

**ECE-320: Linear Control Systems**  
**Homework 4**

Due: **Tuesday** March 27

**Exam #1**, Thursday March 29

1) Consider the plant

$$G_p(s) = \frac{\alpha_0}{s + \alpha_1} = \frac{3}{s + 0.5}$$

where 3 is the nominal value of  $\alpha_0$  and 0.5 is the nominal value of  $\alpha_1$ . In this problem we will investigate the sensitivity of closed loop systems with various types of controllers to these two parameters. We will assume we want the settling time of our system to be 0.5 seconds and the steady state error for a unit step input to be less than 0.1.

a) (*ITAE Model Matching*) Since this is a first order system, we will use the first order ITAE model,

$$G_o(s) = \frac{\omega_o}{s + \omega_o}$$

i) For what value of  $\omega_o$  will we meet the settling time requirements and the steady state error requirements?

ii) Determine the corresponding controller  $G_c(s)$ .

iii) Show that the closed loop transfer function (using the parameterized form of  $G_p(s)$  and the controller from part ii) is

$$G_o(s) = \frac{\frac{8}{3}\alpha_0(s+0.5)}{s(s+\alpha_1) + \frac{8}{3}\alpha_0(s+0.5)}$$

iv) Show that the sensitivity of  $G_o(s)$  to variations in  $\alpha_0$  is given by

$$S_{\alpha_0}^{G_o} = \frac{s}{s+8}$$

v) Show that the sensitivity of  $G_o(s)$  to variations in  $\alpha_1$  is given by

$$S_{\alpha_1}^{G_o} = \frac{-0.5s}{s^2 + 8.5s + 4}$$

b) (*Proportional Control*) Consider a proportional controller, with  $k_p = 2.5$ .

i) Show that the closed loop transfer function is

$$G_o(s) = \frac{2.5\alpha_0}{s + \alpha_1 + 2.5\alpha_0}$$

ii) Show that the sensitivity of  $G_o(s)$  to variations in  $\alpha_0$  is given by

$$S_{\alpha_0}^{G_o} = \frac{s + 0.5}{s + 8}$$

iii) Show that the sensitivity of  $G_o(s)$  to variations in  $\alpha_1$  is given by

$$S_{\alpha_1}^{G_o} = \frac{-0.5}{s + 8}$$

c) (*Proportional+Integral Control*) Consider a PI controller with  $k_p = 4$  and  $k_i = 40$ .

i) Show that the closed loop transfer function is

$$G_o(s) = \frac{4\alpha_0(s + 10)}{s(s + \alpha_1) + 4\alpha_0(s + 10)}$$

ii) Show that the sensitivity of  $G_o(s)$  to variations in  $\alpha_0$  is given by

$$S_{\alpha_0}^{G_o} = \frac{s(s + 0.5)}{s^2 + 12.5s + 120}$$

iii) Show that the sensitivity of  $G_o(s)$  to variations in  $\alpha_1$  is given by

$$S_{\alpha_1}^{G_o} = \frac{-0.5s}{s^2 + 12.5s + 120}$$

d) Using Matlab, simulate the unit step response of each type of controller. Plot all responses on one graph. Use different line types and a legend. Turn in your plot and code.

g) Using Matlab and subplot, plot the sensitivity to  $\alpha_0$  for each type of controller on one graph at the top of the page, and the sensitivity to  $\alpha_1$  on one graph on the bottom of the page. Be sure to use different line types and a legend. Turn in your plot and code. Only plot up to about 8 Hz (50 rad/sec) using a semilog scale with the sensitivity in dB (see below). **Do not make separate graphs for each system!**

In particular, these results should show you that the model matching method, which essentially tries and cancel the plant, are generally more sensitive to getting the plant parameters correct than the PI controller for low frequencies. However, for higher frequencies the methods are all about the same.

*Hint: If  $T(s) = \frac{2s}{s^2 + 2s + 10}$ , plot the magnitude of the frequency response using:*

```
T = tf([2 0],[1 2 10]);  
w = logspace(-1,1.7,1000);  
[M,P]= bode(T,w);  
Mdb = 20*log10(M(:));  
semilogx(w,Mdb); grid;  
xlabel('Frequency (rad/sec)');  
ylabel('dB');
```