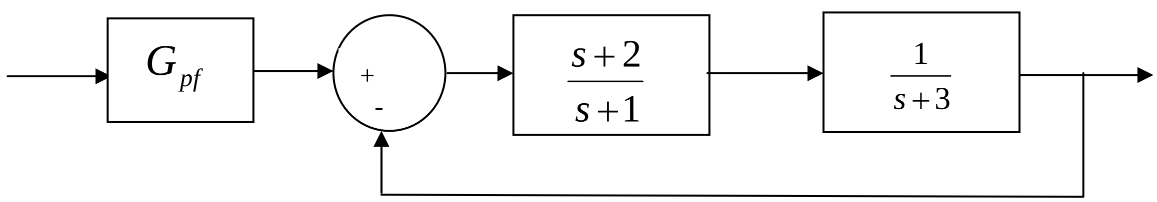
ECE-320: Linear Control Systems
Homework 3

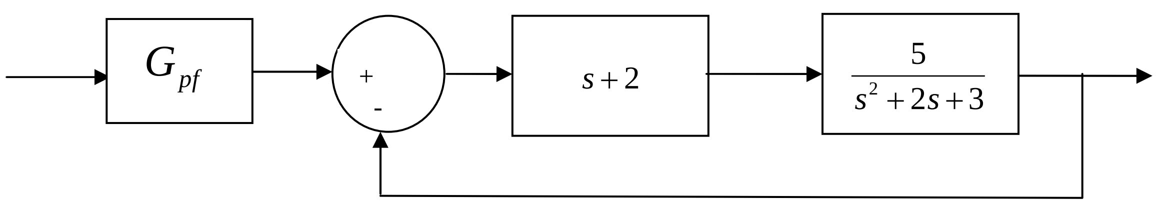
Due: Tuesday September 22 at the beginning of class

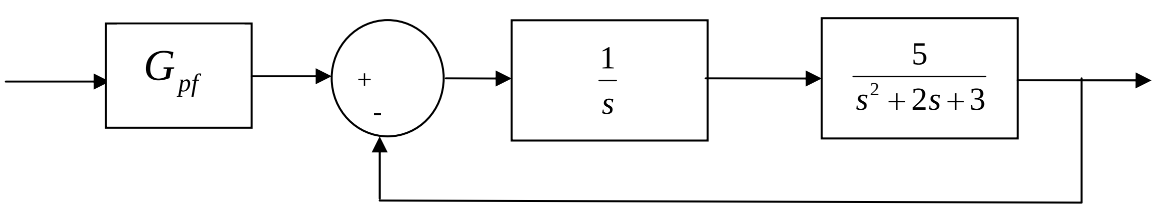
1) For the following systems

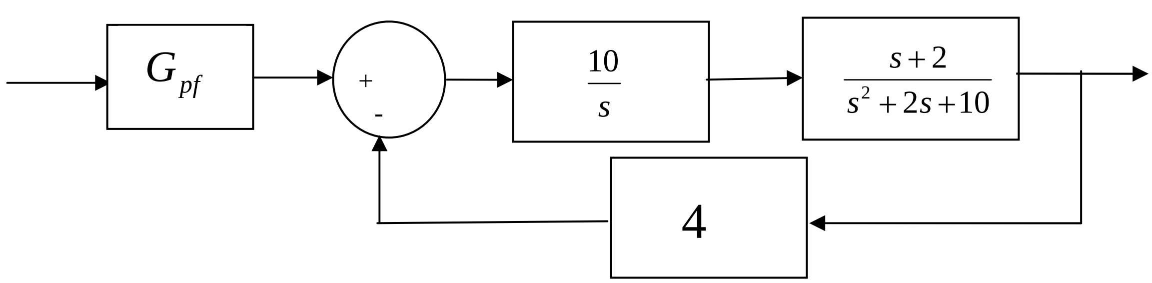
a) Determine the value of the prefilter gain G_{pf} so that the steady state error for a unit step input of the closed loop system is zero. *(Hint: Think about the easiest way to do this, you should never have to write out the closed loop transfer function as the ratio of two polynomials.) Ans. 1,4,1.3,2.5 (though not in that order)*

b) Simulate each system in Matlab (not Simulink) for a step response. Run the simulation until the system comes to steady state. Use **subplot** to put all four plots on one page. You may want to type **orient tall** before your first plot so Matlab will use more of the page.









