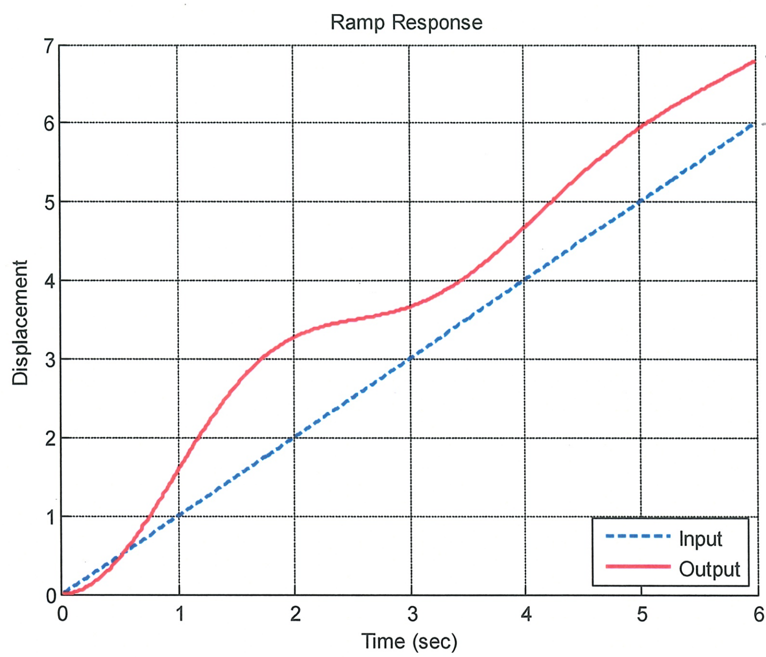
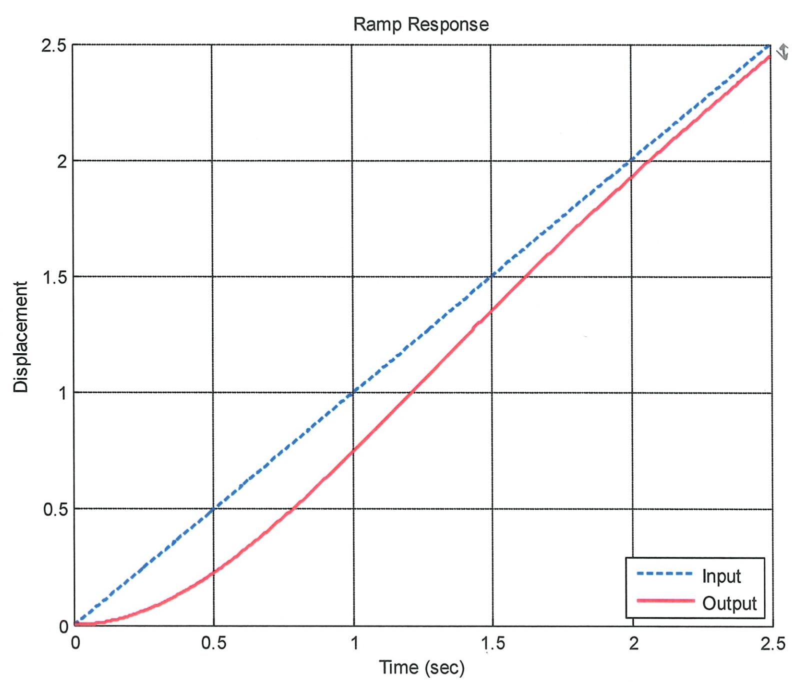
4) Estimate the steady state error for the ramp response of the system shown below:



e_{ss} = input − output
= 6 − 6.8
= $\boxed{-0.8 = e_{ss}}$

5) Estimate the steady state error for the ramp response of the system shown below:



e_{ss} = input − output
= 2.5 − 2.45 = 0.05
$\boxed{0.05 = e_{ss}}$

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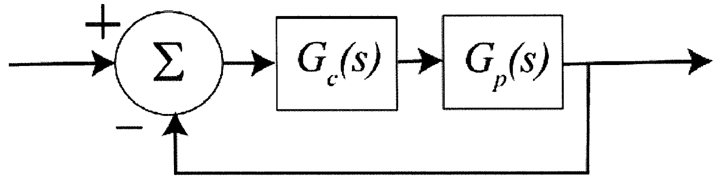
6) (10 points) For this problem assume the following unity feedback system



with $G_p(s) = \frac{3}{(s+1)(s+2)}$ and $G_c(s) = 2(s+3)$

a) Determine the position error constant K_p

$K_p = \lim_{s \to 0} G_c(s) G_p(s) = \frac{18}{2} = \boxed{9 = K_p}$

b) Estimate the steady state error for a unit step using the position error constant.

