ECE-320: Linear Control Systems
Homework 2

Due: Tuesday March 22 at 10 AM

1) For systems with the following transfer functions:

$$H_a(s) = \frac{1}{s+2}$$

$$H_b(s) = \frac{s+6}{(s+2)(s+3)}$$

a) Determine the step and ramp response for each system using Laplace transforms. Your answer should be time domain functions $y_a(t)$ and $y_b(t)$.

b) From these time domain functions, determine the position and velocity errors.

c) Using the equations derived in class (and in the notes), determine the position and velocity errors directly from the transfer functions.

The following Matlab code can be used to estimate the step and ramp response for 5 seconds for transfer function $H_b(s)$.

```
H = tf([1 6],[1 5 6]);              % enter the transfer function
t = [0:0.01:5];                     % t goes from 0 to 5 by increments of 0.01
ustep = ones(1,length(t));          % the step input is all ones, u(t) = 1;
uramp = t;                          % the ramp input is has the input u(t) = t;
ystep = lsim(H,ustep,t);            % find the step response
yramp = lsim(H,uramp,t);            % find the ramp response
figure;                             % make a new figure
orient tall                         % or orient landscape, use more of the page
subplot(2,1,1);                     % put two graphs on one piece of paper
plot(t,ustep,'.-',t,ystep,'-');     % plot input/output with different line types
grid;                               % put a grid on the graph
legend('Step Input','Step Response',4);   % put a legend on the graph
subplot(2,1,2);                     % second of two graphs on one piece of paper
plot(t,uramp,'.-',t,yramp,'-');     % plot input/output with different line types
grid;                               % put a grid on the graph
legend('Ramp Input','Ramp Response',4);   % put a legend on the graph
```

d) Plot the step and ramp response for both systems (a and b) and indicate the position and velocity errors on the graph. Draw on the graph to show you know what the position and velocity error is.

Ans. $e_p = 0.5, 0, e_v = \infty, 0.666$

2) For a system with transfer function

$$H(s) = \frac{a}{s^2 + 2s + 3}$$

what is the range of values of a so that $e_p < 0.2$? Note that we only care about absolute values here. What value of a will produce a finite velocity error? (*Ans.* $2.4 < a < 3.6, 3$)

3) For a system with transfer function

$$H(s) = \frac{bs + 3}{s^2 + 2s + 3}$$

what is the range of values of b so that $e_v < 0.1$? Note that we only care about absolute values here. What is the position error? (*Ans.* $1.7 < b < 2.3, 0$)

4) An ideal second order system has the transfer function $G_o(s)$. The system specifications for a step input are as follows:

a) Percent Overshoot $< 5\%$
b) Settling Time < 4 seconds (2% criteria)
c) Peak Time < 1 second

Show the permissible area for the poles of $G_o(s)$ in order to achieve the desired response.

5) For the transfer function

$$H(s) = \frac{2}{s^2 + 2s + 2}$$

By computing the inverse Laplace transform show that the step response is given by

