ECE-320: Linear Control Systems
Homework 2

Due: Thursday March 17 *at the beginning of class*

Reading: Chapters 4, 5, 6, 8, 9

1) For systems with the following transfer functions:

$$H_a(s) = \frac{1}{s+2}$$

$$H_b(s) = \frac{s+6}{(s+2)(s+3)}$$

a) Determine the unit step and unit ramp response for each system using Laplace transforms. Your answer should be time domain functions $y_a(t)$ and $y_b(t)$.

b) From these time domain functions, determine the steady state errors for a unit step and unit ramp input.

c) Using the equations derived in class (and in the notes), determine the steady state errors for a unit step and a unit ramp input directly from the transfer functions.

The following Matlab code can be used to estimate the step and ramp response for 5 seconds for transfer function $H_b(s)$.

```
H = tf([1 6],[1 5 6]);                  % enter the transfer function
t = [0:0.01:5];                         % t goes from 0 to 5 by increments of 0.01
ustep = ones(1,length(t));              % the step input is all ones, u(t) = 1;
uramp = t;                              % the ramp input is has the input u(t) = t;
ystep = lsim(H,ustep,t);                % find the step response
yramp = lsim(H,uramp,t);                % find the ramp response
figure;                                 % make a new figure
orient tall                             % or orient landscape, use more of the page
subplot(2,1,1);                         % put two graphs on one piece of paper
plot(t,ustep,'.-',t,ystep,'-');         % plot input/output with different line types
grid;                                   % put a grid on the graph
legend('Step Input','Step Response',4); % put a legend on the graph
subplot(2,1,2);                         % second of two graphs on one piece of paper
plot(t,uramp,'.-',t,yramp,'-');         % plot input/output with different line types
grid;                                   % put a grid on the graph
legend('Ramp Input','Ramp Response',4); % put a legend on the graph
```

d) Plot the step and ramp response for both systems (a and b) and indicate the steady state errors on the graph. Draw on the graph to show you know what the steady state errors are.

Ans. Steady state errors for a unit step input: 0.5,0; for a unit ramp input : infinity and 0.666

2) An ideal second order system has the transfer function $G_o(s)$. The system specifications for a step input are as follows:

a) Percent Overshoot < 5%
b) Settling Time < 4 seconds (2% criteria)
c) Peak Time < 1 second

Show the permissible area for the poles of $G_o(s)$ in order to achieve the desired response.

3) For systems A and B, with step responses shown in Figure 1, estimate

- the percent overshoot
- the settling time
- the steady state error for the step input shown

