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ECE-320 Linear Control Systems

Spring 2013, Exam 1

No calculators or computers allowed. However, you may use your computer (or other device) to listen to music.

You must simplify your answers as much as possible, or points will be deducted.

Problem 1	**_______/22**
Problem 2	**_______/12**
Problem 3	**_______/6**
Problem 4	**_______/24**
Problem 5	**_______/12**
Problem 6	**_______/24**
Total	**_______/100**

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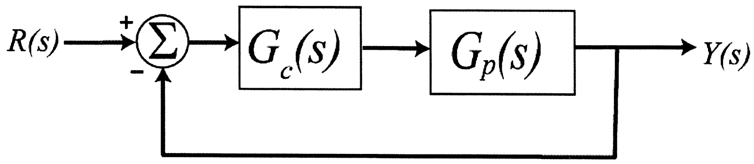
1) (22 points) Consider the following simple feedback control block diagram. The plant is $G_p(s) = \dfrac{7}{s+3}$. The input is a unit step.



a) Determine the settling time, steady state error for a unit step input, and the bandwidth of the plant alone (assuming there is no feedback) $\boxed{T_s = \frac{4}{3}}$ $e_{ss} = 1 - 7/3 = \boxed{-\frac{4}{3} = e_{ss}}$ $\boxed{BW = 3 \text{ rad/sec}}$

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

$$\boxed{G_0(s) = \frac{7K_p}{s+3+7K_p}}$$

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the steady state error for a unit step is 1/8, and the corresponding settling time for the system.

$$e_{ss} = 1 - \frac{7K_p}{3+7K_p} = \frac{3}{3+7K_p} = \frac{1}{8} \qquad 24 = 3 + 7K_p \qquad \boxed{K_p = 3}$$

d) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the settling time is 4/38 seconds, and the corresponding steady state error.

$$T_s = \frac{4}{3+7K_p} = \frac{4}{38} \qquad 3 + 7K_p = 38 \qquad \boxed{K_p = 5}$$

e) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the bandwidth is 17 rad/sec.

$$BW = \frac{1}{3+7K_p} = \frac{1}{17} \qquad \boxed{K_p = 2}$$

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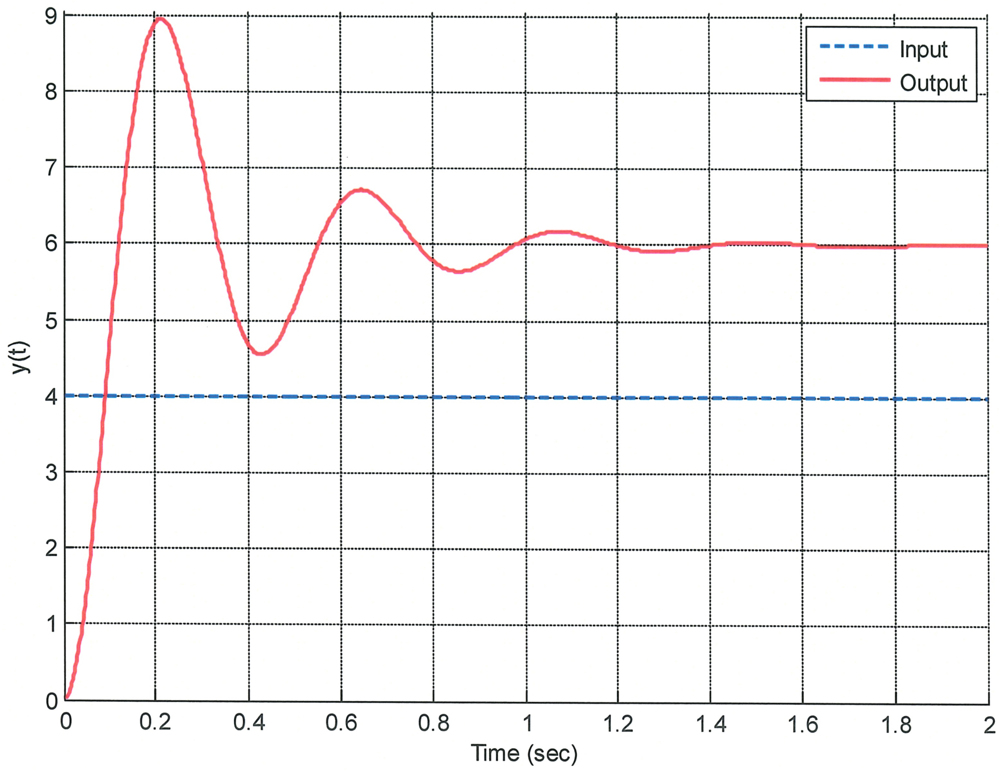
2) (12 points) For the following questions, refer to the following graph showing the input and output of a second order system. For this system the input is a step of amplitude 4. (You can leave your answers as fractions.)



