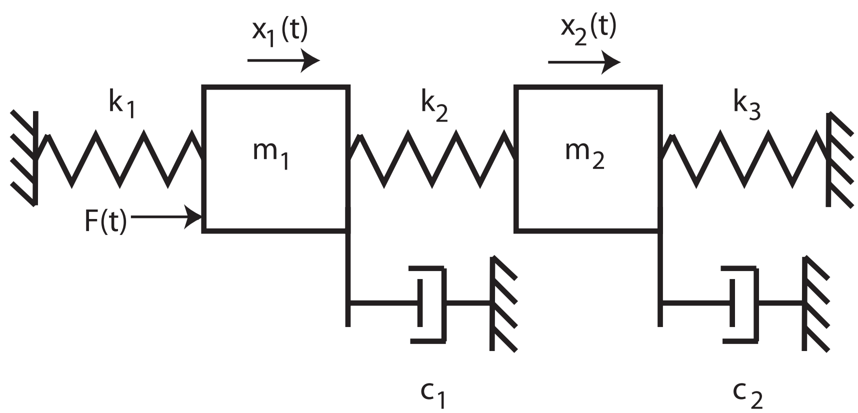
Lab 4: Time and Frequency Domain Modeling of a Two Degree of Freedom System(Rectilinear)

Overview

In this lab you will be modeling one two degree of freedom system using time-domain analysis and frequency domain analysis. The steps we will go through in this lab are very commonly used in system identification (determining the transfer function) of unknown systems. We will utilize these models in later labs so do a good job in this lab, your results in later labs will be affected by how well you perform in this lab. ***Take your time!***

Background

For the following generic two degree of freedom configuration (one of the springs many be missing



we get the following state equations

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\left(\frac{k_1+k_2}{m_1}\right) & -\left(\frac{c_1}{m_1}\right) & \left(\frac{k_2}{m_1}\right) & 0 \\ 0 & 0 & 0 & 1 \\ \left(\frac{k_2}{m_2}\right) & 0 & -\left(\frac{k_2+k_3}{m_2}\right) & -\left(\frac{c_2}{m_2}\right) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\frac{1}{m_1}\right) \\ 0 \\ 0 \end{bmatrix} F$$

We need to identify all of these quantities to get the A *and* B *matrices for the state variable description. For our system* $D = 0$ *and* C *is determined by whatever we want the output to be.*

You will need to set up a folder for Lab 4 and copy all files from the folder ***basic_files*** *into this folder.*

Step 0: Set Up the System. Both the first and second carts should move. The third cart should be fixed in place. In addition:

- Either the carts should have an equal amount of weight on them, or the first cart should have more weight than the second cart. You need at least one mass on each cart.

- Either all springs connecting carts should have equal stiffness, or the springs should get less stiff from left to right. You need to use at least two springs.

- If you want to use the active damper, unscrew the screw in the damper.

You will be using this configuration throughout the remainder of the course so be sure you write down all of the information you need to duplicate this configuration.

Step 1a) Initial Estimates of ω_1, ζ_1, ω_2, *and* ζ_2

From the equations of motion we have

$$\begin{aligned}\ddot{x}_1 + 2\zeta_1\omega_1\dot{x}_1 + \omega_1^2 x_1 &= \frac{k_2}{m_1}x_2 + \frac{1}{m_1}F \\ \ddot{x}_2 + 2\zeta_2\omega_2\dot{x}_2 + \omega_2^2 x_2 &= \frac{k_2}{m_2}x_1\end{aligned}$$

a) If there is no applied force ($F = 0$) and the second cart is fixed in place ($x_2 = 0$), we have

$$s^2 + 2\zeta_1\omega_1 s + \omega_1^2 = 0$$

Use the log decrement method to get our initial estimate of ω_1 and ζ_1.

b) If the second cart is free to move and the first cart is fixed in place ($x_1 = 0$), we have

$$s^2 + 2\zeta_2\omega_2 s + \omega_2^2 = 0$$

Use the log decrement method to get our initial estimate of ω_2 and ζ_2.

For the log-decrement analysis you will go through the following steps for **each** cart:

- Be sure only one cart is free to move
- Reset the system using **ECPDSPresetmdl.mdl.**
- Modify **Model210_Openloop.mdl** so the input has ***zero*** amplitude.
- Compile **Model210_Openloop.mdl** if necessary.

- Connect **Model210_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Displace whichever cart is free to move, and hold it.
- Start **(play) Model210_Openloop.mdl** and let the cart go.
- Run the m-file **Log_Dec.m.** This should be in the same directory as **Model210_Openloop.mdl** and **Log_Dec.fig**. This routine assumes the position of the first cart is labeled *x1*, the position of the second cart is labeled *x2*, and the time is labeled *time*. (These are the defaults in **Model210_Openloop.mdl**.)

You will need to include these **two** log-decrement results in your memo.