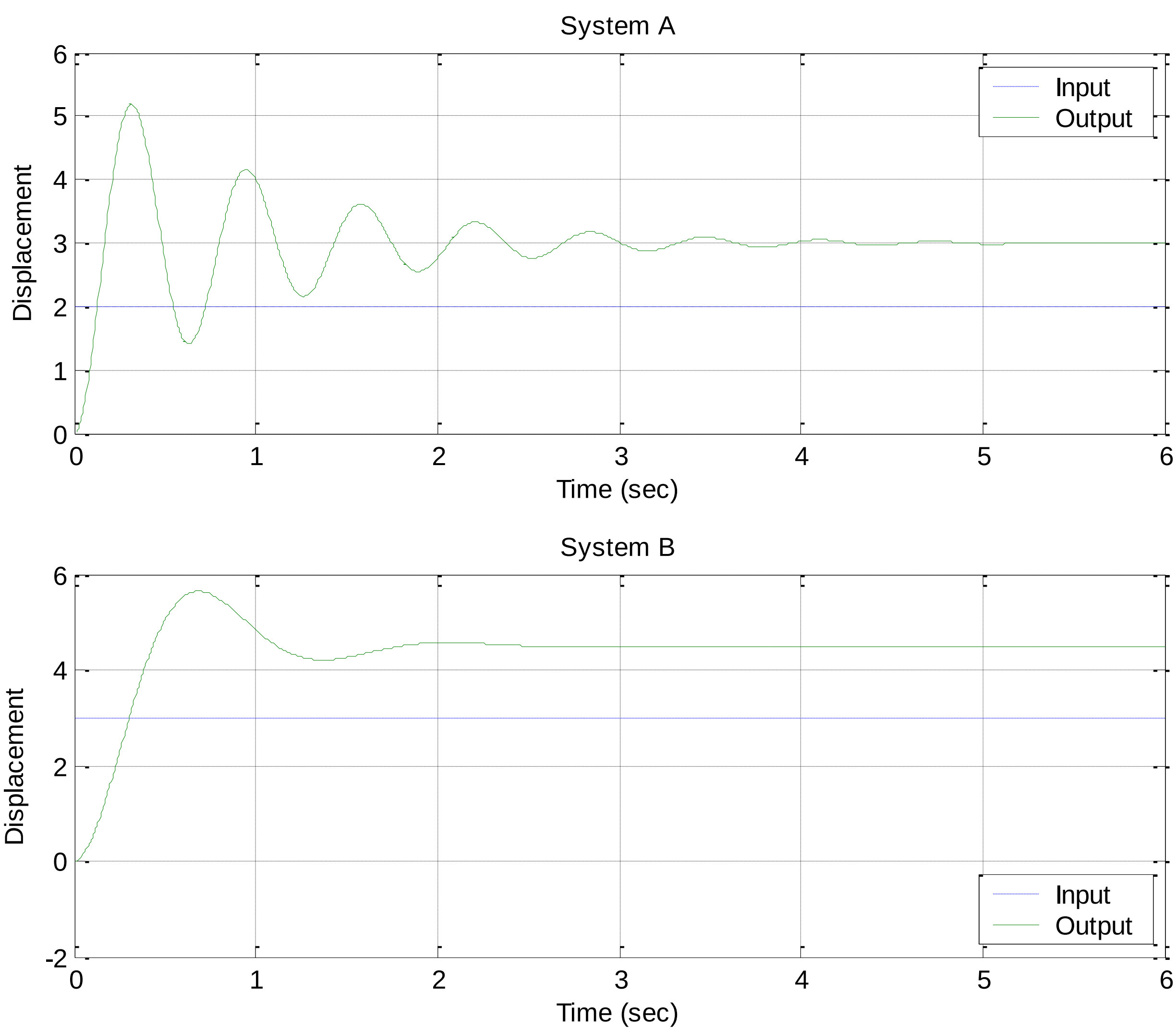


Figure 1. Step responses of system A and system B.

4) For systems C and D, with ramp responses shown in Figure 2, determine the steady state error for the ramp input shown