$e_{ss} = \frac{1}{1+K_p} = \boxed{\frac{1}{10} = e_{ss}}$

c) Determine the velocity error constant K_v

$K_v = \lim_{s \to 0} s\, G_c(s) G_p(s) = \boxed{0 = K_v}$

d) Estimate the steady state error for a unit ramp using the velocity error constant.

$e_{ss} = \frac{1}{K_v} = \boxed{\infty = e_{ss}}$

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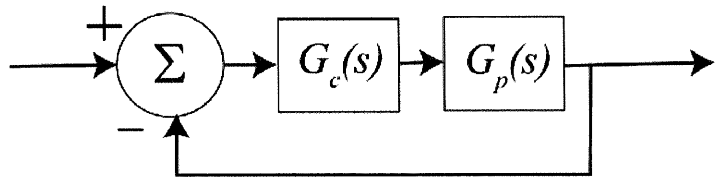
7) (10 points) For this problem assume the following unity feedback system



with $G_p(s) = \frac{3}{(s+2)(s+4)}$ and $G_c(s) = \frac{5(s+1)}{s}$

a) Determine the position error constant K_p

$K_p = \lim_{s \to 0} G_p(s) G_c(s) = \boxed{\infty = K_p}$

b) Estimate the steady state error for a unit step using the position error constant.

$e_{ss} = \frac{1}{1+K_p} = \boxed{0 = e_{ss}}$

c) Determine the velocity error constant K_v

$K_v = \lim_{s \to 0} s G_c(s) G_p(s) = \boxed{\frac{15}{8} = K_v}$

d) Estimate the steady state error for a unit ramp using the velocity error constant.

$e_{ss} = \frac{1}{K_v} = \boxed{\frac{8}{15} = e_{ss}}$

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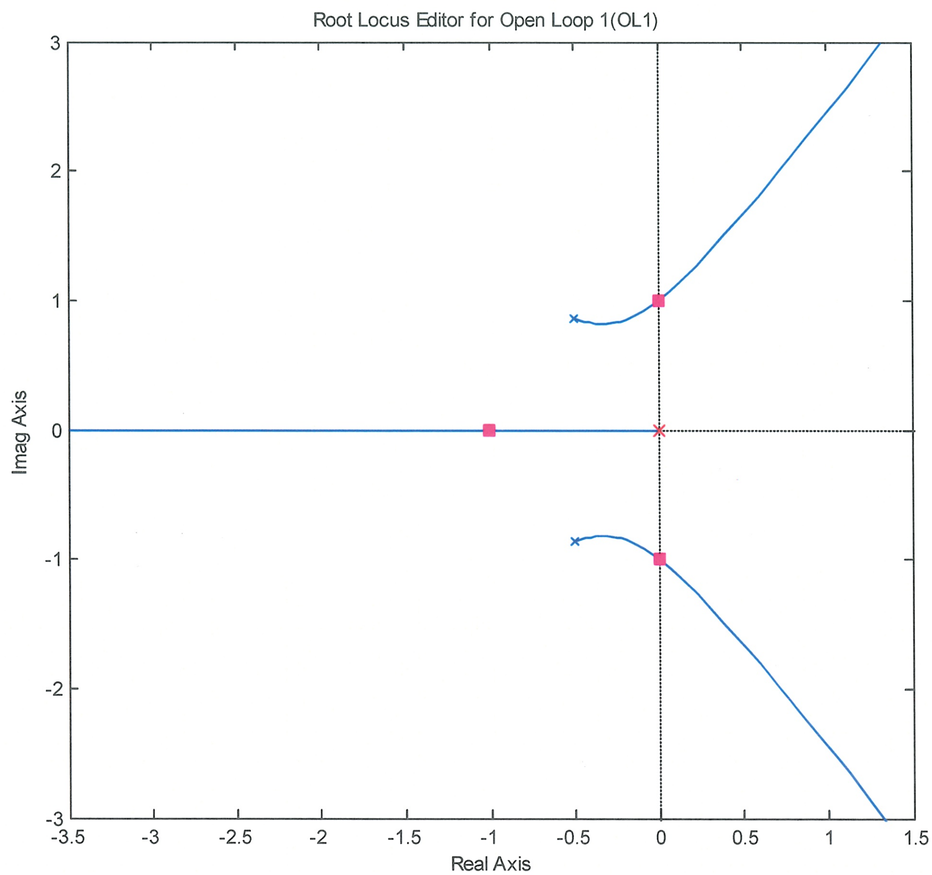
Problems 8-10 refer to the following root locus plot (from sisotool)