a) What is the static gain of the system? $K(4) = 6$ $\boxed{K = 1.5}$

b) What is the percent overshoot? $\frac{9-6}{6} \times 100\% = \boxed{50\% = PO}$

c) What is the steady state error? $e_{ss} = 4 - 6 = \boxed{-2 = e_{ss}}$

d) What is the steady state error for a ramp input? $\boxed{\infty}$ type 0 system

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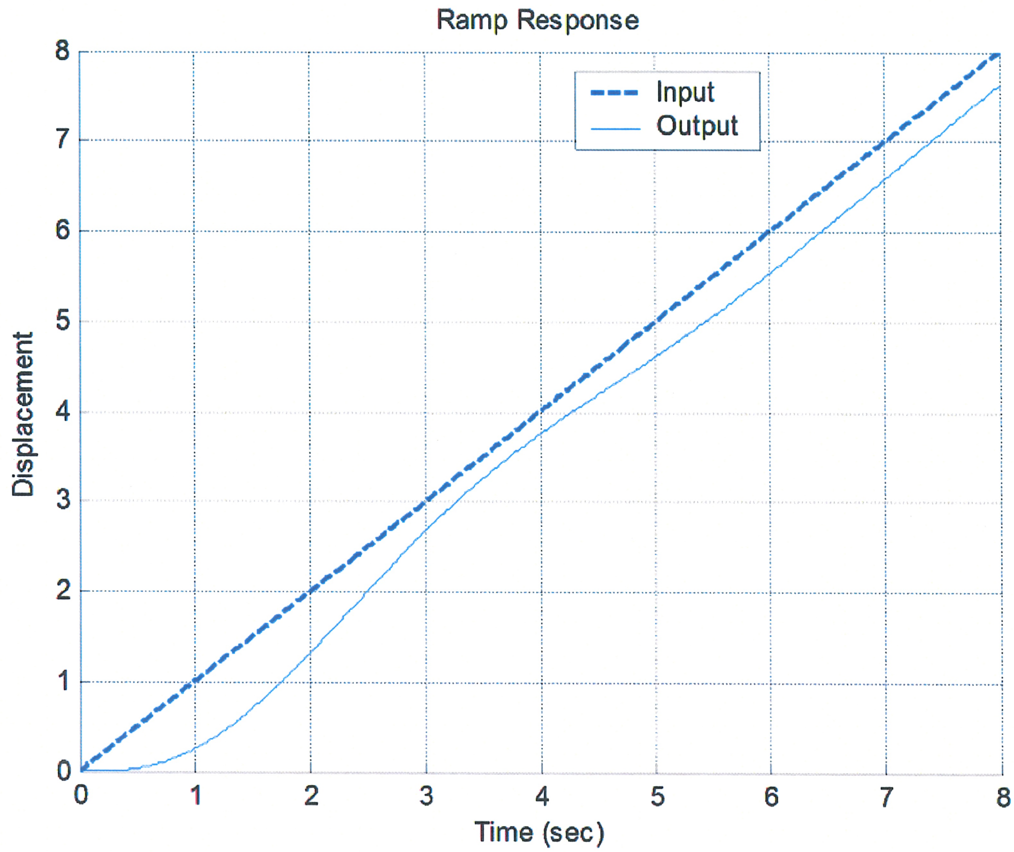
3) (6 points) For the following questions, refer to the following graph showing the input and output of a system.



