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ECE-320 Linear Control Systems

Spring 2012, Exam 2

No calculators or computers allowed.

Problem 1-15	**______/30**
Problem 16	**______/20**
Problem 17	**______/25**
Problem 18	**______/25**
Total	**______/100**

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1) Which of the following transfer functions represents an (asymptotically) **unstable** systems? (circle all of them)

a) $G(z)=\frac{z}{z+0.6}$ b) $G(z)=\frac{z}{z-0.8}$ c) $G(z)=\frac{z}{z+0.2}$ d) $G(z)=\frac{z}{z-1.2}$

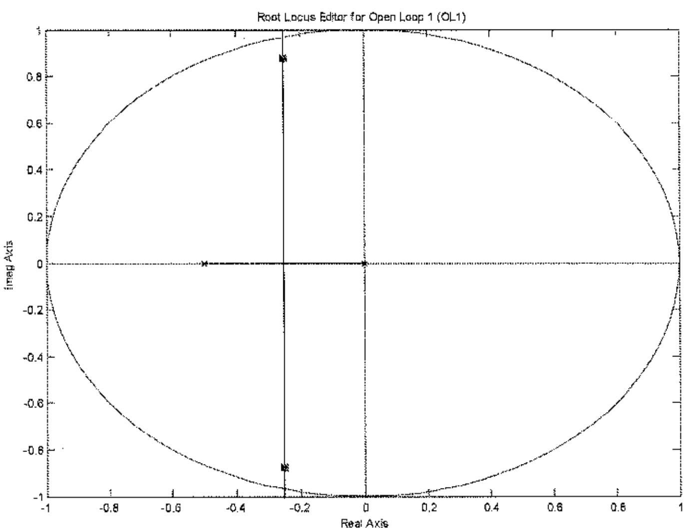
2) Which of the following systems will have a smaller settling time?

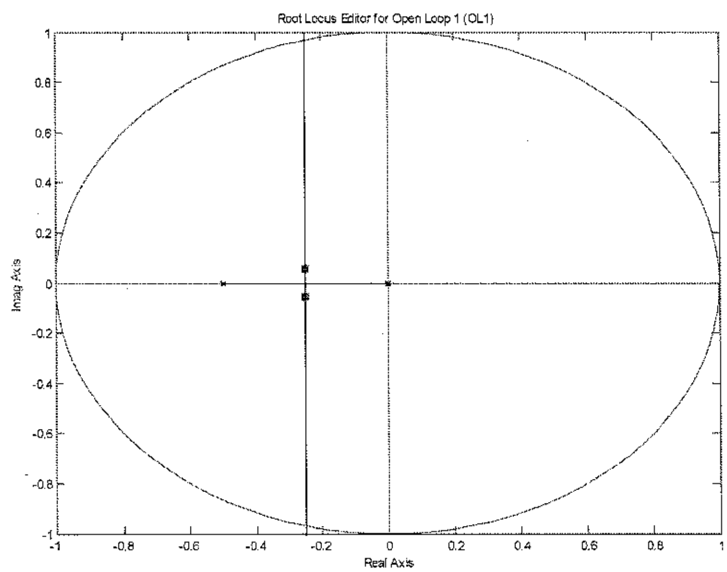
a) $G(z)=\frac{z}{z-0.9}$ b) $G(z)=\frac{z}{z-0.7}$ c) $G(z)=\frac{z}{z+0.5}$ d) $G(z)=\frac{z}{z+0.2}$

3) Which of the following systems will have a smaller **settling time?**

a) $G(z)=\frac{1}{(z-0.2+j0.5)(z-0.2-j0.5)}$ b) $G(z)=\frac{1}{(z-0.1+j0.5)(z-0.1-j0.5)}$ c) $G(z)=\frac{1}{(z+0.5)}$

Problems 4 and 5 refer to the following two root locus plot for a discrete-time system





4) For which system is the settling time likely to be smallest?

a) The system on the left b) the system on the right c) the settling time will be the same

5) Is this a type 1 system?