$$y(t) = u(t) - e^{-t} \cos(t) u(t) - e^{-t} \sin(t) u(t)$$

6) For systems A and B, with step responses shown in Figure 1, estimate
- the percent overshoot
- the settling time
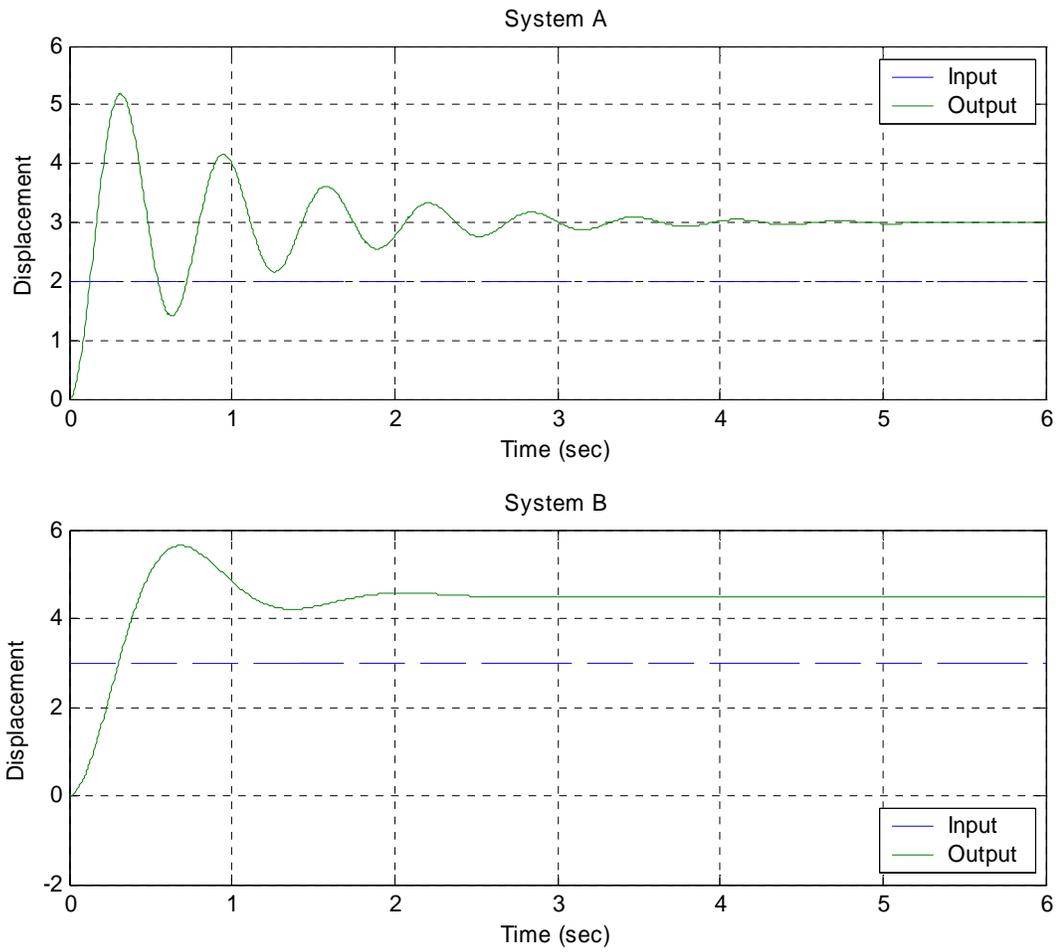- the position error
- the velocity error:

Figure 1. Step responses of system A and system B.

