

**ECE-320: Linear Control Systems**  
**Homework 1**

Due: Thursday March 10 *at the beginning of class*

**Reading: Chapters 1-6, 8.1, 9**

1) Determine the impulse response of the following using partial fractions as necessary. You are expected to be able to do all of these with the Laplace transform Table in the notes, and the properties above. You may *check* your answers with Maple.

a)  $H(s) = \frac{e^{-s}}{s^2 + 3}$     b)  $H(s) = \frac{s+3}{(s+1)(s+2)}$     c)  $H(s) = \frac{s}{(s+1)^2(s+3)}$   
d)  $H(s) = \frac{s+1}{s^2(s+2)}$     e)  $H(s) = \frac{e^{-2s}(s+1)}{s(s+2)}$     f)  $H(s) = \frac{1}{(s+1)(s+2)(s+3)(s+4)}$

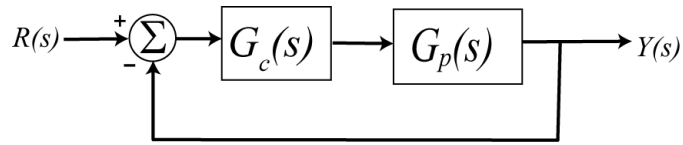
2) For the following transfer functions

$$H(s) = \frac{2}{s^2 + 2s + 2} \quad H(s) = \frac{3}{s^2 + 4s + 6}$$
$$H(s) = \frac{5}{s^2 + 6s + 10} \quad H(s) = \frac{4}{s^2 - 4s + 7}$$

By computing the inverse Laplace transform show that the step responses are given by

$$y(t) = \left[ 1 - e^{-t} \cos(t) - e^{-t} \sin(t) \right] u(t)$$
$$y(t) = \left[ \frac{1}{2} - \frac{1}{\sqrt{2}} e^{-2t} \sin(\sqrt{2}t) - \frac{1}{2} e^{-2t} \cos(\sqrt{2}t) \right] u(t)$$
$$y(t) = \left[ \frac{1}{2} - \frac{3}{2} e^{-3t} \sin(t) - \frac{1}{2} e^{-3t} \cos(t) \right] u(t)$$
$$y(t) = \left[ \frac{4}{7} + \frac{8\sqrt{3}}{21} e^{2t} \sin(\sqrt{3}t) - \frac{4}{7} e^{2t} \cos(\sqrt{3}t) \right] u(t)$$

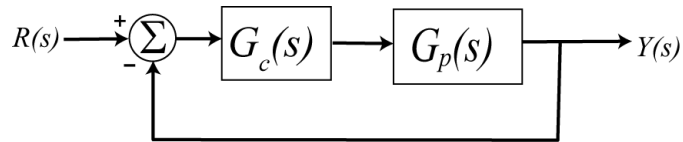
3) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function  $G_p(s) = \frac{3}{s+2}$  and the controller is a proportional controller, so  $G_c(s) = k_p$ .



- Determine the settling time of the plant alone (assuming there is no feedback)
- Determine the closed loop transfer function,  $G_o(s)$
- Determine the value of  $k_p$  so the settling time of the system is 0.5 seconds.
- If the input to the system is a unit step, determine the output of the system.
- The steady state error is the difference between the input and the output as  $t \rightarrow \infty$ . Determine the steady state error for this system.

*Partial Answer:*  $y(t) = \frac{3}{4} [1 - e^{-8t}] u(t)$ ,  $e_{ss} = 0.25$

4) **(Model Matching)** Consider the following closed loop system, with plant (the system we are trying to control)  $G_p(s)$  and controller  $G_c(s)$ .



