ECE-320: Linear Control Systems
Homework 3

Due: Tuesday March 29 at 10 AM

1) Read (and understand) chapter 7 from the notes. You are responsible for this and it will be on exams. (It's also pretty simple, so stop whining....)

2) For the following transfer functions, determine

- the characteristic polynomial
- the characteristic modes
- if the system is stable, unstable, or marginally stable

a) $H(s) = \dfrac{s-1}{s(s+2)(s+10)}$

b) $H(s) = \dfrac{s(s-1)}{(s+1)^2(s^2+s+1)}$

c) $H(s) = \dfrac{1}{s^2(s+1)}$

d) $H(s) = \dfrac{s^2-1}{(s-1)(s+2)(s^2+1)}$

e) $H(s) = \dfrac{1}{(s^2+2)(s+1)}$

3) For a system with the following pole locations, estimate the settling time and determine the dominant poles

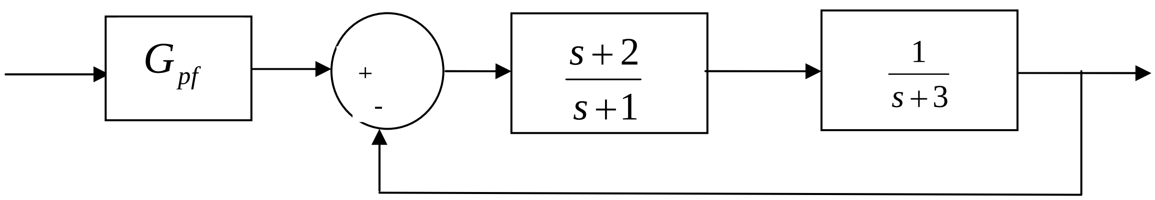
a) -1,-1,-4,-5
b) -4, -6, -6, -8
c) -1+j, -1-j, -2, -2
d) -3-2j, -3+2j, -4+j, -4-j

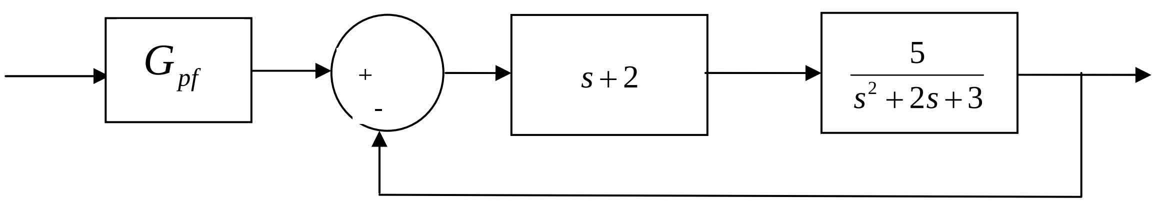
Ans. 4/3, 4, 4, 1 (though not in that order)

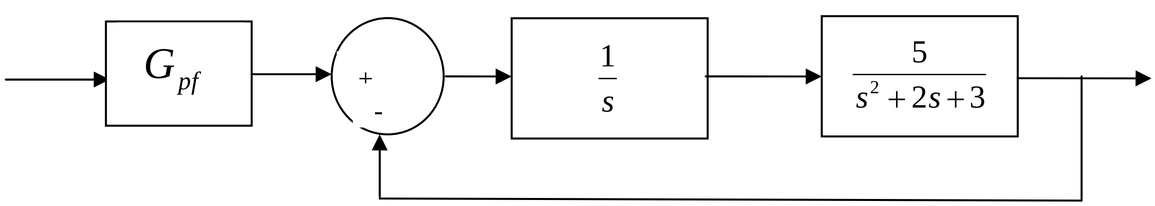
4) For the following systems

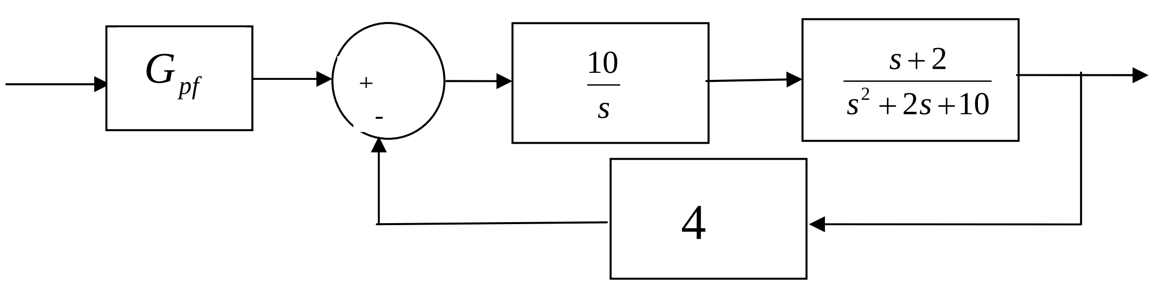
a) Determine the value of the prefilter gain G_{pf} so that the position error of the closed loop system is zero. *(Hint: Think about the easiest way to do this, you should never have to write out the closed loop transfer function as the ratio of two polynomials.) Ans. 1,4,1.3,2.5 (though not in that order)*