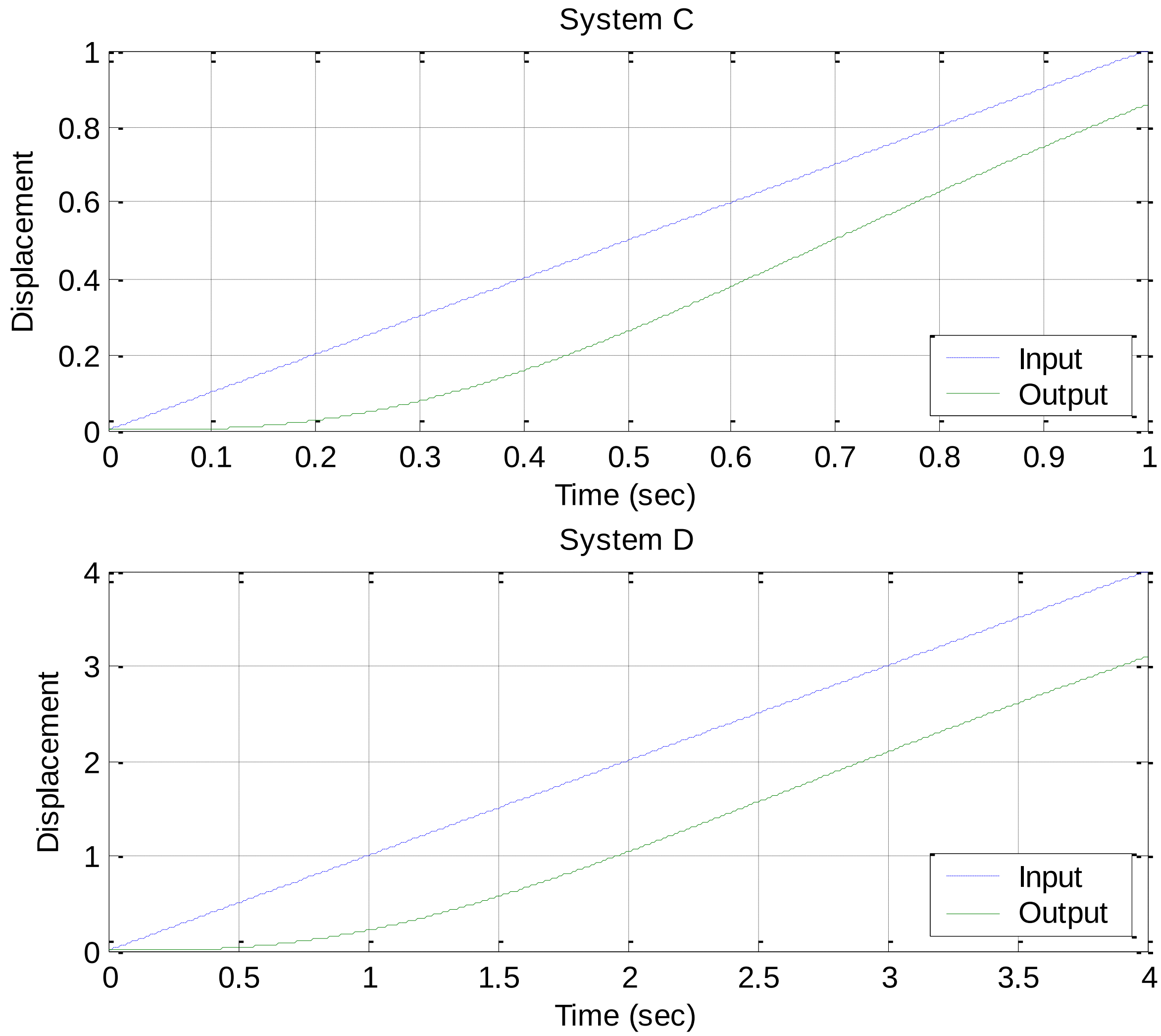
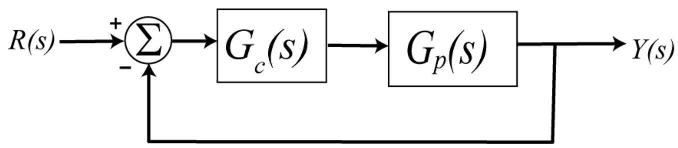


Figure 2. Ramp responses for systems C and D.

Answers for 3 and 4 (in no particular order, your approximations should be close, but they probably won't match.) 73%, 25%, 2, 3.5, -1, -1.2,1,0.16

5) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{3}{s+2}$ and the controller is an integral controller $G_c(s) = \dfrac{k_i}{s}$



a) Determine the closed loop transfer function, $G_0(s)$

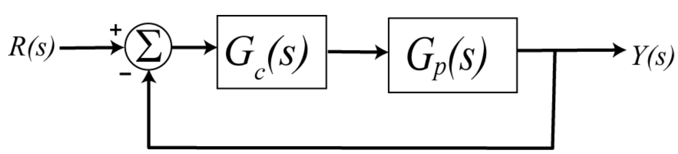
b) Determine the poles of value of $G_0(s)$ and show they are only real if $0 < k_i < \dfrac{1}{3}$. Note that the best possible setting time is 4 seconds. Use $k_i = \dfrac{1}{3}$ for the remainder of this problem.

c) If the input to the system is a unit step, determine the output of the system.

d) The steady state error is the difference between the input and the output as $t \to \infty$. Determine the steady state error for this system.

Partial Answer: $y(t) = \left[1 - e^{-t} - te^{-t}\right]u(t), \quad e_{ss} = 0$

6) Consider the following simple feedback control block diagram. The plant is $G_p(s) = \dfrac{2}{s+4}$. The input is a unit step.



a) Determine the settling time and steady state error of the plant alone (assuming there is no feedback)

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the steady state error for a unit step is 0.1, and the corresponding settling time for the system.

