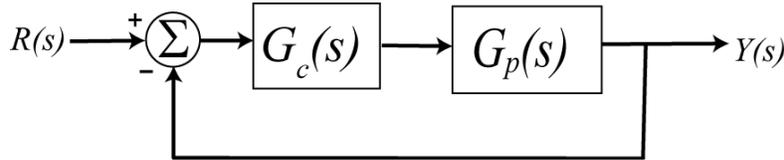# ECE-320: Linear Control Systems
Homework 2

Due: Thursday March 15 *at the beginning of class*

Reading: Chapters 4, 5, 6, 10

1) Consider the following simple feedback control block diagram. The plant is $G_p(s) = \dfrac{2}{s+4}$. The input is a unit step.

a) Determine the settling time and steady state error of the plant alone (assuming there is no feedback)

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the steady state error for a unit step is 0.1, and the corresponding settling time for the system.

d) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the settling time is 0.5 seconds, and the corresponding steady state error.

e) Assuming an integral controller, $G_c(s) = k_i / s$, determine closed loop transfer function, $G_0(s)$

f) Assuming an integral controller, $G_c(s) = k_i / s$, determine the value of k_i so the steady state error for a unit step is less than 0.1 and the system is stable.

Partial Answers: $T_s = 1$, $e_{ss} = 0.5$, $k_p = 18$, $k_p = 2$, $T_s = 0.1$, $e_{ss} = 0.5$, $k_i > 0$

2) Consider the following simple feedback control block diagram. The plant is $G_p(s) = \dfrac{4}{s+7}$.
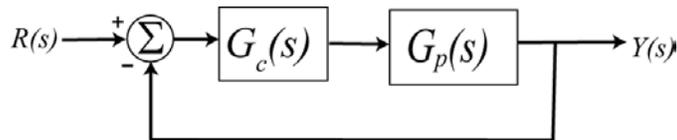
a) What is the bandwidth of the plant alone (assuming there is no feedback)

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the bandwidth of the closed loop system is 27 rad/sec.

d) Assuming the proportional controller from problem c, determine the settling time and the steady state error for a unit step.

Partial Answers: 7, 5, 7/27, 4/27

3) An ideal second order system has the transfer function $G_o(s)$. The system specifications for a step input are as follows:

a) Percent Overshoot $< 5\%$

b) Settling Time < 4 seconds (2% criteria)

c) Peak Time < 1 second

Sketch, in the complex plane, the permissible area for the poles of $G_o(s)$ in order to achieve the desired response.

4) For systems with the following transfer functions:

$$H_a(s) = \frac{1}{s+2} \quad H_b(s) = \frac{s+6}{(s+2)(s+3)}$$

a) Determine the unit step and unit ramp response for each system using Laplace transforms. Your answer should be time domain functions $y_a(t)$ and $y_b(t)$.

b) From these time domain functions, determine the steady state errors for a unit step and unit ramp input.

c) Using the equations derived in class (and in the notes), determine the steady state errors for a unit step and a unit ramp input directly from the transfer functions.

The following Matlab code can be used to estimate the step and ramp response for 5 seconds for transfer function $H_b(s)$.

```
H = tf([1 6],[1 5 6]);                  % enter the transfer function
t = [0:0.01:5];                         % t goes from 0 to 5 by increments of 0.01
ustep = ones(1,length(t));              % the step input is all ones, u(t) = 1;
uramp = t;                              % the ramp input is has the input u(t) = t;
ystep = lsim(H,ustep,t);                % find the step response
yramp = lsim(H,uramp,t);                % find the ramp response
figure;                                 % make a new figure
orient tall                             % or orient landscape, use more of the page
subplot(2,1,1);                         % put two graphs on one piece of paper
plot(t,ustep,'.-',t,ystep,'-');         % plot input/output with different line types
grid;                                   % put a grid on the graph
legend('Step Input','Step Response',4); % put a legend on the graph
subplot(2,1,2);                         % second of two graphs on one piece of paper
plot(t,uramp,'.-',t,yramp,'-');         % plot input/output with different line types
grid;                                   % put a grid on the graph
legend('Ramp Input','Ramp Response',4); % put a legend on the graph
```

d) Plot the step and ramp response for both systems (a and b) and indicate the steady state errors on the graph. Draw on the graph to show you know what the steady state errors are.

Ans. Steady state errors for a unit step input: 0.5,0; for a unit ramp input : infinity and 0.666

5) For the following systems

a) Determine the system type (0, 1, 2, ...)

b) If the system is type 0 assume $G_{pf} = 1$ and determine the position error constant K_p and the steady state error for a unit step input. Then determine the value of G_{pf} to make this error zero. If the system is type 1, assume $G_{pf} = 1$ and determine the steady state error for a unit step, the velocity error constant K_v, and the steady state error for a unit ramp. Is there any constant value of G_{pf} that can change the steady state error for a ramp?