2) For the following transfer functions, determine

- the characteristic polynomial
- the characteristic modes
- if the system is stable, unstable, or marginally stable

a) $H(s) = \dfrac{s-1}{s(s+2)(s+10)}$

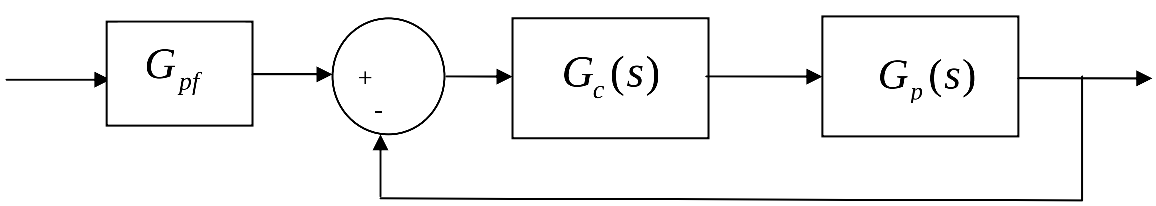
b) $H(s) = \dfrac{s(s-1)}{(s+1)^2(s^2+s+1)}$

c) $H(s) = \dfrac{1}{s^2(s+1)}$

d) $H(s) = \dfrac{s^2-1}{(s-1)(s+2)(s^2+1)}$

e) $H(s) = \dfrac{1}{(s^2+2)(s+1)}$

3) For the following sy stem, with prefilter G_{pf} , plant $G_p(s) = \dfrac{1}{s+1}$, and controller $G_c(s)$



a) Assume the prefilter is absent (i.e., it's a 1), and determine the controller so that the closed loop system matches a second order ITAE optimal system, i.e., so that the closed loop transfer function is

$$G_0(s) = \frac{\omega_0^2}{s^2 + 1.4\omega_0 s + \omega_0^2}$$

Ans. $G_c(s) = \dfrac{\omega_0^2(s+1)}{s(s+1.4\omega_0)}$ *, note that there is a pole/zero cancellation between the controller and the plant and there is a pole at zero in the controller.*

b) Show that the damping ratio for this system is 0.7, the closed loop poles of this system are at $-0.7\omega_0 \pm j0.714\omega_0$. For faster response should ω_0 be large or small?

c) If we want the steady state error for a step input to be zero, what should the value of the prefilter be?

d) Assume the prefilter is absent (i.e., it's a 1) and determine the controller so that the closed loop system matches a third order deadbeat system, i.e., so that the closed loop transfer function is

$$G_0(s) = \frac{\omega_0^3}{s^3 + 1.90\omega_0 s^2 + 2.20\omega_0^2 s + \omega_0^3}$$

Ans. $G_c(s) = \dfrac{\omega_0^3 (s+1)}{s(s^2 + 1.9\omega_0 s + 2.20\omega_0^2)}$ *, note that there is a pole/zero cancellation between the controller and the plant and there is a pole at zero in the controller.*

e) If we want the steady state error for a step input to be zero, what should the value of the prefilter be?

Preparation for Lab 3

As you undoubtedly recall, if we have a *stable* system with transfer function $H(s)$, and the input to the system is $u(t) = A\cos(\omega t + \theta)$, then the steady state output is given by

$$y(t) = |H(j\omega)| A\cos(\omega t + \theta + \angle H(j\omega))$$

This is really nothing more than a phasor relationship

$$Y = \left[|H(j\omega)| \angle H(j\omega)\right]\left[|U(j\omega)| \angle U(j\omega)\right]$$

or

$$\begin{aligned} |Y| &= |H(j\omega)||U(j\omega)| \\ \angle Y &= \angle H(j\omega) + \angle U(j\omega) \end{aligned}$$

4) Assume

$$H(s) = \frac{s}{s+2}$$

a) If the input to this system is $u(t) = 3\cos(2t)$ determine the steady state output.

b) If the input to this system is $u(t) = 5\sin(5t + 10^o)$ determine the steady state output.

(*Ans.* $2.12\cos(2t + 45^o), 4.64\sin(5t + 31.8^o)$)

5) In addition to determining $|H(j\omega)|$ and $\angle H(j\omega)$ analytically, we can read these values from a Bode plot of the transfer function. Of course the magnitude portion of a Bode plot is in dB, and we need the actual amplitude $|H(j\omega)|$. For the system with Bode plot given in Figure 5, determine the steady state output of this system if the input is $u(t) = 5\sin(3t + 20^o)$ and $u(t) = 5\sin(2t + 20^o)$.