7) For systems C and D, with ramp responses shown in Figure 2, determine
- the velocity error
- the position error
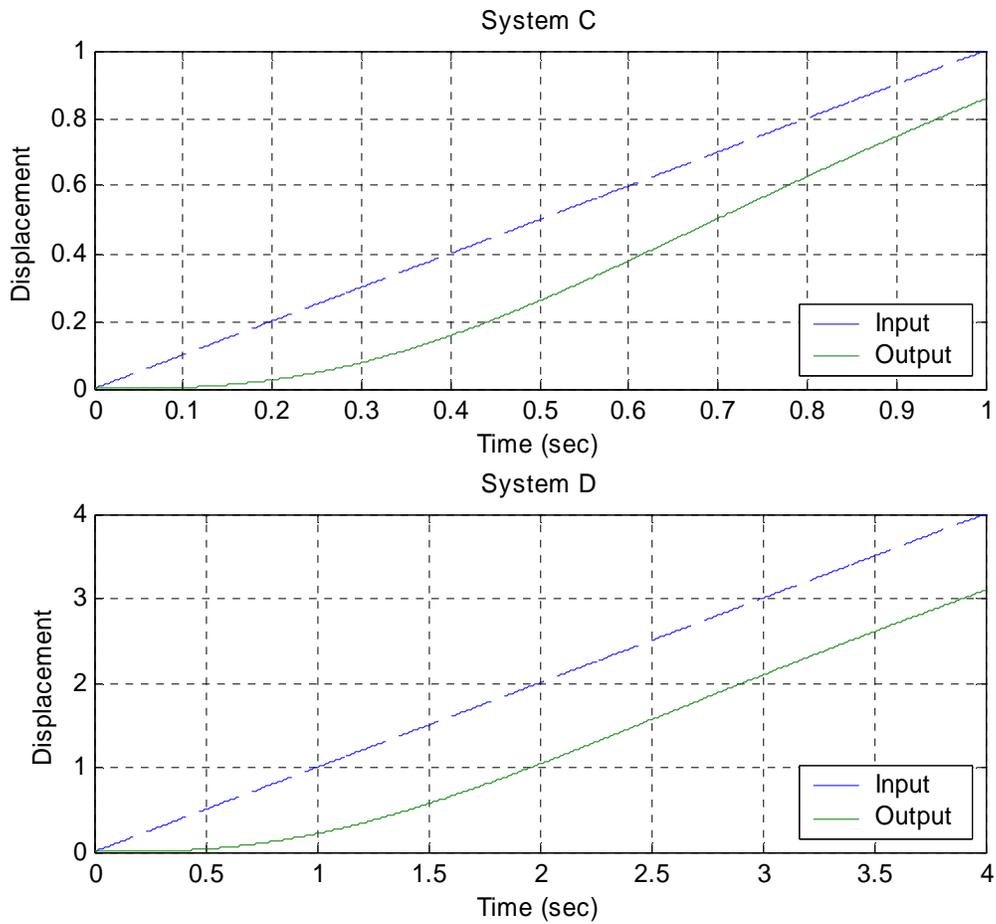
System C

System D

Figure 2. Ramp responses for systems C and D.

Answers for 6 and 7 (in no particular order, your approximations should be close, but they probably won't match.) 0,0,73%, 25%, 2, 3.5, 1, 1.2,1,0.16, ∞ *,* ∞

Preparation for Lab 3.

As you undoubtedly recall, if we have a *stable* system with transfer function $H(s)$, and the input to the system is $u(t) = A\cos(\omega t + \theta)$, then the steady state output is given by

$$y(t) = |H(j\omega)| A\cos(\omega t + \theta + \angle H(j\omega))$$

This is really nothing more than a phasor relationship

$$Y = \left[|H(j\omega)| \angle H(j\omega)\right]\left[|U(j\omega)| \angle U(j\omega)\right]$$

or

$$|Y| \quad = \quad |H(j\omega)||U(j\omega)|$$
$$\angle Y \quad = \quad \angle H(j\omega) + \angle U(j\omega)$$

8) Assume

$$H(s) = \frac{s}{s+2}$$

a) If the input to this system is $u(t) = 3\cos(2t)$ determine the steady state output.
b) If the input to this system is $u(t) = 5\sin(5t + 10^o)$ determine the steady state output.
(*Ans.* $2.12\cos(2t + 45^o), 4.64\sin(5t + 31.8^o)$)

9) In addition to determining $|H(j\omega)|$ and $\angle H(j\omega)$ analytically, we can read these values from a Bode plot of the transfer function. Of course the magnitude portion of a Bode plot is in dB, and we need the actual amplitude $|H(j\omega)|$. For the system with Bode plot given below, determine the steady state output of this system if the input is $u(t) = 5\sin(3t + 20^o)$ and $u(t) = 5\sin(2t + 20^o)$.