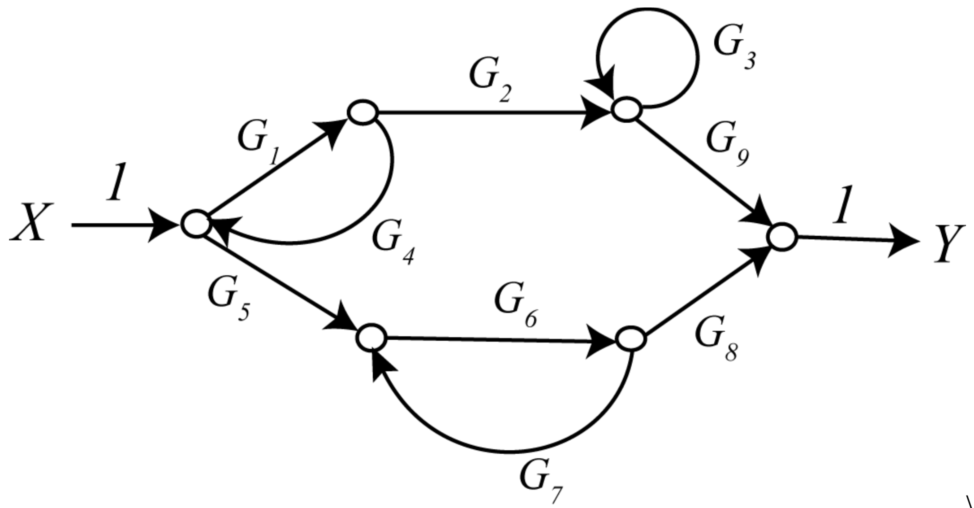
d) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the settling time is 0.5 seconds, and the corresponding steady state error.

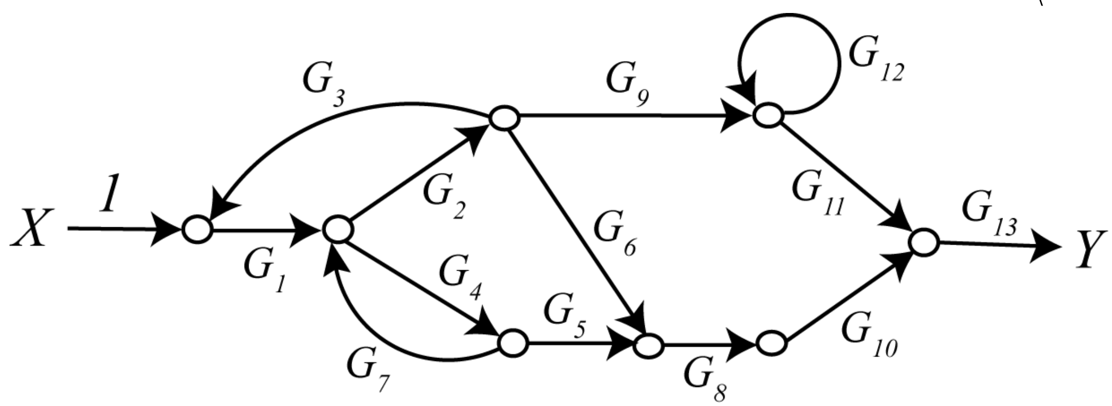
e) Assuming an integral controller, $G_c(s) = k_i / s$, determine closed loop transfer function, $G_0(s)$

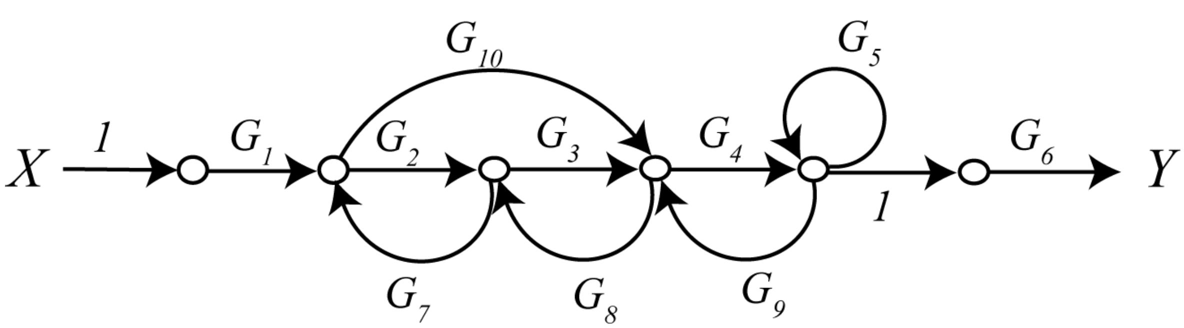
f) Assuming an integral controller, $G_c(s) = k_i / s$, determine the value of k_i so the steady state error for a unit step is less than 0.1 and the system is stable.