a) yes b) no c) not enough information

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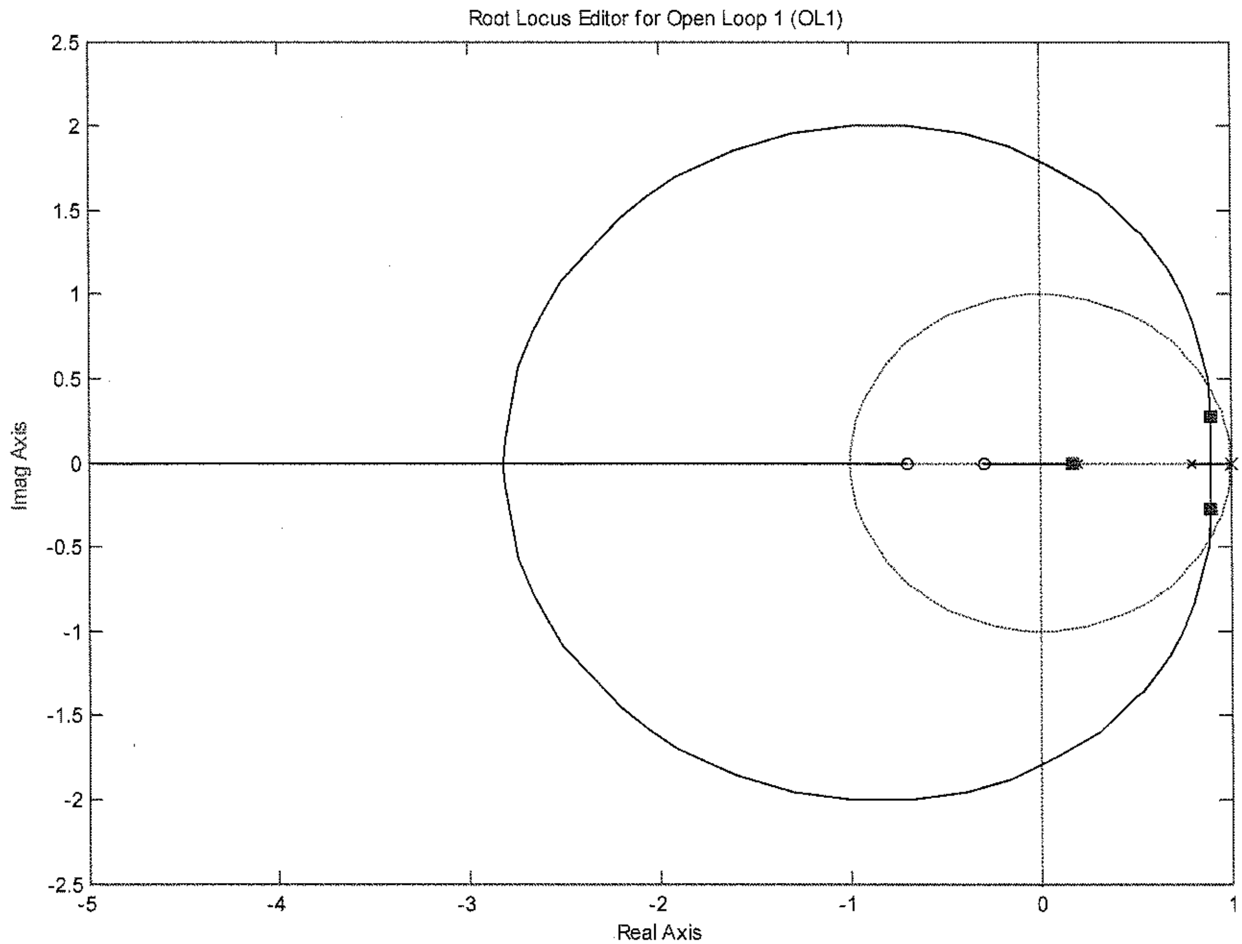
Problems 6-8 refer to the following root locus plot for a discrete-time system



6) With the closed loop pole locations shown in the figure, is the closed loop system stable?

a) yes b) no c) not enough information

7) Is there any value of k for which the closed loop system is stable?

a) yes b) no c) not enough information

8) Is this a type one system?

a) yes b) no c) not enough information

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9) Is the following system ***controllable***? $G(s) = \dfrac{G_{pf}}{(s - k_1 k_2)^2}$

a) Yes b) No c) impossible to determine

10) Is the following system ***controllable***? $G(s) = \dfrac{8G_{pf}}{s^2 + 12s + (k_1 + k_2 + 20)}$

a) Yes b) No c) impossible to determine

11) Is the following system **controllable**? $G(s) = \dfrac{G_{pf}}{s^2 + (k_2 + k_1 - 1)s + (k_2 + 2)}$

a) Yes b) No c) impossible to determine

12) Consider a plant that is unstable but is a controllable system. Is it possible to use state variable feedback to make this system stable?

a) Yes b) No

13) Is it possible for a system with state variable feedback to change the zeros of the plant (other than by pole-zero cancellation) ?

a) Yes b) No

14) Is it possible for a system with state variable feedback to introduce zeros into the closed loop system?

a) Yes b) No

15) If a plant has *n* poles, then a system with state variable feedback with no pole-zero cancellations will have

a) more than *n* poles b) less than *n* poles c) n poles d) it is not possible to tell

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16) For the state variable model

$$\dot{q} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} q + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} q + [0] u$$

Determine the closed loop transfer function with ***state variable feedback***.

$$\tilde{A} = A - BK = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} [K_1 \ K_2] = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1-K_1 & 1-K_2 \end{bmatrix}$$

$$sI - \tilde{A} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1-K_1 & 1-K_2 \end{bmatrix} = \begin{bmatrix} s & -1 \\ K_1-1 & s+K_2-1 \end{bmatrix}$$

$$(sI - \tilde{A})^{-1} = \frac{1}{s(s+K_2-1)+(K_1-1)} \begin{bmatrix} s+K_2-1 & 1 \\ 1-K_1 & s \end{bmatrix}$$

$$G_0(s) = C(sI-\tilde{A})^{-1}\tilde{B} = \frac{[1 \ 0]}{\Delta(s)} \begin{bmatrix} s+K_2-1 & 1 \\ 1-K_1 & s \end{bmatrix} \begin{bmatrix} 0 \\ G_{pf} \end{bmatrix}$$

$$= \frac{G_{pf}}{\Delta(s)}$$

$$= \boxed{\frac{G_{pf}}{s^2 + (K_2-1)s + (K_1-1)} = G_0(s)}$$

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17) For impulse response $h(n)=\left(\frac{1}{2}\right)^{n+1}u(n-1)$ and input $x(n)=\left(\frac{1}{4}\right)^{n-1}u(n-2)$, determine the system output by evaluating the convolution sum $y(n)=\sum_{k=-\infty}^{\infty}h(n-k)x(k)$