8) Is it possible for -2 to be a closed loop pole for this system ? (Yes or No)

yes, but the system will be unstable

9) When the gain is approximately 1 the closed loop poles are as shown in the figure. If we want the system to be stable what conditions do we need to place on the gain k?

poles $\longrightarrow$ zeros

$K=0$ $\quad$ $K=\infty$

$\boxed{0 < K < 1}$

10) Is this a type one system? (Yes or No) yes, pole at the origin

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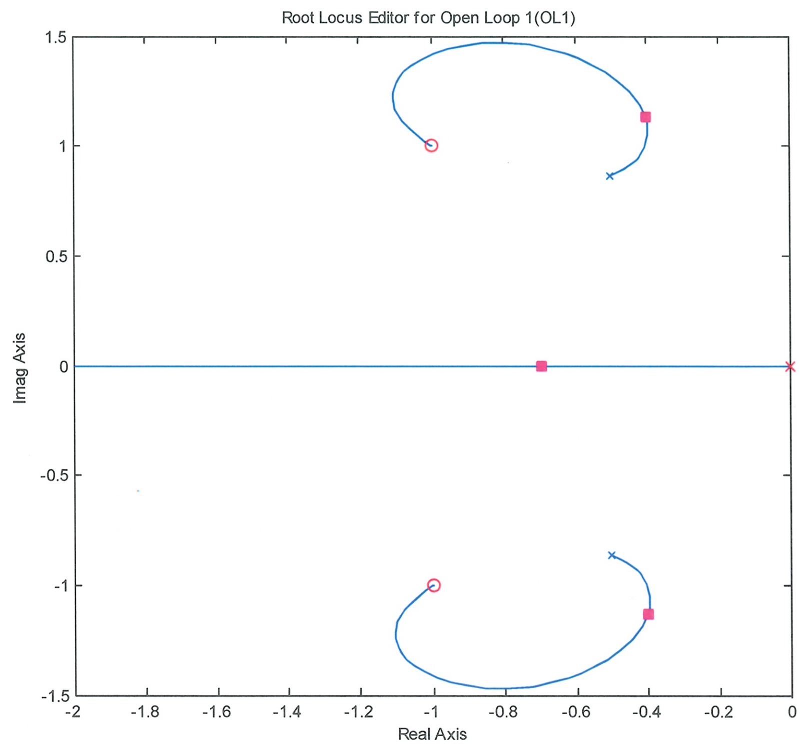
Problems 11 -13 refer to the following root locus plot (from sisotool)



11) When $k = 0.5$ the poles are as they are shown in the figure. Estimate the closed loop poles.

$-.7, \; -0.4 \pm j\,1.2$

12) Estimate the settling time as the gain $k \rightarrow \infty$ $\quad T_s \approx \frac{4}{1} = \boxed{4 = T_s}$

13) Is this a type one system? (Yes or No) yes, pole at the origin

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14) (6 points) For the following two PID controllers, determine k_p, k_i, and k_d

$$G_c(s) = \frac{0.2(s^2+4s+5)}{s} = 0.8 + \frac{1}{s} + 0.2s$$

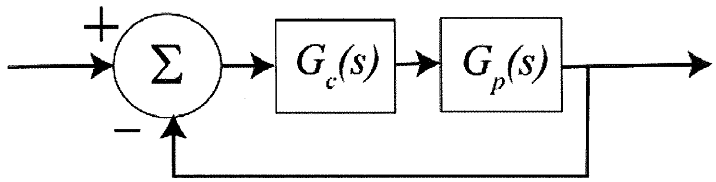
$$G_c(s) = \frac{(s+2)(s+3)}{s} = 5 + \frac{6}{s} + 1s$$

$K_p = 0.8 \quad K_i = 1 \quad K_d = 0.2$

$K_p = 5 \quad K_i = 6 \quad K_d = 1$

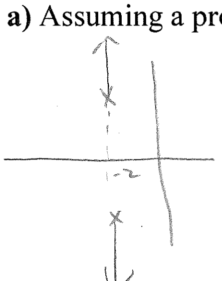
15) (10 points) For this problem assume the closed loop system below and assume

$$G_p(s) = \frac{3}{(s+2+2j)(s+2-2j)}$$



For each of the following problems you should sketch the root locus to answer the following questions. (You ***will not*** be graded on your root locus sketches, just your answers.)