Partial Answers: $T_s = 1, e_{ss} = 0.5, k_p = 18, k_p = 2, T_s = 0.1, e_{ss} = 0.5, k_i > 0$

7) For the following signal flow diagrams determine the system transfer function. *You may use Maple.*







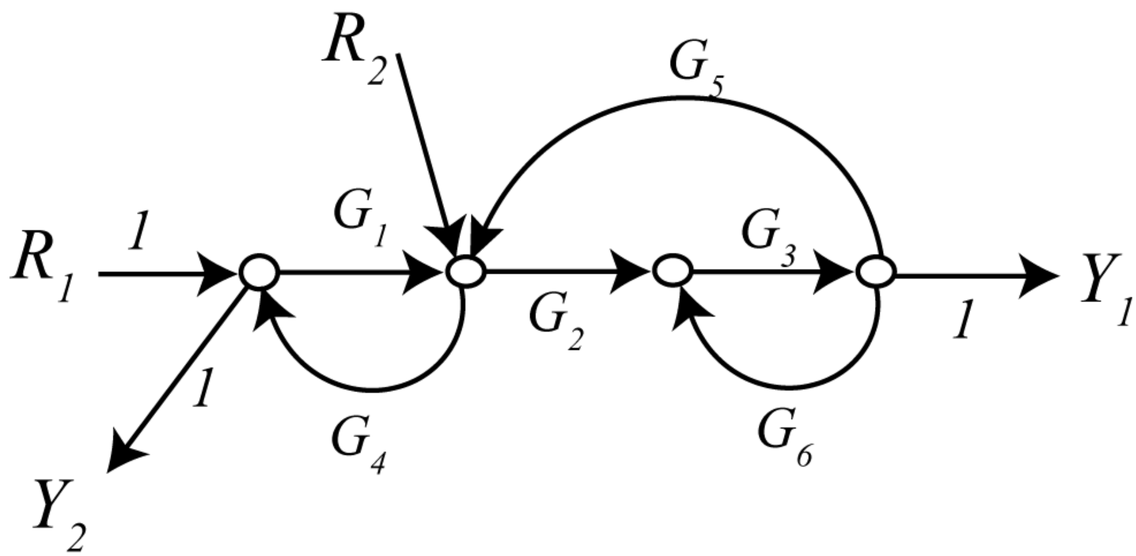
Answers:

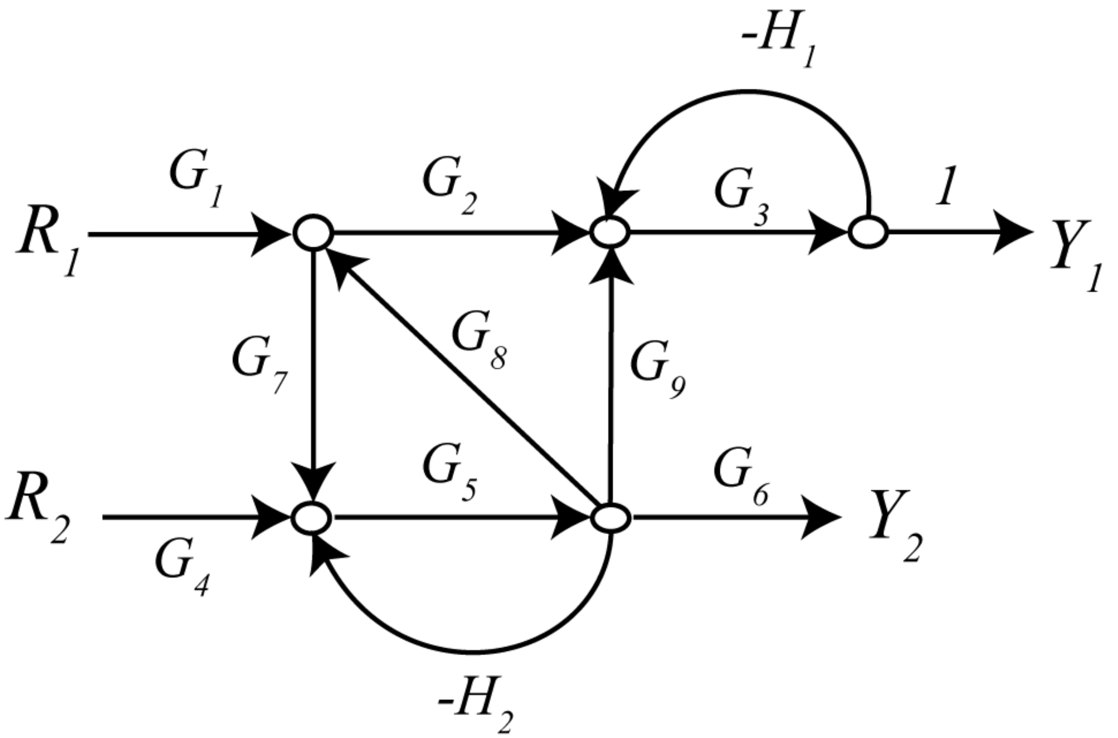
$$\frac{Y}{X}=\frac{G_1G_2G_9(1-G_6G_7)+G_5G_6G_8(1-G_3)}{1-G_1G_4-G_6G_7-G_3+G_1G_4G_6G_7+G_1G_3G_4+G_3G_6G_7-G_1G_3G_4G_6G_7}$$

