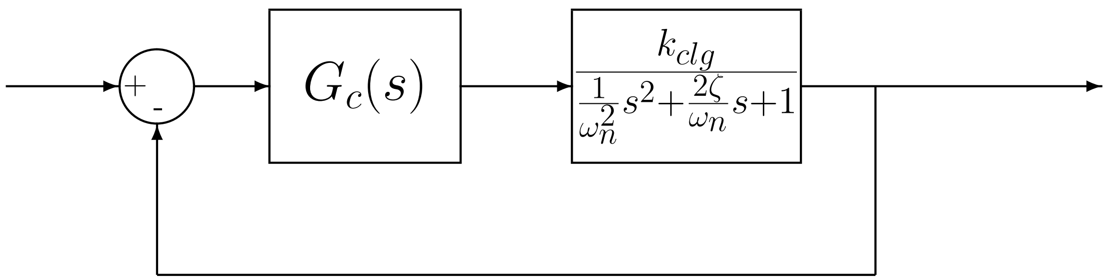
ECE-320 Linear Control Systems
Homework 5

Due: Tuesday October 12, 2004

1 Consider the following system, with plant $G_p(s)$ and controller $G_c(s)$. Assume unit inputs. I want you to find e_p and e_v for each of the following controller types (I am telling you the answers, you are supposed to show how you got them.)



a) Show that for a proportional (P) controller, where

$$G_c(s) = k_p$$

$$e_p = \frac{1}{1+k_p k_{clg}}$$
$$e_v = \infty$$

b) Show that for an integral (I) controller, where

$$G_c(s) = \frac{k_i}{s}$$

$$e_p = 0$$
$$e_v = \frac{1}{k_i k_{clg}}$$

c) Show that for a proportional+integral (PI) controller, where

$$G_c(s) = k(s+z)/s$$

$$e_p = 0$$
$$e_v = \frac{1}{kzk_{clg}}$$

d) Show that for a proportional+derivative (PD) controller, where

$$G_c(s) = k(s+z)$$

$$e_p = \frac{1}{1+kzk_{clg}}$$
$$e_v = \infty$$

e) Show that for a proportional+integral+derivative (PID) controller, where

$$G_c(s) = k(s+z_1)(s+z_2)/s$$

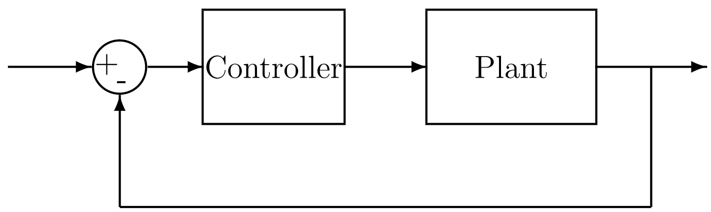
$$e_p = 0$$
$$e_v = \frac{1}{kz_1z_2k_{clg}}$$

f) Show that for a lead or lag controller, where

$$G_c(s) = k(s+z)/(s+p)$$

$$e_p = \frac{1}{1+k\frac{z}{p}k_{clg}}$$
$$e_v = \infty$$

For problems 2-4, we will be examining the root locus for three different plants (systems we are trying to control) with various common controller types. We will assume a unity feedback configuration as shown below:



For each part of each problem, you need to sketch the root locus *with arrows showing the direction of travel as k increases.* If there are any poles going to zeros at infinity, you need to

- compute the centroid of the asymptotes (σ)
- compute the angles of the asymptotes

You may check your answers using a computer, but you should do this assignment without the use of a computer. Keep in mind that we usually want to move the roots of the closed loop transfer function away from the $j\omega$ axis for faster response.

2 Assume the system/device we wish to control has a transfer function given by

$$G(s) = \frac{1}{s^2 + 4s + 29} = \frac{1}{(s + 2 - 5j)(s + 2 + 5j)}$$

sketch the root locus of the closed loop system with each of the controller types listed below:

a) $G_c(s) = k$ (a proportional controller)

b) $G_c(s) = k/s$ (an integral controller)

c) $G_c(s) = k(s + z)/s$ (a proportional + integral controller) Write σ as a function of z. For what values of z will two of the the asymptotes be in the right half plane?

d) $G_c(s) = k(s + z)$ (a proportional + derivative controller)

e) $G_c(s) = k(s + z_1)(s + z_2)/s$ (a proportional+integral+derivative controller). Sketch this for the case when z_1 and z_2 are complex conjugates, and then z_1 and z_2 are both real.

f) $G_c(s) = k(s + z)/(s + p)$ ($p > z$, a lead controller). Write an expression for σ when $p = z + l$. What happens to the asymptotes as l gets larger?

3 Assume the system/device we wish to control has a transfer function given by

$$G(s) = \frac{1}{s(s+1)}$$

sketch the root locus of the closed loop system with each of the controller types listed below:

a) $G_c(s) = k$ (a proportional controller)

b) $G_c(s) = k/s$ (an integral controller). Is this system stable for any values of k?

c) $G_c(s) = k(s+z)/s$ (a proportional + integral controller) Write σ as a function of z. For what values of z will two of the the asymptotes be in the right half plane?

d) $G_c(s) = k(s+z)$ (a proportional + derivative controller). There are two cases here, the first case is when the zero is between the poles, and the other case is when the zero is to the left both of the poles.

e) $G_c(s) = k(s+z_1)(s+z_2)/s$ (a proportional+integral+derivative controller). There are four cases here.

1. z_1 and z_2 are complex conjugates

2. z_1 and z_2 are real and
   (a) both zeros are to the left of all the poles
   (b) the pattern is two poles, two zeros, and a pole
   (c) the pattern is two poles, a zero, a pole, and a zero

f) $G_c(s) = k(s+z)/(s+p)$ ($p > z$, a lead controller). Assume the zero of the lead is to the left of both poles of $G(s)$. Write an expression for σ when $p = z + l$. What happens to the asymptotes as l gets larger?

4 Assume the system/device we wish to control has a transfer function given by

$$G(s) = \frac{1}{(s+4)(s+1)}$$

sketch the root locus of the closed loop system with each of the controller types listed below:

a) $G_c(s) = k$ (a proportional controller)

b) $G_c(s) = k/s$ (an integral controller)

c) $G_c(s) = k(s+z)/s$ (a proportional + integral controller) Write σ as a function of z. There are three cases

1. the zero between 0 and -1
2. the zero between -1 and -4
3. the zero to the left of -4

For what values of z will two of the the asymptotes be in the right half plane?

d) $G_c(s) = k(s+z)$ (a proportional + derivative controller). There are three cases

1. the zero between 0 and -1
2. the zero between -1 and -4
3. the zero to the left of -4

e) $G_c(s) = k(s+z_1)(s+z_2)/s$ (a proportional+integral+derivative controller). Sketch this for the case when z_1 and z_2 are complex conjugates, and then z_1 and z_2 are both real and to the left of all of the poles (there are more cases than this!).

f) $G_c(s) = k(s+z)/(s+p)$ ($p > z$, a lead controller). Assume the zero of the lead is to the left of both poles of G(s). Write an expression for σ when $p = z + l$. What happens to the asymptotes as l gets larger?