The transfer function for this closed loop system is

$$G_o(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)}$$

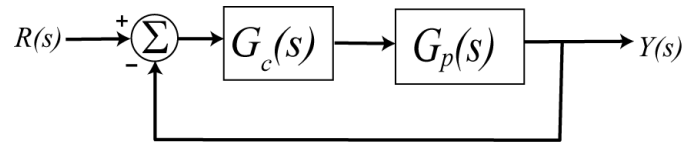
One way to choose the controller is to try and make your closed loop system match a transfer function that you choose (hence the name model matching). Let's assume that our **desired** closed loop transfer function  $G_o(s)$ ,

our plant can be written in terms of numerators and denominators as  $G_o(s) = \frac{N_o(s)}{D_o(s)}$   $G_p(s) = \frac{N_p(s)}{D_p(s)}$

Show that our controller is then  $G_c(s) = \frac{N_o(s)D_p(s)}{N_p(s)[D_o(s) - N_o(s)]}$

*Note that there are some restrictions here, in that for implementation purposes the controller must be stable, and it must be proper.*

5) (Model Matching) For the following system, with plant  $G_p(s) = \frac{1}{s+1}$ , and controller  $G_c(s)$



a) Using the results from problem 4, determine the controller so that the closed loop system matches a second order ITAE (Integral of Time and Absolute Error) optimal system, i.e., so that the closed loop transfer function is

$$G_0(s) = \frac{\omega_0^2}{s^2 + 1.4\omega_0 s + \omega_0^2}$$

Ans.  $G_c(s) = \frac{\omega_0^2(s+1)}{s(s+1.4\omega_0)}$ , note that there is a pole/zero cancellation between the controller and the plant and there is a pole at zero in the controller.

b) Determine the controller so that the closed loop system matches a third order **deadbeat** system, i.e., so that the closed loop transfer function is

$$G_0(s) = \frac{\omega_0^3}{s^3 + 1.90\omega_0 s^2 + 2.20\omega_0^2 s + \omega_0^3}$$

Ans.  $G_c(s) = \frac{\omega_0^3(s+1)}{s(s^2 + 1.9\omega_0 s + 2.20\omega_0^2)}$ , note that there is a pole/zero cancellation between the controller and the plant and there is a pole at zero in the controller.

**Preparation for Lab 1 (to be done individually and turned in with your homework)**

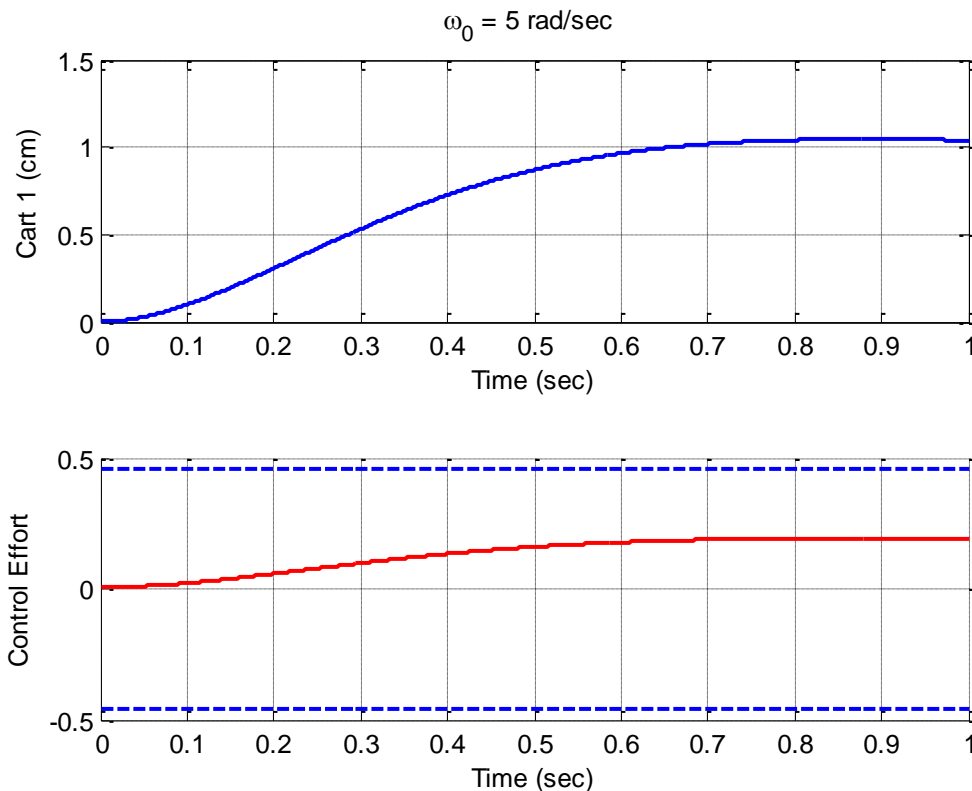
6) Download the file **homework1.rar** from the class website and extract the files. You will be using and modifying the files **closedloop\_driver.m** and **closedloop.mdl** for much of this course.