$$\frac{Y}{X}=\frac{G_1G_2G_3G_4G_6+G_1G_4G_6G_{10}}{1-G_2G_7-G_3G_8-G_4G_9-G_7G_8G_{10}-G_5+G_2G_4G_7G_9+G_2G_5G_7+G_3G_5G_8+G_5G_7G_8G_{10}}$$

$$\frac{Y}{X}=\frac{G_1G_2G_9G_{11}G_{13}+G_1G_2G_6G_8G_{10}G_{13}(1-G_{12})+G_1G_4G_5G_8G_{10}G_{13}(1-G_{12})}{1-G_1G_2G_3-G_{12}-G_4G_7+G_1G_2G_3G_{12}+G_4G_7G_{12}}$$

8) We can also use Mason's rule for systems with multiple inputs and multiple outputs. To do this, we use superposition and assume only one input is non-zero at a time. The only things that changes are the paths (which depends on the input and the output) and the cofactors (which depends on the path). The determinant does not change, since it is intrinsic to the system. For the following systems, determine the transfer functions from all inputs (R) to all outputs (Y). *You may use Maple.*





Answers:

$$\frac{Y1}{R1}=\frac{G_1G_2G_3}{1-G_1G_4-G_2G_3G_5-G_3G_6+G_1G_3G_4G_6},$$
$$\frac{Y2}{R1}=\frac{1-G_2G_3G_5-G_3G_6}{1-G_1G_4-G_2G_3G_5-G_3G_6+G_1G_3G_4G_6}$$
$$\frac{Y1}{R2}=\frac{G_2G_3}{1-G_1G_4-G_2G_3G_5-G_3G_6+G_1G_3G_4G_6},$$
$$\frac{Y2}{R2}=\frac{G_4(1-G_3G_6)}{1-G_1G_4-G_2G_3G_5-G_3G_6+G_1G_3G_4G_6}$$

$$\frac{Y1}{R1}=\frac{G_1G_2G_3(1+G_5H_2)+G_1G_3G_5G_7G_9}{1+G_5H_2-G_5G_7G_8+H_1G_3+G_3G_5H_1H_2-H_1G_3G_5G_7G_8},$$
$$\frac{Y2}{R1}=\frac{G_1G_5G_6G_7(1+G_3H_1)}{1+G_5H_2-G_5G_7G_8+H_1G_3+G_3G_5H_1H_2-H_1G_3G_5G_7G_8}$$
$$\frac{Y1}{R2}=\frac{G_3G_4G_5G_9+G_2G_3G_4G_5G_8}{1+G_5H_2-G_5G_7G_8+H_1G_3+G_3G_5H_1H_2-H_1G_3G_5G_7G_8},$$
$$\frac{Y2}{R2}=\frac{G_4G_5G_6(1+H_1G_3)}{1+G_5H_2-G_5G_7G_8+H_1G_3+G_3G_5H_1H_2-H_1G_3G_5G_7G_8}$$

9) For the block diagram shown below, determine a corresponding signal flow diagram and show that the closed loop transfer function is

$$H_{system}=\frac{G_1G_2G_3+G_4(1-G_1G_2H_1+G_2H_1+G_2G_3H_2)}{1-G_1G_2H_1+G_2H_1+G_2G_3H_2}$$