(*Ans.* $5\sin(3t + 20^o), 1.7\sin(2t + 65^o)$)

Bode Diagram
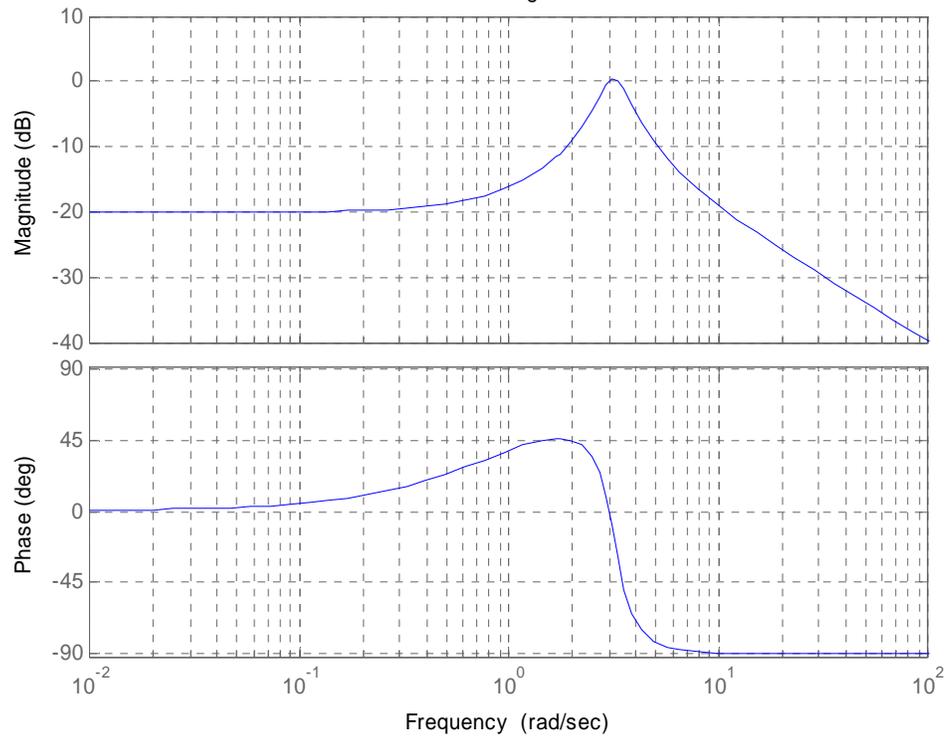
10) Now we want to use the Bode plot to identify the system. Let's assume the input to an unknown system is a sequence of sinusoids, $u(t) = A\cos(\omega t + \theta)$ at different frequencies and different amplitudes. Once the transients have died out and the system is in steady state we measure the output $y(t)$. We then have the following data:

$$u(t) = 4\cos(2\pi * 0.25t) \quad y(t) = 0.089\cos(2\pi * 0.25t - 0.3^o)$$
$$u(t) = 4\cos(2\pi * 0.5t) \quad y(t) = 0.092\cos(2\pi * 0.5t - 0.6^o)$$
$$u(t) = 4\cos(2\pi * t) \quad y(t) = 0.104\cos(2\pi * t - 1.4^o)$$
$$u(t) = 3\cos(2\pi * 2t) \quad y(t) = 0.178\cos(2\pi * 2t - 6.2^o)$$
$$u(t) = 3\cos(2\pi * 2.25t) \quad y(t) = 0.321\cos(2\pi * 2.25t - 12.7^o)$$
$$u(t) = 2\cos(2\pi * 2.4t) \quad y(t) = 0.429\cos(2\pi * 2.4t - 28.1^o)$$
$$u(t) = 1\cos(2\pi * 2.5t) \quad y(t) = 0.424\cos(2\pi * 2.5t - 75.5^o)$$
$$u(t) = 1\cos(2\pi * 2.6t) \quad y(t) = 0.258\cos(2\pi * 2.6t - 142.2^o)$$
$$u(t) = 2\cos(2\pi * 2.75t) \quad y(t) = 0.218\cos(2\pi * 2.75t - 164.1^o)$$
$$u(t) = 4\cos(2\pi * 3t) \quad y(t) = 0.207\cos(2\pi * 3t - 171.9^o)$$
$$u(t) = 8\cos(2\pi * 4t) \quad y(t) = 0.115\cos(2\pi * 4t - 176.9^o)$$
$$u(t) = 10\cos(2\pi * 5t) \quad y(t) = 0.075\cos(2\pi * 5t - 178.0^o)$$
$$u(t) = 10\cos(2\pi * 6t) \quad y(t) = 0.047\cos(2\pi * 6t - 178.5^o)$$
$$u(t) = 10\cos(2\pi * 7t) \quad y(t) = 0.033\cos(2\pi * 7t - 178.8^o)$$
$$u(t) = 10\cos(2\pi * 8t) \quad y(t) = 0.024\cos(2\pi * 8t - 180^o)$$