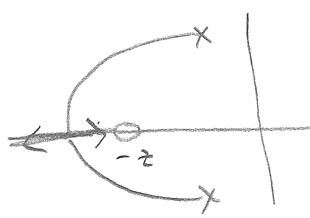
a) Assuming a proportional controller $G_c(s) = k_p$, what is the settling time as $k_p \to \infty$?



$\sigma_c = \frac{[(-2)+(-2)]}{2} = -2 \qquad \theta = \frac{180 + i360}{2} = \pm 90^\circ$

$T_s = \frac{4}{2} = \boxed{2 = T_s}$

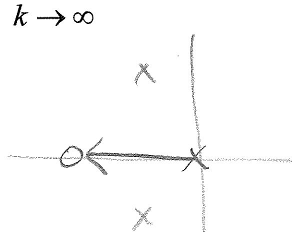
b) Assuming a proportional + derivative controller $G_c(s) = k(s+z)$, what is the value of z so that the settling time $T_s = \frac{1}{2}$ as $k \to \infty$



$T_s = \frac{1}{2} = \frac{4}{z}$

$\boxed{z = 8}$

c) Assuming a the PI controller $G_c(s) = \frac{k(s+z)}{s}$, what is the value of z so that the settling time $T_s = 4$ as $k \to \infty$



$\sigma_c = \frac{[(-2)+(-2)] - [-z]}{3-1} = \frac{-4+z}{2} \qquad \theta = \frac{180 + i360}{3-1} = \pm 90^\circ$

$T_s = \frac{4}{\sigma} \quad \sigma = 1 \quad \boxed{z = 1} \quad \sigma_c = -3/2$

$\boxed{z = 2 \quad \sigma_c = -1}$

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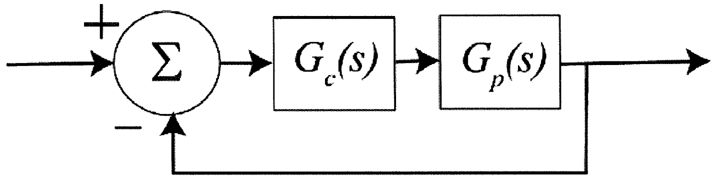
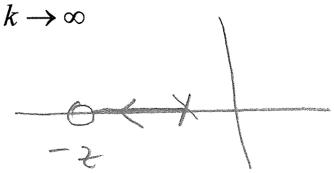
16) (10 points) For this problem assume the following unity feedback system



with $G_p(s) = \dfrac{1}{(s+2)}$

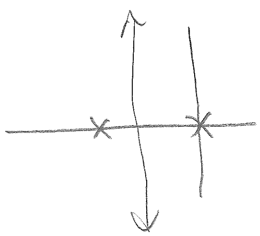
For each of the following problems you should sketch the root locus to answer the following questions. (You will not be graded on your root locus sketches, just your answers.)

a) Assuming a proportional + derivative controller $G_c(s) = k(s+z)$, what is the value of z so that the settling time $T_s = \dfrac{1}{2}$ as $k \to \infty$



$T_s = \frac{4}{z} = \frac{1}{2}$ $\boxed{z=8}$

b) Assuming an integral controller $G_c(s) = \dfrac{k_i}{s}$, is the closed loop system stable for all values of k_i

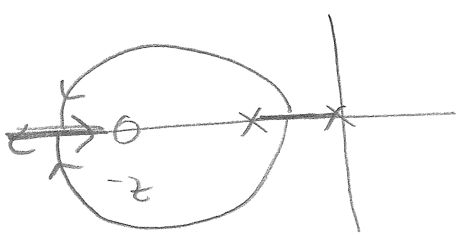


$\sigma = \frac{[(-2)+0]}{2} = -1$ $\theta = \frac{180+i360}{2}$

$= \pm 90°$

$\boxed{\text{always stable}}$

c) Assuming a the PI controller $G_c(s) = \dfrac{k(s+z)}{s}$, what is the value of z so that the settling time $T_s = 0.25$ as $k \to \infty$