a) What is the steady state error? $e_{ss} = 8 - 7.6 \approx \boxed{0.4 = e_{ss}}$

b) What is the steady state error for a step input? $\boxed{e_{ss} = 0}$

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4) (24 points) For the following transfer functions, determine ***both*** the *impulse response* and the *unit step response* of the system. *Do not forget any necessary unit step functions.*

a) $H(s) = \dfrac{1}{(s+2)}$

b) $H(s) = \dfrac{1}{(s+1)^2}$

c) $H(s) = \dfrac{s}{s^2+4s+5}$

a) $\boxed{h(t) = e^{-2t}u(t)}$ $\quad Y(s) = \dfrac{1}{s(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+2}$ $\quad A = \frac{1}{2} \quad B = -\frac{1}{2}$

$\boxed{y(t) = \left[\frac{1}{2} - \frac{1}{2}e^{-2t}\right]u(t)}$

b) $\boxed{h(t) = te^{-t}u(t)}$ $\quad Y(s) = \dfrac{1}{s(s+1)^2} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{(s+1)^2}$ $\quad A = 1 \quad C = -1$

$0 = A + B \quad B = -1$

$\boxed{y(t) = \left[1 - e^{-t} - te^{-t}\right]u(t)}$

c) $H(s) = \dfrac{s}{(s+2)^2+1} = \dfrac{s+2}{(s+2)^2+1} - \dfrac{2}{(s+2)^2+1}$

$\boxed{h(t) = e^{-2t}\cos(t)u(t) - 2e^{-2t}\sin(t)u(t)}$

$Y(s) = \dfrac{1}{(s+2)^2+1}$

$\boxed{y(t) = e^{-2t}\sin(t)u(t)}$

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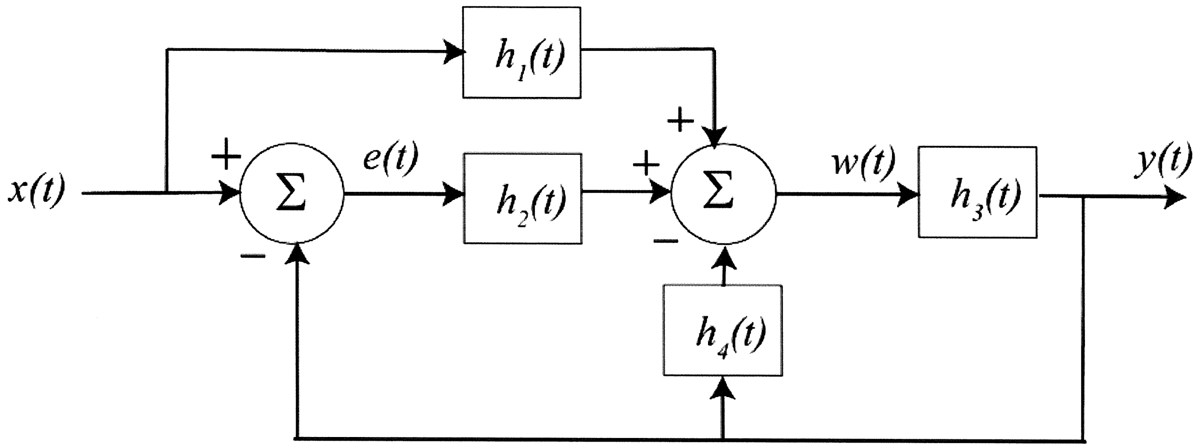
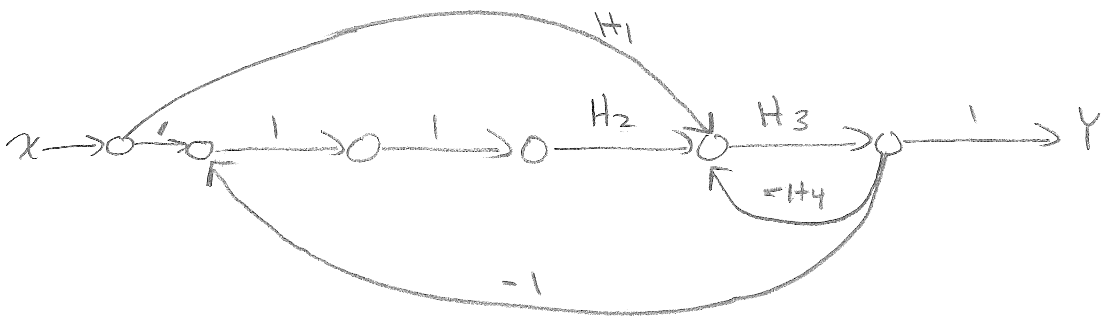
5) (12 points) For the following block diagram



