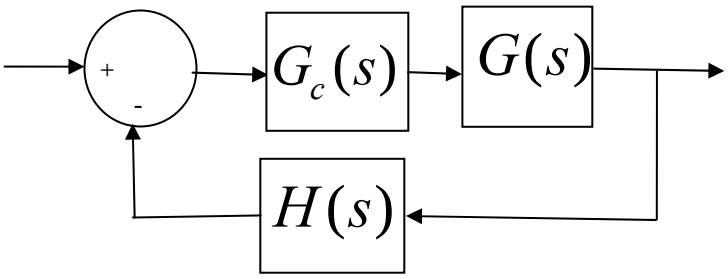
Phase Lead

Consider the following feedback system



The primary function of the phase lead compensator is to reshape the frequency response curve to add phase to the system, thereby increasing the phase margin. This usually results in an increased bandwidth, and hence a faster response time.

Basic Procedure

1) Assume the phase lead compensator has the form

$$G_c(s) = K\frac{(Ts+1)}{(\alpha Ts+1)} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

where $K_c = \frac{K}{\alpha}$

Determine K to satisfy any steady state error requirements.

2) Using the value of K determined in step 1, draw the Bode plot of $KG(s)H(s)$.

3) Determine the necessary phase to be added to the system to achieve the desired phase margin. Add an additional 5^o to 12^o to the necessary phase to account for the fact that the phase lead compensator shifts the gain crossover frequency to the right and decreases the phase margin. The total phase we need to add is ϕ_m.

4) Determine α using the relationship $\alpha = \frac{1-\sin(\phi_m)}{1+\sin(\phi_m)}$. α should be larger than 0.05, or you need two or more compensators in series.

5) Determine the frequency ω_m at which

$$20\log_{10}|KG(j\omega_m)H(j\omega_m)| = -20\log_{10}(\frac{1}{\sqrt{\alpha}}) = 10\log_{10}(\alpha)$$

This is the new gain crossover frequency $\omega_m = \frac{1}{T\sqrt{\alpha}}$. Since we know ω_m and α, we can compute $T = \frac{1}{\omega_m\sqrt{\alpha}}$

6) Determine the pole and zero of the compensator $z = \frac{1}{T}$ and $p = \frac{1}{T}\frac{1}{\alpha}$

7) Determine $K_c = \frac{K}{\alpha}$

8) Check the phase margin to see if it is acceptable. We will use *sisotool.*

When you use sisotool to tweak your compensators, you should

1) enter the transfer function for $G(s)H(s)$

2) instruct sisotool to use the Natural Frequency or Time Constant method of displaying the controller. To do this

edit → sisotool preferences →options
and select Natural Frequency or Time Constant

3) enter the pole and zero of the compensator

4) Set the gain to be K, the required gain for the steady state errors. This gain cannot change!

5) Play with the pole and/or zero to get the required phase margin