b) Simulate each system in Matlab (not Simulink) for a step response. Run the simulation until the system comes to steady state. Use **subplot** to put all four plots on one page. You may want to type **orient tall** before your first plot so Matlab will use more of the page.

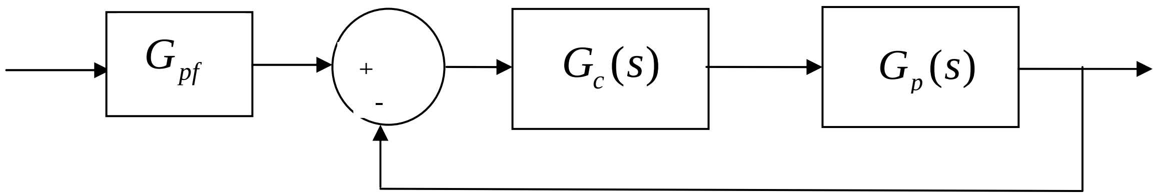








5) For the following system, with prefilter G_{pf}, plant $G_p(s) = \frac{1}{s+1}$, and controller $G_c(s)$



a) Determine the controller so that the closed loop system matches a second order zero position error ITAE optimal system, i.e., so that the closed loop transfer function is

$$G_0(s) = \frac{\omega_0^2}{s^2 + 1.4\omega_0 s + \omega_0^2}$$

Ans. $G_c(s) = \frac{\omega_0^2(s+1)}{s(s+1.4\omega_0)}$, *note that there is a pole/zero cancellation between the controller and the plant and there is a pole at zero in the controller.*

b) For zero position error, what should the value of the prefilter be?

c) Determine the controller so that the closed loop system matches a third order zero deadbeat system, i.e., so that the closed loop transfer function is

$$G_0(s) = \frac{\omega_0^3}{s^3 + 1.90\omega_0 s^2 + 2.20\omega_0^2 s + \omega_0^3}$$

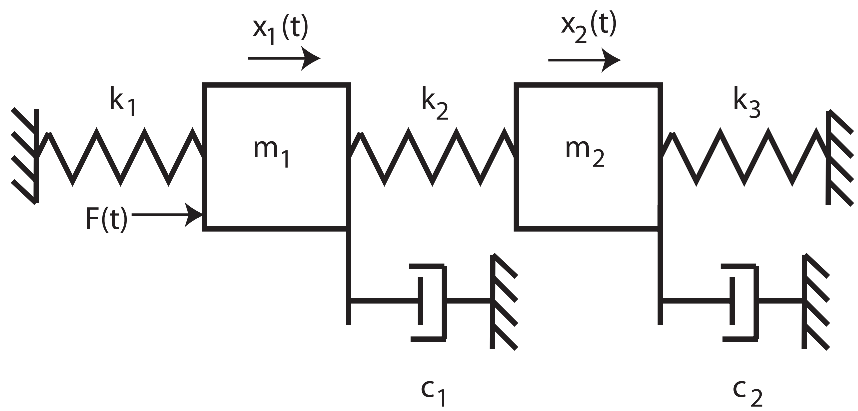
Ans. $G_c(s) = \frac{\omega_0^3(s+1)}{s(s^2 + 1.9\omega_0 s + 2.20\omega_0^2)}$, *note that there is a pole/zero cancellation between the controller and the plant and there is a pole at zero in the controller.*

d) For zero position error, what should the value of the prefilter be?

Preparation for Lab 4.

In the following, we are developing a state variable model for a 2 degree of freedom system. In order to do the frequency domain system identification, we will try to look for transfer functions that are in a particular form that will help identify important system parameters. We then need to be able to relate these parameters to the state variable model. This must all be done by hand, no Maple.