Step 1b) Estimating the Gains K_1 *and* K_2

You will go through the following steps:

- Be sure both carts are free to move
- Reset the system using **ECPDSPresetmdl.mdl.**
- Modify **Model210_Openloop.mdl** so the input is a step. Set the amplitude to something small, like 0.01 or 0.02 cm.
- Compile **Model210_Openloop.mdl**
- Connect **Model210_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model210_Openloop.mdl.** If the carts do not seem to move much, increase the amplitude of the step. If the carts move too much, decrease the amplitude of the step. You may have to recompile.
- Estimate the static gains as

$$K_1 = \frac{x_{1,ss}}{A} \quad K_2 = \frac{x_{2,ss}}{A}$$

where x_{ss} is the steady state value of the cart position, and A is the input amplitude.

You should do this in Matlab, don't use the X-Y Graph. The variables *x1*, *x2*, and *time* should be in your workspace.

You need to increase the value of the input amplitude until the first cart is moving about 2 cm or so Use the static gains associated with this input amplitude as your estimates of the static gain.

Step 1c) Fitting the Estimated Frequency Response to the Measured Frequency Response

We will be constructing the magnitude portion of the Bode plot and fitting this measured frequency response to the frequency response of the expected transfer function to determine the parameters we need. For each frequency $\omega = 2\pi f$ we have as input

$u(t) = A\cos(\omega t)$ where, for out systems, A is measured in centimeters. After a transition period, the steady state output will be $x_1(t) = B_1 \cos(\omega t + \theta_1)$ for the first cart and $x_2(t) = B_2 \cos(\omega t + \theta_2)$ for the second cart, where both B_1 and B_2 are also measured in cm. Since we will be looking only at the magnitude portion of the Bode plot, we will ignore the phase angles θ_1 and θ_2.

You will go through the following steps

For frequencies $f = 0.5, 1, 1.5 ... 7.5$ Hz

- Make sure both carts are free to move, and the third cart is fixed.
- Modify **Model210_Openloop.mdl** so the input is a sinusoid. You may have to set the mode to **Normal**.
- Set the frequency and amplitude of the sinusoid. Try a small amplitude to start, like 0.01 cm. Generally this amplitude should be as large as you can make it without the system hitting a limit. This amplitude will probably vary with each frequency.
- Compile **Model210_Openloop.mdl,** if necessary. (Assume it is not necessary. The system will let you know if it is necessary.)
- Connect **Model210_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model210_Openloop.mdl.** If the carts do not seem to move much, increase the amplitude of the input sinusoid. If the carts move too much, decrease the amplitude of the input sinusoid.
- Record the input frequency (f), the amplitude of the input (A), and the amplitude of the output (B_1 and B_2) when the system is in steady state. Modify the program **get_A.m** to help record these amplitudes accurately. Be sure the plot from **get_A** shows the system in steady state. Attach this modified code to your memo. Be sure to look at the graph and understand what the code is doing before you use it!!!
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Enter the values of f , A , B_1 and B_2 into the program **process_data_2dof.m** (you need to edit the file)

At the Matlab prompt, type **data = process_data_2dof**;

Run the program **model_2dof.m.** There are ten input arguments to this program:

- **data**, the measured data as determined by **process_data_2dof.m**
- the estimated value of K_2
- ω_a, the estimated frequency of the first resonance, when both carts are moving, in radians/sec

- ζ_a, the estimated first damping ratio when both carts are moving. Assume $\zeta_a = 0.1$.
- ω_b, the estimated frequency of the second resonance, when both carts are moving, in radians/sec
- ζ_b, the estimated second damping ratio when both carts are moving. Assume $\zeta_b = 0.1$.
- ω_1 the estimated natural frequency of the first cart when it is the only cart moving (from the log decrement analysis)
- ζ_1 the estimated damping ratio of the first cart when it is the only cart moving (from the log decrement analysis)
- ω_2 the estimated natural frequency of the second cart when it is the only cart moving (from the log decrement analysis)
- ζ_2 the estimated damping ratio of the second cart when it is the only cart moving (from the log decrement analysis)

The program **model_2dof.**m will produce the following:

- A graph indicating the fit of the identified transfer function to the measured data for the first cart (You need to include the <u>*final*</u> graph of this fit in your memo.)
- A graph indicating the fit of the identified transfer function to the measured data for the second cart (You need to include the <u>*final*</u> graph of this fit in your memo.)
- The optimal estimates of all parameters (written at the top of the graphs)
- A file **state_model_2dof.mat** in your directory**.** This file contains the A, B, C, and D matrices for the state variable model of the system. If you subsequently type **load state_model_2dof** you will load these matrices into your workspace.

You need to be sure you have 4 points close to the resonant peaks of the transfer functions. This is particularly true if you have very small values of ζ (which correspond to very sharp peaks) At this point you probably should go back and <u>add a few points near both the resonant peaks and nulls</u>.

Your memo should include a description the system (so you can set them up again), a table comparing the estimated values of the static gains, the natural frequencies, and the damping ratios using the two different methods (time domain and optimized frequency domain), and a brief comparison of the values. The damping ratios are often quite different, so that's OK. The other values should be close. You should include as attachments 4 graphs (2 log-decrement and2 frequency response graphs for the rectilinear system), each with a Figure number and caption. You should also include the data used for estimating the static gains.