(*Ans.* $5\sin(3t + 20^o), 1.7\sin(2t + 65^o)$)

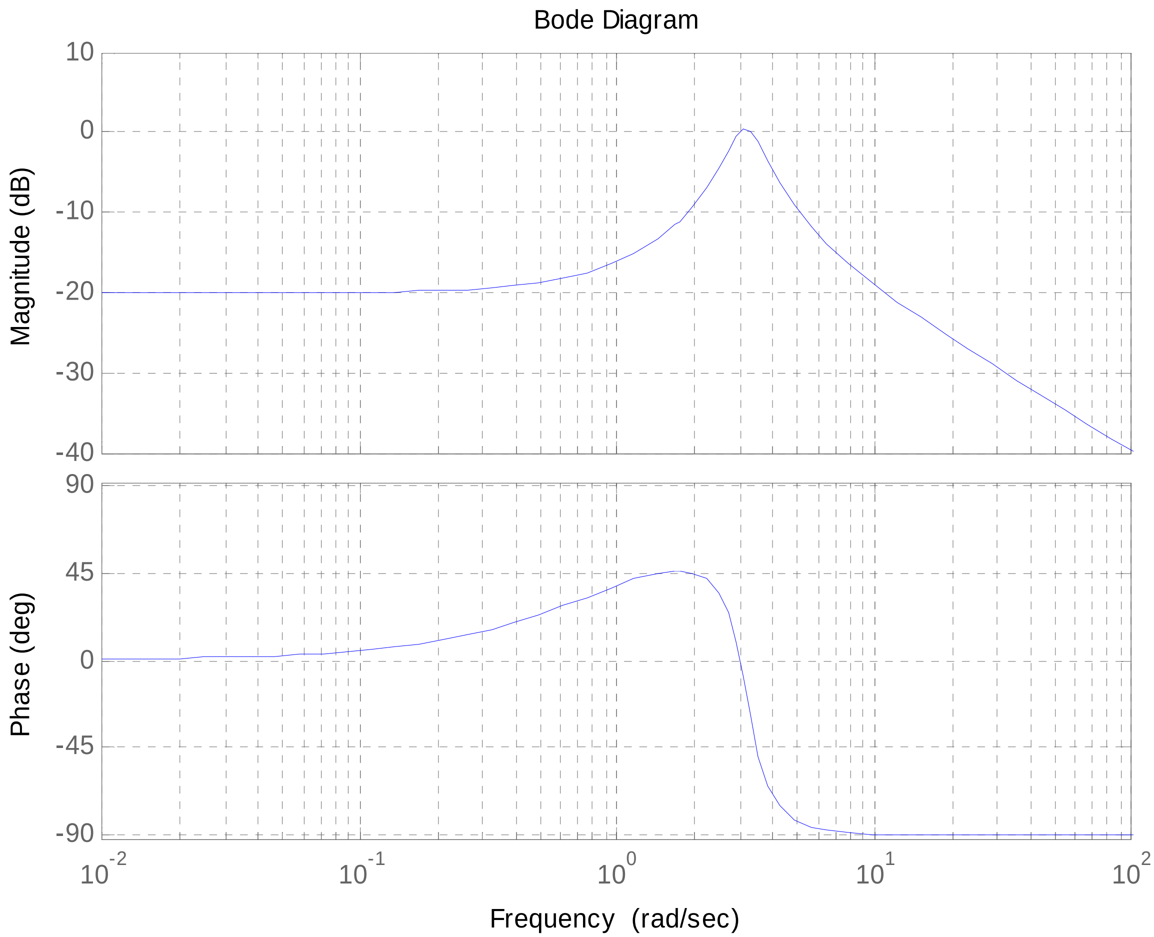


Figure 5: Bode plot of an unknown system.

6) Now we want to use the Bode plot to identify the system. Let's assume the input to an unknown system is a sequence of sinusoids, $u(t) = A\cos(\omega t + \theta)$ at different frequencies and different amplitudes. Once the transients have died out and the system is in steady state we measure the output $y(t)$. We then have the following data:

$$u(t) = 4\cos(2\pi * 0.25t) \quad y(t) = 0.089\cos(2\pi * 0.25t - 0.3^o)$$
$$u(t) = 4\cos(2\pi * 0.5t) \quad y(t) = 0.092\cos(2\pi * 0.5t - 0.6^o)$$
$$u(t) = 4\cos(2\pi * t) \quad y(t) = 0.104\cos(2\pi * t - 1.4^o)$$
$$u(t) = 3\cos(2\pi * 2t) \quad y(t) = 0.178\cos(2\pi * 2t - 6.2^o)$$
$$u(t) = 3\cos(2\pi * 2.25t) \quad y(t) = 0.321\cos(2\pi * 2.25t - 12.7^o)$$
$$u(t) = 2\cos(2\pi * 2.4t) \quad y(t) = 0.429\cos(2\pi * 2.4t - 28.1^o)$$
$$u(t) = 1\cos(2\pi * 2.5t) \quad y(t) = 0.424\cos(2\pi * 2.5t - 75.5^o)$$
$$u(t) = 1\cos(2\pi * 2.6t) \quad y(t) = 0.258\cos(2\pi * 2.6t - 142.2^o)$$
$$u(t) = 2\cos(2\pi * 2.75t) \quad y(t) = 0.218\cos(2\pi * 2.75t - 164.1^o)$$
$$u(t) = 4\cos(2\pi * 3t) \quad y(t) = 0.207\cos(2\pi * 3t - 171.9^o)$$
$$u(t) = 8\cos(2\pi * 4t) \quad y(t) = 0.115\cos(2\pi * 4t - 176.9^o)$$
$$u(t) = 10\cos(2\pi * 5t) \quad y(t) = 0.075\cos(2\pi * 5t - 178.0^o)$$
$$u(t) = 10\cos(2\pi * 6t) \quad y(t) = 0.047\cos(2\pi * 6t - 178.5^o)$$
$$u(t) = 10\cos(2\pi * 7t) \quad y(t) = 0.033\cos(2\pi * 7t - 178.8^o)$$
$$u(t) = 10\cos(2\pi * 8t) \quad y(t) = 0.024\cos(2\pi * 8t - 180^o)$$

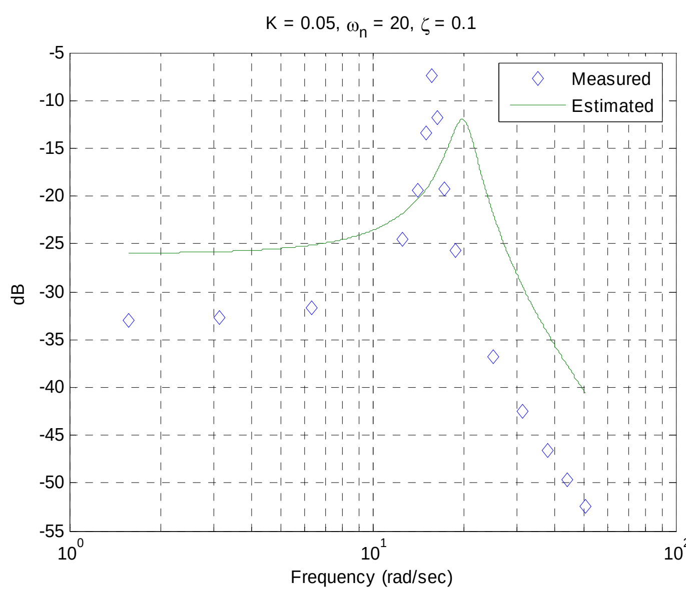
a) From this data, construct a table with the i^{th} input frequency f_i (in Hz), and the corresponding magnitude of the transfer function at that frequency , $|H_i| = |H(j2\pi f_i)|$. Note that the amplitude of the input is changing. Note that we could also utilize the phase, but we don't need that for the systems we are trying to model.

b) Now we need to try and fit a transfer function to this data, i.e., determine a transfer function that will have the same Bode plot (at least the same magnitude portion). You will need to go though the following steps (you probably want to put this in an m-file...)

```
%
% I won't tell you how to do this again, so pay attention!
%
%  Enter the measured frequency response data
%
  w = 2*pi*[f1   f2   ...   fn]      % frequencies in radians/sec
  H = [|H1|  |H2|  ...  |Hn|]        % corresponding amplitudes of the transfer function
%
%   generate 1000 points (for a smooth curve) between min(w) and max(w)
```

```
%   space them out logarithmically
%
  ww = logspace(log10(min(w)),log10(max(w)),1000);
%
%  next guess the parameters for a second order system, you will have to change these
%  to fit the data
%
  omega_n = 20;
  zeta = 0.1;
  K = 0.05
  HH = tf(K,[1/omega_n^2 2*zeta/omega_n 1];
%
%  get the frequency response, this is one of many possible ways
%
[M,P] = bode(HH,ww);
M = M(:);
%
%  Now plot them both on the same graph and make it look pretty
%
semilogx(w,20*log10(H),'d',ww,20*log10(M),'-'); grid; legend('Measured','Estimated');
ylabel('dB'); xlabel('Frequency (rad/sec)');
title(['K = ' num2str(K) ',  \omega_n = ' num2str(omega_n) ', \zeta =  ' num2str(\zeta)]);
%
%  Note there are spaces between the single quote (') and the num2str function
```

If you have not screwed up, you should get the following graph:



c) Now you need to adjust the parameters of the estimated transfer function to get the best fit. Turn in your final plot with the estimates of the parameters at the top (as in the figure above.)