6) Consider the following model of the two degree of freedom system we will be using in lab 2.



a) Draw free body diagrams for each mass and show that the equations of motion can be written as

$$\begin{aligned} m_1\ddot{x}_1 + c_1\dot{x}_1 + (k_1+k_2)x_1 &= F + k_2x_2 \\ m_2\ddot{x}_2 + c_2\dot{x}_2 + (k_2+k_3)x_2 &= k_2x_1 \end{aligned}$$

b) Defining $q_1 = x_1$, $q_2 = \dot{x}_1$, $q_3 = x_2$, and $q_4 = \dot{x}_2$, show that we get the following state equations

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\left(\frac{k_1+k_2}{m_1}\right) & -\left(\frac{c_1}{m_1}\right) & \left(\frac{k_2}{m_1}\right) & 0 \\ 0 & 0 & 0 & 1 \\ \left(\frac{k_2}{m_2}\right) & 0 & -\left(\frac{k_2+k_3}{m_2}\right) & -\left(\frac{c_2}{m_2}\right) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\frac{1}{m_1}\right) \\ 0 \\ 0 \end{bmatrix} F$$

In order to get the A and B matrices for the state variable model, we need to determine all of the quantities in the above matrices. The C matrix will be determined by what we want the output of the system to be.

c) If we want the output to be the position of the first cart, what should C be? If we want the output to be the position of the second cart what should C be?

d) Now we will rewrite the equations from part (a) as

$$\begin{aligned}
\ddot{x}_1 + 2\zeta_1\omega_1\dot{x}_1 + \omega_1^2 x_1 &= \frac{k_2}{m_1}x_2 + \frac{1}{m_1}F \\
\ddot{x}_2 + 2\zeta_2\omega_2\dot{x}_2 + \omega_2^2 x_2 &= \frac{k_2}{m_2}x_1
\end{aligned}$$

We will get our initial estimates of ζ_1, ω_1, ζ_2, and ω_2 using the log-decrement method (assuming only one cart is free to move at a time). Assuming we have measured these parameters, show how $A_{2,1}$, $A_{2,2}$, $A_{4,3}$, and $A_{4,4}$ can be determined.

e) By taking the Laplace transforms of the equations from part (d), show that we get the following transfer function

$$\frac{X_2(s)}{F(s)} = \frac{\left(\frac{k_2}{m_1 m_2}\right)}{(s^2 + 2\zeta_1\omega_1 s + \omega_1^2)(s^2 + 2\zeta_2\omega_2 s + \omega_2^2) - \frac{k_2^2}{m_1 m_2}}$$

f) It is more convenient to write this as

$$\frac{X_2(s)}{F(s)} = \frac{\left(\frac{k_2}{m_1 m_2}\right)}{(s^2 + 2\zeta_a\omega_a s + \omega_a^2)(s^2 + 2\zeta_b\omega_b s + \omega_b^2)}$$

By equating powers of s in the denominator of the transfer function from part (e) and this expression you should be able to write down four equations. The equations corresponding to the coefficients of s^3, s^2, and s do not seem to give us any new information, but they will be used to get consistent estimates of ζ_1 and ω_1. The equation for the coefficient of s^0 will give us a new relationship for $\frac{k_2^2}{m_1 m_2}$ in terms of the parameters we will be measuring.

g) We will actually be fitting the frequency response data to the following transfer function

$$\frac{X_2(s)}{F(s)} = \frac{K_2}{\left(\frac{1}{\omega_a^2}s^2 + \frac{2\zeta_a}{\omega_a}s + 1\right)\left(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s + 1\right)}$$

What is K_2 in terms of the parameters of part (f)?

h) Using the transfer function in (f) and the Laplace transform of the second equation in part (d), show that the transfer function between the input and the position of the first cart is given as

$$\frac{X_1(s)}{F(s)} = \frac{\frac{1}{m_1}\left(s^2 + 2\zeta_2\omega_2 s + \omega_2^2\right)}{(s^2 + 2\zeta_a\omega_a s + \omega_a^2)(s^2 + 2\zeta_b\omega_b s + \omega_b^2)}$$

i) This equation is more convenient to write in the form

$$\frac{X_1(s)}{F(s)} = \frac{K_1\left(\frac{1}{\omega_2^2}s^2 + \frac{2\zeta_2}{\omega_2}s + 1\right)}{(\frac{1}{\omega_a^2}s^2 + \frac{2\zeta_a}{\omega_a}s + 1)(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s + 1)}$$

What is K_1 in terms of the quantities given in part (h)?

j) Verify that $A_{4,1} = \frac{k_2}{m_2} = \frac{K_2}{K_1}\omega_2^2$

k) Verify that $A_{2,3} = \frac{k_2}{m_1} = \frac{\omega_1^2\omega_2^2 - \omega_a^2\omega_b^2}{A_{4,1}}$

l) Verify that $B_2 = \frac{1}{m_1} = \frac{K_2\omega_a^2\omega_b^2}{A_{4,1}}$. Note that this term contains all of the scaling and unit conversions.