$T_s = \frac{4}{z} = \frac{1}{4}$

$\boxed{z=16}$

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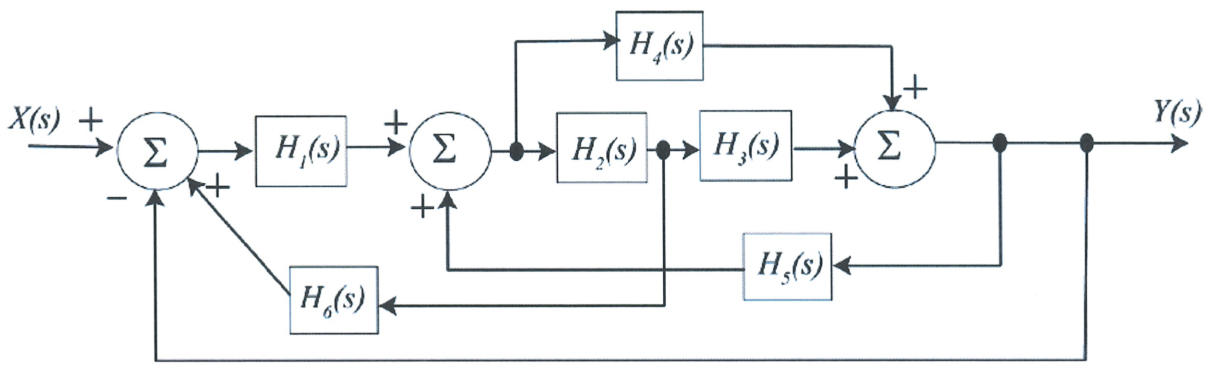
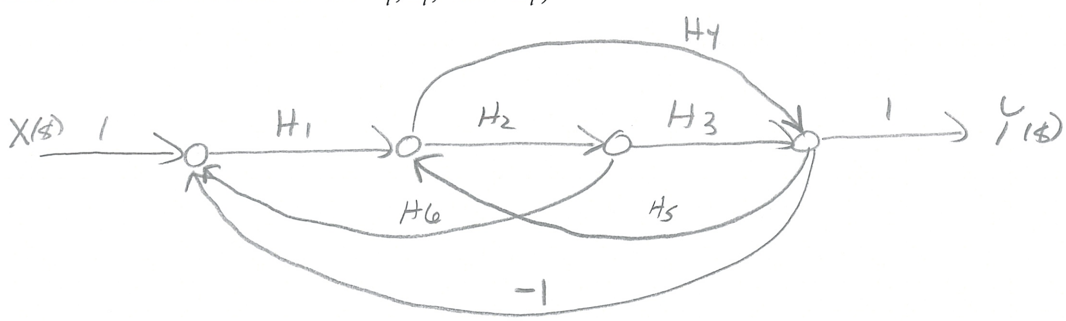
17) (11 points) For the following block diagram



a) Draw the corresponding signal flow graph, labeling each branch and direction. *Feel free to insert as many branches with a gain of 1 as you think you may need.*

b) Determine the system transfer function using Mason's gain rule. You must clearly indicate all of the paths, the loops, the determinant and the cofactors, but you do not need to simplify your final answer (it can be written in terms of the $P_i, L_i,$ and Δ_i)

a) 

b)

$P_1 = H_1 H_2 H_3$ $\quad L_1 = H_2 H_3 H_5$ $\quad L_3 = H_1 H_2 H_6$ $\quad L_5 = -H_1 H_2 H_3$

$P_2 = H_1 H_4$ $\quad L_2 = H_4 H_5$ $\quad L_4 = -H_1 H_4$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5)$ $\qquad \Delta_1 = \Delta_2 = 1$

$$\frac{Y}{X} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

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18) (10 points) Determine ***both*** the *impulse response* and the *unit step response* of a system with transfer function

$$H(s) = \frac{s}{(s+2)^2 + 3^2}$$

a) (easy way) $H(s) = \frac{s+2-2}{(s+2)^2+3^2} = \frac{s+2}{(s+2)^2+3^2} - \frac{2}{3}\,\frac{3}{(s+2)^2+3^2}$

$$\boxed{h(t) = \left[e^{-2t}\cos(3t) - \tfrac{2}{3}e^{-2t}\sin(3t)\right]u(t)}$$

(hard way) $H(s) = \frac{s}{(s+2)^2+3^2} = \frac{A(s+2)}{(s+2)^2+3^2} + B\,\frac{3}{(s+2)^2+3^2}$

$\times s$, let $s \to \infty$ $\quad \boxed{1 = A}$

let $s = -2$ $\quad -\frac{2}{9} = \frac{B}{3}$ $\quad \boxed{B = -\tfrac{2}{3}}$

b) $Y(s) = H(s)\frac{1}{s} = \frac{1}{(s+2)^2+3^2} = \frac{1}{3}\,\frac{3}{(s+2)^2+3^2}$

$$\boxed{y(t) = \tfrac{1}{3}e^{-2t}\sin(3t)\,u(t)}$$