ECE-320 Linear Control Systems
Homework 6

Due: Tuesday October 19, 2004

For the following three problems, it is useful to remember

- our compensator has the form $G_c(s) = \frac{B(s)}{A(s)}$, where $B(s) = B_0 + B_1 s + ...$ and $A(s) = A_0 + A_1 s + ...$
- the plant has the form $G_p(s) = N(s)/D(s)$, where $N(s) = N_0 + N_1 s + ...$ and $D(s) = D_0 + D_1 s + ...$
- The desired characteristic polynomial for the closed loop transfer function is $D_0(s) = F_0 + F_1 s + F_2 s^2 + ...$
- To determine the equations to solve set

  $$A(s)D(s) + B(s)N(s) = D_0(s)$$

  and equate coefficients of s. This will give you the system of equations to solve.
- To use the compensator to make a system a type one system, set $A_0 = 0$

1 For the plant

$$G_p(s) = \frac{1}{s(s+2)}$$

show that the first order compensator that will put the closed loop poles at $-1 \pm j$ and -3 is $G_c(s) = 2$.

2 For the plant

$$G_p(s) = \frac{1}{s+2}$$

Assume we want to place both closed loop poles at -4 and also have a type 1 system. Show that the first order compensator that will do this is given by

$$G_c(s) = \frac{6(s+2.667)}{s}$$

3 For a plant like the systems we have in lab with transfer function given by

$$G_p(s) = \frac{6000}{s^2 + 3.2s + 400}$$

a) Show that the first order compensator that places all three closed loop poles at -5 is given by

$$G_c(s) = \frac{-0.0605s - 0.7658}{s + 11.8}$$

Note that the numerator of this compensator make it pretty useless for tracking a step input.

b) Show that the second order system that places all four closed loop poles at -5 and produces a type 1 system is given by

$$G_c(s) = \frac{-0.05063s^2 - 1.0367s + 0.1042}{s(s + 16.8)}$$

4 For the systems described by

$$G_{p1}(s) = \frac{28.6}{0.0012s^2 + 0.0073s + 1}$$

$$G_{p2}(s) = \frac{20.6}{0.0024s^2 + 0.0018s + 1}$$

use the program **diophantine.m** on the web site to construct both a first and second order controller to meet the desired constraints:

- the settling time is less than 0.5 sec
- the position error is less than 0.2 cm
- the percent overshoot less than 25%
- the magnitude of the real part of the closed loop poles is less than 100.

The program **diophantine** has the following input arguments

- the amplitude of the step input (in cm)
- the system transfer function, in the form

$$G_p(s) \quad = \quad \frac{K_{clg}}{\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1}$$

- the location of the desired closed loop poles as a row vector (three poles for a first order controller, four poles for a second order controller)
- the order of the controller (1 or 2). If a second order controller is chosen, one pole will be automatically placed at the origin, making a first order system
- the final time to run the simulation to
- the filename containing the ECP data. If the ECP data is not yet available, enter '' (two single quotes).