Draw the corresponding signal flow graph, labeling each branch and direction. *Feel free to insert as many branches with a gain of 1 as you think you may need.*

Determine the system transfer function using Mason's gain rule. *You must clearly indicate all of the paths, the loops, the determinant and the cofactors.* ***You need to simplify your final answer!***



$P_1 = H_1 H_3 \qquad P_2 = H_2 H_3$

$L_1 = -H_3 H_4 \qquad L_2 = -H_2 H_3$

$\Delta = 1 - (L_1 + L_2) = 1 + H_2 H_3 + H_3 H_4$

$\Delta_1 = \Delta_2 = 1$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{H_1(s) H_3(s) + H_2(s) H_3(s)}{1 + H_2(s) H_3(s) + H_3(s) H_4(s)}}$$

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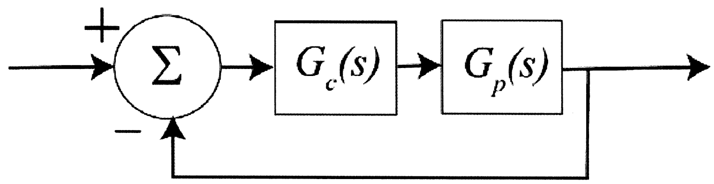
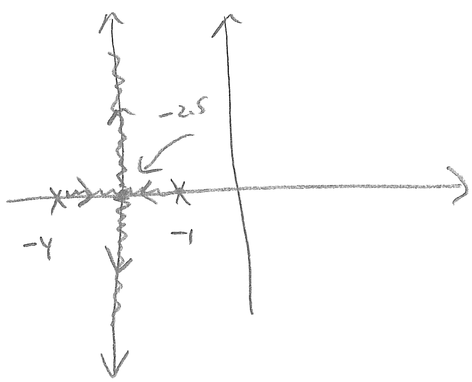
6) (24 points) For the following problem, assume the closed loop system below and assume
$G_p(s) = \dfrac{3}{(s+1)(s+4)}$



a) Assume we are using a proportional controller, so $G_c(s) = k_p$. Sketch the root locus, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary, and answer the following questions.

$\sigma = \frac{[-1\ -4]}{2} = -2.5 \qquad \theta = \frac{180 + i360}{2} = \pm 90^\circ$



i) Is the system always stable? $\boxed{\text{yes}}$

ii) As $k_p \to \infty$, what do you expect the settling time to be? $T_s = \frac{4}{5/2} = \boxed{8/5 \text{ sec} = T_s}$

iii) Determine the position error constant K_p and the steady state error for a unit step in terms of k_p

$\boxed{K_p = \frac{3k_p}{4}} \quad e_{ss} = \frac{1}{1 + \frac{3k_p}{4}} = \boxed{\frac{4}{4+3k_p} = e_{ss}}$

