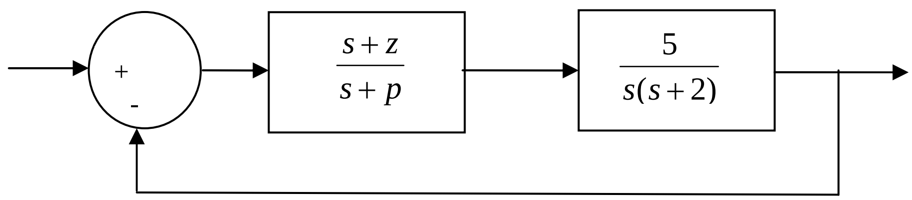
ECE-320: Linear Control Systems
Homework 6

Due: Wednesday April 26, 2006 at 3:30 PM

1) For the system shown below, with the lag compensator ($z > p$):



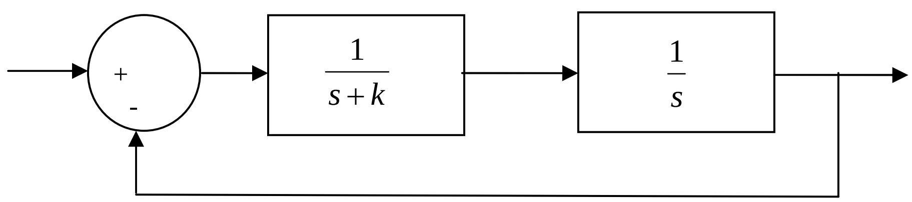
a) Show that without the lag compensator, $K_v = \frac{5}{2}$ and the steady state error for a unit ramp input is $e_{ss} = \frac{2}{5}$.

b) Include the lag compensator, with z = 0.1, so the steady state error will be 0.01. (Answer: p = 0.0025).

2) Standard root locus form for determining the poles of the closed loop transfer function is

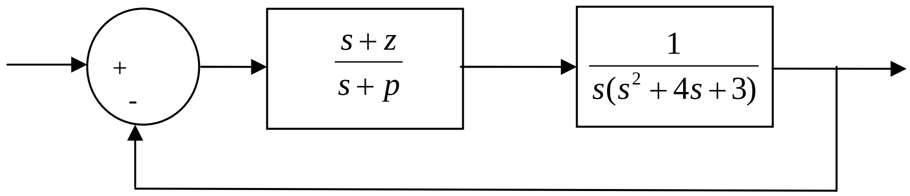
$$1 + kG(s) = 0$$

If we want to use the root locus to determine the possible pole locations for the following system,



what is $G(s)$?

3) For the system shown below:

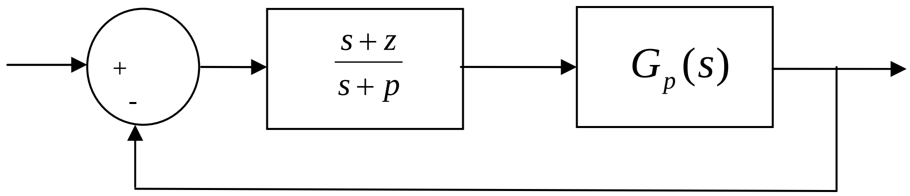


Assume we want to use the lag compensator so that the steady state error for a unit rampe is $e_{ss} = 0.1$ We will be varying the locations of the pole and zero of the lag compensator to accomplish this, and will look at the effects of these changes on both the unit step response and the unit ramp response. For each of the simulations below, run the simulation to 35 seconds. For z = 0.1, 0.01, and 0.001

- Find the correct value for p to produce the required steady state error.
- Using Matlab, simulate the unit step response for the original system (without the lag compensator) and with the lag compensator. Plot both results on one graph, as well as the input signal, using different line styles and a legend. Use the subplot command to put this on the top of the page.
- Using Matlab, simulate the unit ramp response for both the original system and the system with the lag compensator. Plot both results on one graph, as well as the input signal, using different line styles and a legend. Use the subplot command to put this on the bottom of the page.

You should notice that the large the value of z, the quicker the steady state velocity error is reduced. However, this is at the expense of large changes in the step response.

4) We have used lag compensators for obtaining the desired steady state error for a ramp input. It is possible to use a lag compensator to achieve a desired steady state error for a step input, but this is seldom done since the lag compensator tends to slow down the system. In this problem we will consider the following system

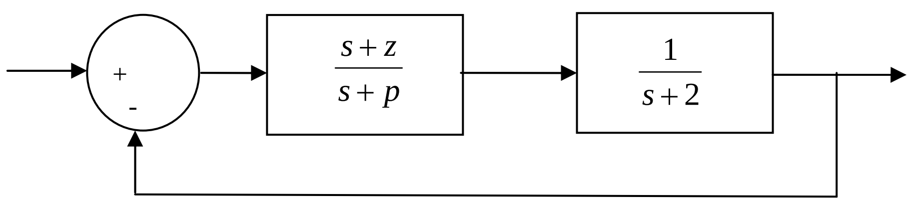


Assume that without the lag compensator, we have a known position error constant K_p.

a) Now we add the lag compensator. Show that to achieve the desired steady state error for a unit step input e_{ss}, p and z are related by

$$p = \frac{e_{ss} K_p z}{1 - e_{ss}}$$

b) Using Matlab, determine the unit step response of the following system without the lag compensator.



c) For the system above, assume we want $e_{ss} = 0.1$ for a unit step input. For $z = 1$ and 0.1 determine the corresponding value of p and simulate the system to determine the unit step input. Plot all of your results on one graph, along with the original (no lag) step response. Be sure to use different line types and a legend. Run the simulation two times, first for a final time of 2 seconds (to see the initial shape of the response is not really changing) and then for 100 seconds, to verify that the system has reached 0.9.

d) For this system, we can actually get reasonable results if we make z large enough. However, note that we are using the lag controller to significantly alter the root locus. For $z = 10$ and 100 determine the corresponding value of p and simulate the system to determine the unit step input. Plot all of your results on one graph, along with the original step response. Be sure to use different line types and a legend. Run the simulation for a final time of 2 seconds.

5) For the plant

$$G_p(s) = \frac{1}{s(s+2)}$$

show that the first order controller that put the closed loop poles at $-1 \pm j$ and -3 is $G_c(s) = 2$.

6) For the plant

$$G_p(s) = \frac{1}{s+1}$$

a) Determine a controller so the closed loop pole is at -10, then determine a prefilter so the steady state error for a unit step is zero.

b) Determine a controller $G_c(s)$ so the system is a type 1 system and the closed loop poles are at $-2 \pm j$. Show that the resulting closed loop transfer function is

$$G_o(s) = \frac{3s+5}{s^2+4s+5}$$

and determine the steady state error for a unit ramp.

7) For the plant

$$G_p(s) = \frac{s-1}{s+1}$$

a) Determine a **strictly proper** controller $G_c(s)$ so the closed loop poles are at $-2 \pm j$, determine the closed loop transfer function, and find a constant prefilter so the steady state error for a unit step is zero. Show that the resulting closed loop transfer function (without the prefilter) is

$$G_o(s) = \frac{-s+1}{s^2+4s+5}$$

b) Determine a **strictly proper** controller $G_c(s)$ so the closed loop poles are at $-2 \pm j$ and -5 ($D_o(s) = s^3 + 9s^2 + 25s + 25$), and the system is a type one system. Show that the resulting closed loop transfer function is

$$G_o(s) = \frac{(-25-21s)(s-1)}{s^3+9s^2+25s+25}$$