Ans. (steady state errors) $-\dfrac{3}{2}, \dfrac{3}{13}, -\dfrac{3}{5}, \dfrac{1}{2}$; (prefilers) $\dfrac{2}{5}, \dfrac{13}{10}, G_{pf}$ has no effect
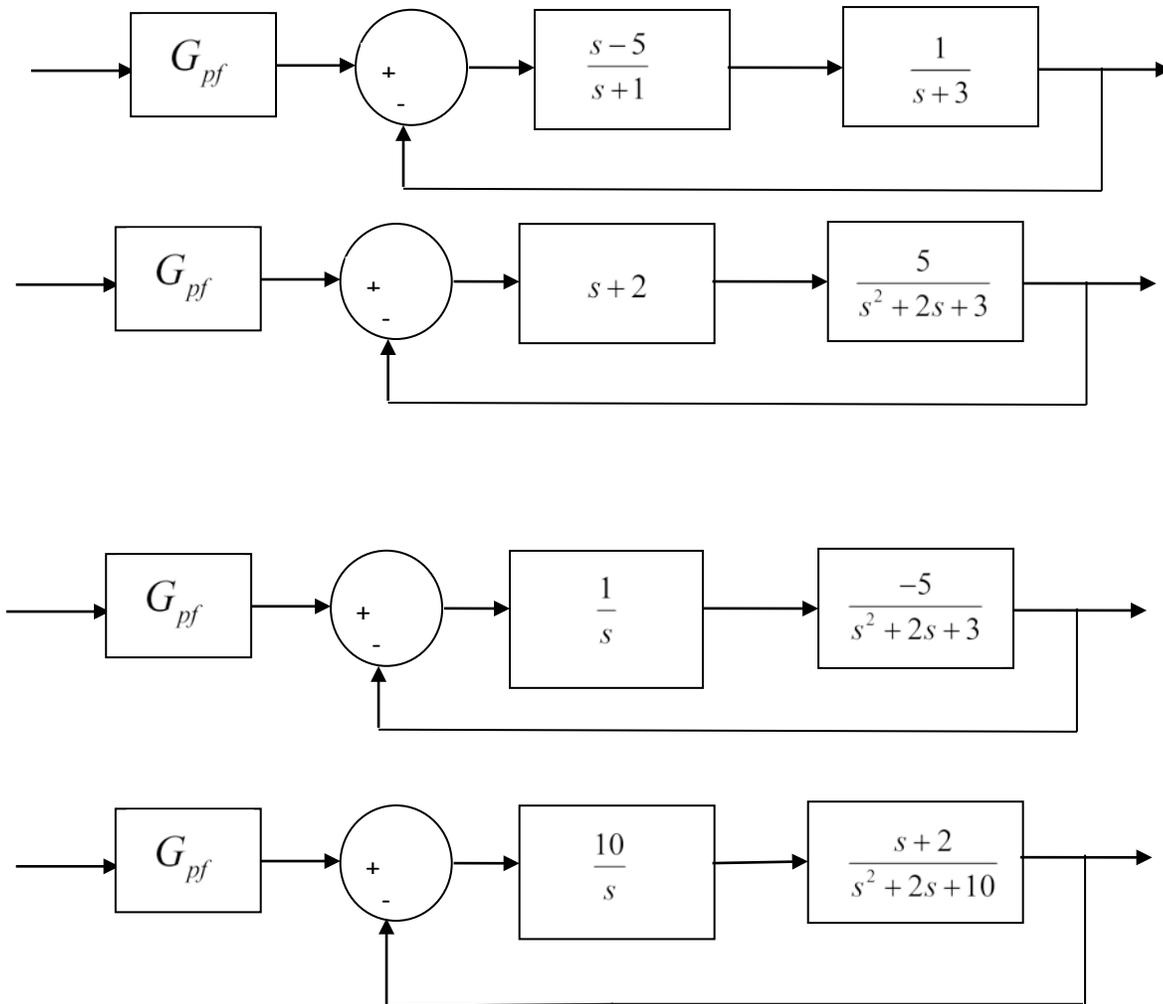
6) Consider the following control system.
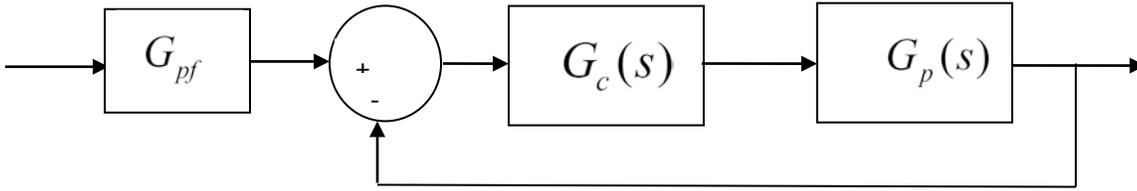
We can compute the position error constant K_p as $K_p = G_c(0)G_p(0)$

a) Determine an expression for the closed loop transfer function (from input to output) $G_o(s)$ in terms of G_{pf}, $G_c(s)$, and $G_p(s)$.

b) For a steady state error of zero for a step input we want $G_o(0) = 1$. Use this information to show that we can determine the prefilter gain to be
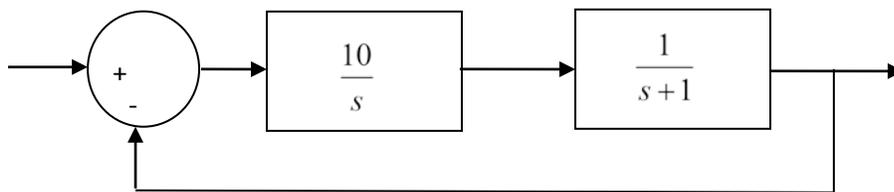
$$G_{pf} = 1 + \frac{1}{K_p}$$

c) We can find the steady state error for a unit step input as $e_{ss} = \frac{1}{1 + K_p}$. Using this, show that we can

determine the prefilter gain to be

$$G_{pf} = \frac{1}{1 - e_{ss}}$$

Note that if $K_p = \infty$ (or equivalently $e_{ss} = 0$), we get $G_{pf} = 1$)

7) Consider the following control system:

a) If the input to the system is $r(t) = 8u(t)$, what is the steady state output?

b) If the input to the system is $r(t) = 8\sin(3t)u(t)$, what is the output in steady state?
What is the time lag between the input signal and the output signal?

Hint: you can write $\omega t - \theta = \omega(t - t_d)$ if θ is measured in radians.
Answers: $y(t) = 8$, $y(t) = 8\sqrt{10}\sin(3t - 71.57°), t_d = 0.416\,\text{sec}$

4