a) From this data, construct a table with the ith input frequency f_i (in Hz), and the corresponding magnitude of the transfer function at that frequency , $|H_i| = |H(j2\pi f_i)|$. Note that the amplitude of the input is changing. Note that we could also utilize the phase, but we don't need that for the systems we are trying to model.

b) Now we need to try and fit a transfer function to this data, i.e., determine a transfer function that will have the same Bode plot (at least the same magnitude portion). You will need to go though the following steps (you probably want to put this in an m-file...)

```
%
% I won't tell you how to do this again, so pay attention!
%
%  Enter the measured frequency response data
%
```
$\quad w = 2 * pi * [f_1 \quad f_2 \quad \cdots \quad f_n]$ % frequencies in radians/sec

$\quad H = [|H_1| \quad |H_2| \quad \cdots \quad |H_n|]$ % corresponding amplitudes of the transfer function
```
%
%   generate 1000 points (for a smooth curve) between min(w) and max(w)
```

```
%    space them out logarithmically
%
  ww = logspace(log10(min(w)),log10(max(w)),1000);
%
%  next guess the parameters for a second order system, you will have to change these
%  to fit the data
%
  omega_n = 20;
  zeta = 0.1;
  K = 0.05
  HH = tf(K,[1/omega_n^2 2*zeta/omega_n 1]);
%
%  get the frequency response, this is one of many possible ways
%
[M,P] = bode(HH,ww);
M = M(:);
%
%  Now plot them both on the same graph and make it look pretty
%
semilogx(w,20*log10(H)),'d',ww,20*log10(M),'-'); grid; legend('Measured','Estimated');
ylabel('dB'); xlabel('Frequency (rad/sec)');
title(['K = ' num2str(K) ',  \omega_n = ' num2str(omega_n) ', \zeta =  ' num2str(\zeta)]);
%
%  Note there are spaces between the single quote (') and the num2str function
```

If you have not screwed up, you should get the following graph:
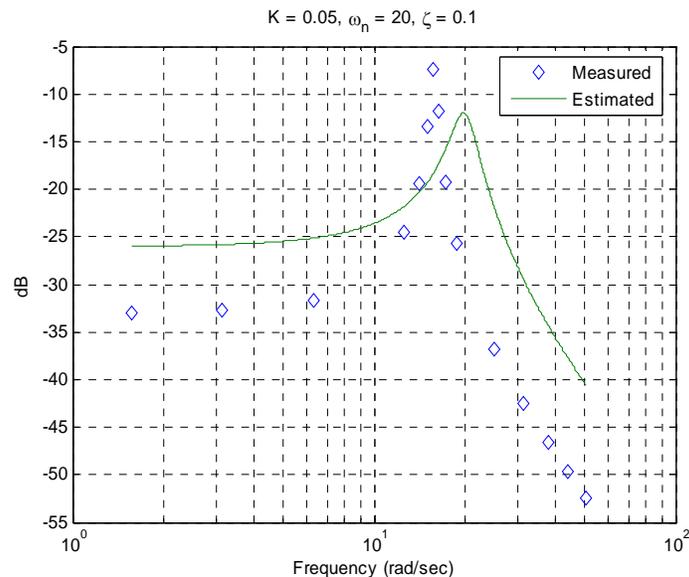
c) Now you need to adjust the parameters of the estimated transfer function to get the best fit. Turn in your final plot with the estimates of the parameters at the top (as in the figure above.)