Note: you do not have to simplify your answer, but you must remove all sums and include a unit step function of some sort.

$$y(n)=\sum_{k=-\infty}^{\infty}\left(\frac{1}{2}\right)^{n-k+1}u(n-k-1)\left(\frac{1}{4}\right)^{k-1}u(k-2)$$

$u(n-k-1)=1$ for $n-k-1\geq 0$
$n-1\geq k$

$u(k-2)=1$ for $k-2\geq 0$
$k\geq 2$

$n-1\geq k\geq 2$

$\boxed{n-3\geq 0}$

$$=\sum_{k=2}^{n-1}\left(\frac{1}{2}\right)^{n-k+1}\left(\frac{1}{4}\right)^{k-1}$$

$$=\left(\frac{1}{4}\right)^{-1}\left(\frac{1}{2}\right)^{n+1}\sum_{k=2}^{n-1}\left(\frac{1}{2}\right)^{-k}\left(\frac{1}{4}\right)^{k}$$

$$=4\left(\frac{1}{2}\right)^{n+1}\sum_{k=2}^{n-1}\left(\frac{1}{2}\right)^{k}$$

let $l=k-2$ $\quad l+2=k$

$$=4\left(\frac{1}{2}\right)^{n+1}\sum_{l=0}^{n-3}\left(\frac{1}{2}\right)^{l+2}=4\left(\frac{1}{2}\right)^{n-1}\left(\frac{1}{2}\right)^{2}\sum_{l=0}^{n-3}\left(\frac{1}{2}\right)^{l}$$

$$=\left(\frac{1}{2}\right)^{n+1}\sum_{l=0}^{n-3}\left(\frac{1}{2}\right)^{l}=\left(\frac{1}{2}\right)^{n+1}\left[\frac{1-\left(\frac{1}{2}\right)^{n-2}}{1-1/2}\right]u(n-3)$$

$$\boxed{y(n)=\left(\frac{1}{2}\right)^{n}\left[1-\left(\frac{1}{2}\right)^{n-2}\right]u(n-3)}$$

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18) For impulse response $h(n)=\left(\frac{1}{2}\right)^{n-3}u(n-1)$ and input $x(n)=\left(\frac{1}{3}\right)^{n+1}u(n-1)$,

a) determine the z-transform of $h(n)$, $H(z)$

b) determine the z-transform of $x(n)$, $X(z)$

c) determine $y(n)$

Hint: Assume $Y(z)=z^{-1}G(z)$, determine $g(n)$ and then $y(n)$

(a) $h(n)=\left(\frac{1}{2}\right)^{n-3}u(n-1)=\left(\frac{1}{2}\right)^{n-1}\left(\frac{1}{2}\right)^{-2}u(n-1)=4z^{-1}\frac{z}{z-1/2}$

$$\boxed{H(z)=\frac{4}{z-1/2}}$$

(b) $x(n)=\left(\frac{1}{3}\right)^{n+1}u(n-1)=\left(\frac{1}{3}\right)^{n-1}\left(\frac{1}{3}\right)^{2}u(n-1)=\frac{1}{9}z^{-1}\frac{z}{z-1/3}$

$$\boxed{X(z)=\frac{1/9}{z-1/3}}$$

(c) $Y(z)=\dfrac{4/9}{(z-1/2)(z-1/3)}$ $\qquad \dfrac{G(z)}{z}=\dfrac{4/9}{(z-1/2)(z-1/3)}=\dfrac{A}{z-1/2}+\dfrac{B}{z-1/3}$

$Y(z)=z^{-1}G(z)$ $\qquad A=\dfrac{4/9}{1/2-1/3}=\dfrac{4/9}{1/6}=\dfrac{24}{9}=\dfrac{8}{3}$

$B=\dfrac{4/9}{1/3-1/2}=\dfrac{4/9}{-1/6}=-8/3$

$G(z)=\frac{8}{3}\frac{z}{z-1/2}-\frac{8}{3}\frac{z}{z-1/3}$ $\qquad g(n)=\frac{8}{3}\left(\frac{1}{2}\right)^{n}u(n)-\frac{8}{3}\left(\frac{1}{3}\right)^{n}u(n)$

$$y(n)=g(n-1)=\boxed{\frac{8}{3}\left(\frac{1}{2}\right)^{n-1}u(n-1)-\frac{8}{3}\left(\frac{1}{3}\right)^{n-1}u(n-1)=y(n)}$$