The program **closedloop\_driver.m** does the following things (try to find these in the program, it will help you later)

- sets some system parameters,
- reads in the model file **bobs\_210\_model.mat**, which contains a state variable model of a particular system,
- determines the transfer function representation of the system,
- determines a controller using one of two model matching methods,
- sets the input amplitude and duration of the simulation
- simulates the system using the Simulink model **closedloop.mdl**.
- plots the output of the system with the chosen controller
- plots the control effort required with the chosen controller (do not let it near the dashed blue lines)

In Lab 1 you will be determining two different system models and trying to control the models using model matching algorithms. In this prelab you will just run the file and make some changes.

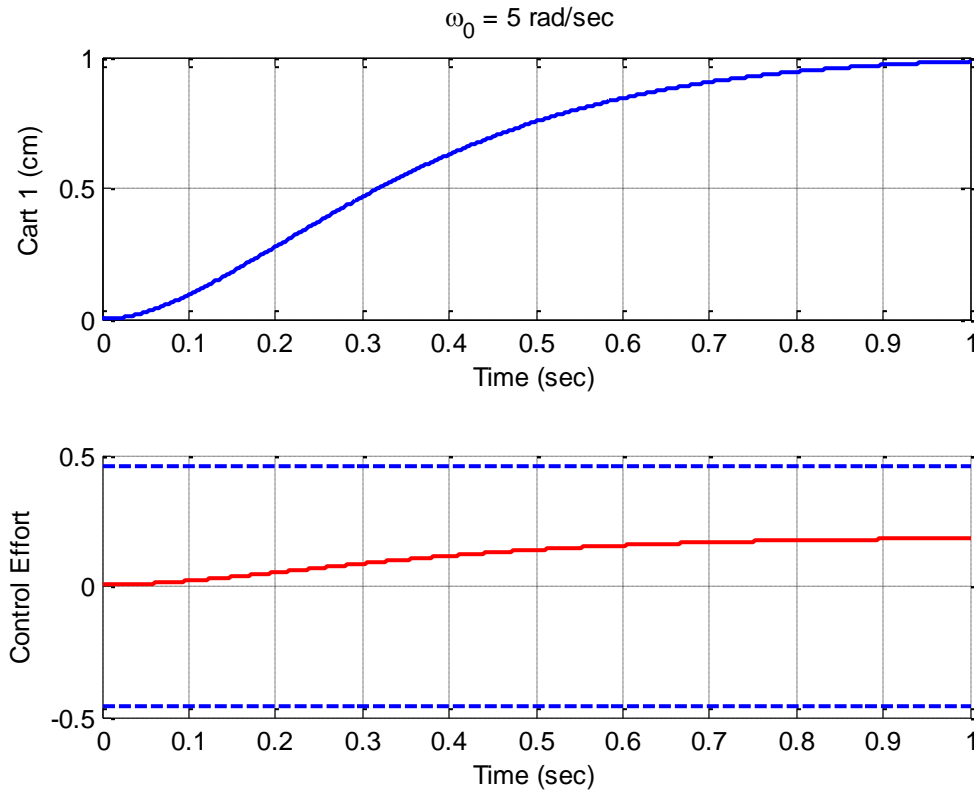
a) Run the simulation as it is, for a zero position error second order ITAE closed loop system. You should get a graph identical the graph below



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**b)** Modify  $\omega_0$  so the settling time of the system is less than 0.5 seconds and the control effort does not saturate the system (the control effort stays between the two dashed blue lines) for the second order ITAE closed loop system. Turn in your graph.

**c)** Modify the simulation so it uses a second order deadbeat closed loop system for model matching and set  $\omega_0 = 5$ . Run the simulation and you should get a result like that shown below.



**d)** Modify  $\omega_0$  so the settling time of the system is less than 0.5 seconds and the control effort does not saturate the system (the control effort stays between the two dashed blue lines) for a second order deadbeat closed loop system. Turn in your graph.