ECE-320: Linear Control Systems
Homework 8

Due: Tuesday May 3 at the beginning of class
Exam 2, Thursday May 5

1) For the z-transform

$$X(z)=\frac{3}{z-2}$$

a) Show that, by multiplying and dividing by z and then using partial fractions, the corresponding discrete-time sequence is

$$x(k)=-\frac{3}{2}\delta(k)+\frac{3}{2}2^{k}u(k)$$

b) By starting with the z-transform

$$Y(z)=\frac{3z}{z-2}$$

and the z-transform properties, show that

$$x(k)=3\,2^{k-1}u(k-1)$$

2) For impulse response $h(n)=\left(\frac{1}{2}\right)^{n-3}u(n-1)$ and input $x(n)=\left(\frac{1}{4}\right)^{n+1}u(n-2)$, use z-transforms of the input and impulse response to show the system output is $y(n)=\left[\left(\frac{1}{2}\right)^{n}-\left(\frac{1}{4}\right)^{n-1}\right]u(n-3)$

3) Consider the following difference equation

$$x(k+2)-4x(k+1)+4x(k)=f(k)$$

Assume all initial conditions are zero.

a) Determine the *impulse response* of the system, i.e., the response $x(k)$ when $f(k)=\delta(k)$.

b) Determine $x(0), x(1), x(2), x(3),$ and $x(4)$ from your answer to **a.** Compare this answer with the known values of $x(0)$ and $x(1)$. Using the difference equation compute $x(3)$ and $x(4)$ and compare these values to those in your solution.

c) Determine the *step response* of the system, i.e., the response $x(k)$ when $f(k)=u(k)$

d) Determine $x(0), x(1), x(2), x(3),$ and $x(4)$ from your answer to **c.** Compare this answer with the known values of $x(0)$ and $x(1)$. Using the difference equation compute $x(3)$ and $x(4)$ and compare these values to those in your solution.

4) Consider the difference equation

$$x(k+2)-5x(k+1)+6x(k)=f(k)$$

where $f(k)=u(k)$, a unit step. Assume $x(0)=1$ and $x(1)=1$.

a) Show that the *Zero Input Response* (ZIR) is given by $x_{ZIR}(n)=\left[2^{n+1}-3^{n}\right]u(n)$

b) Show that the *Zero State Response* (ZSR) is given by $x_{ZSR}(n)=\left[\frac{1}{2}-2^{n}+\frac{1}{2}3^{n}\right]u(n)$

c) Find the transfer function and the impulse response.

5) For each of the following transfer functions, determine if the system is asymptotically stable, and if so, the estimated 2% settling time for the system. Assume the sampling interval is $T=0.1$ s

a) $H(z)=\frac{z+2}{(z-0.1)(z+0.2)}$ **d)** $H(z)=\frac{1}{z^{2}+z+0.5}$

b) $H(z)=\frac{1}{(z-2)(z+0.5)}$ **e)** $H(z)=\frac{z-1}{z^{2}+0.5z+0.2}$

c) $H(z)=\frac{1}{(z-0.1)(z-0.5)}$ **f)** $H(z)=\frac{1}{z^{2}+z+5}$

Scambled Answers: 0.497, 0.58 , 1.15, 0.24, two unstable systems

6) Consider the continuous-time plant with transfer function

$$G_p(s)=\frac{1}{(s+1)(s+2)}$$

We want to determine the discrete-time equivalent to this plant, $G_p(z)$, by assuming a zero order hold is placed before the continuous-time plant to convert the discrete-time control signal to a continuous time control signal. Show that if we assume a sampling interval of T, the equivalent discrete-time plant is

$$G_p(z)=\frac{z(0.5-e^{-T}+0.5e^{-2T})+(0.5e^{-T}-e^{-2T}+0.5e^{-3T})}{(z-e^{-T})(z-e^{-2T})}$$

Note that we have poles were we expect them to be, but we have introduced a zero in going from the continuous time system to the discrete-time system.