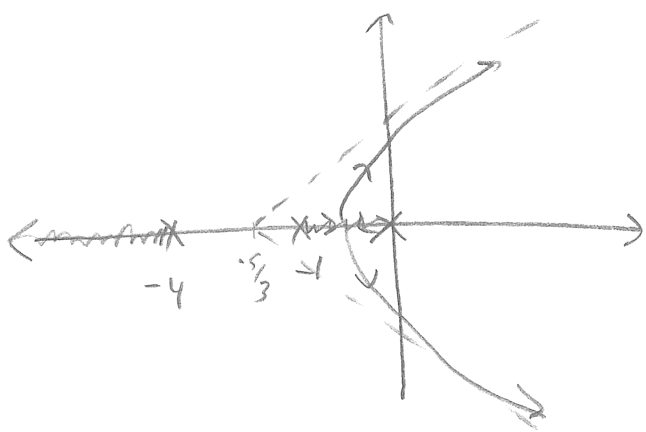
iv) Determine the velocity error constant K_v and the steady state error for a unit ramp in terms of k_p

$\boxed{K_v = 0} \quad \boxed{e_{ss} = \infty}$

Name ____ Mailbox ____________

b) Assume we are using an integral controller, so $G_c(s) = \frac{k_i}{s}$. Sketch the root locus, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary, and answer the following questions.

$\sigma = \frac{[-1-4]}{3} = -5/3 \qquad \theta = \frac{180 + i360}{3} = \pm 60°, 180°$



i) Is the system always stable? $\boxed{\text{no}}$

ii) Determine the position error constant K_p and the steady state error for a unit step in terms of k_i

$\boxed{K_p = \infty}$ $\boxed{e_{ss} = 0}$

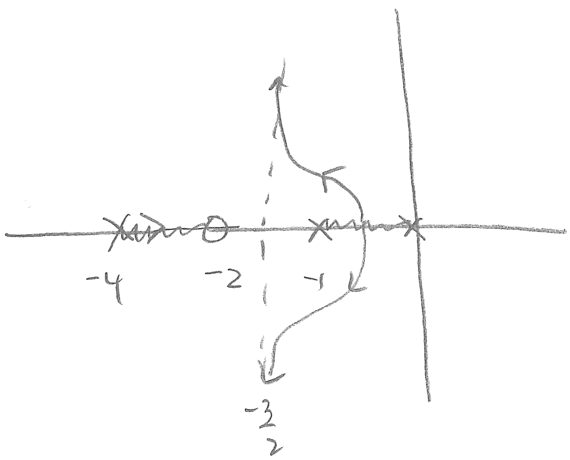
iii) Determine the velocity error constant K_v and the steady state error for a unit ramp in terms of k_i

$\boxed{K_V = \frac{3k_i}{4}}$ $\boxed{e_{ss} = \frac{4}{3k_i}}$

Name ____ Mailbox ____________

c) Assume we are using a proportional + integral controller, so $G_c(s) = \frac{k(s+z)}{s}$. Sketch the root locus assuming $z = 2$, including the direction travelled as the gain increases and the angle of the asymptotes and centroid of the asymptotes, if necessary.

$\sigma = \frac{[-1-4] - [-z]}{3-1} = \frac{-5+z}{2}$ For $z=2$ $\frac{-3}{2} = \sigma$ $\theta = \frac{180 + i360}{2} = \pm 90^0$



i) What do we expect the settling time to be as $k \to \infty$? $T_s = \frac{4}{3/2} = \boxed{\frac{8}{3} = T_s}$

For the remaining questions, assume we are using the original PI controller $G_c(s) = \frac{k(s+z)}{s}$, *where we* ***do not*** *assume that* $z = 2$.

ii) What are the conditions on z so that the system is stable? $\sigma = \frac{-5+z}{2} < 0$ $\boxed{z < 5}$

iii) Determine the position error constant K_p and the steady state error for a unit step in terms of k and z $\boxed{K_p = \infty}$ $\boxed{e_{ss} = 0}$

iv) Determine the velocity error constant K_v and the steady state error for a unit ramp in terms of k and z

$\boxed{K_v = \frac{3kz}{4}}$ $\boxed{e_{ss} = \frac{4}{3kz}}$