**ECE-320** Linear Control Systems
Homework 4

Due: Tuesday September 28

$\boxed{1}$ For a system with plant

$$G_p(s) = \frac{s+3}{s(s-1)}$$

show that the quadratic optimal closed loop transfer function is

$$G_0(s) = \frac{10(s+3)}{s^2 + 12.7s + 30}$$

when $q = 100$.

What are $e_p$ and $e_v$ for this system? (*Ans.* $e_p = 0$, $e_v = 0.09$)

$\boxed{2}$ For a system with plant

$$G_p(s) = \frac{s-1}{s(s-2)}$$

show that the quadratic optimal closed loop transfer function is

$$G_0(s) = \frac{-10(s-1)}{s^2 + 11.1s + 10}$$

when $q = 100$.

What are $e_p$ and $e_v$ for this system? (*Ans.* $e_p = 0$, $e_v = 2.11$)

$\boxed{3}$ For a one degree of freedom system like we have in lab, with plant

$$G_p(s) = \frac{15}{0.0025s^2 + 0.0080s + 1}$$

a) Show that when $q = 0.1$ the quadratic optimal closed loop transfer function is

$$G_0(s) = \frac{1856.6}{s^2 + 55.5s + 1939.1}$$

and the position error is $e_p = 0.043$.

b) Show that the controller is given by

$$G_c(s) = \frac{0.0038s^2 + 0.012s + 1.5}{0.012s^2 + 0.67s + 1}$$

c) Using the *quadratic.m* program, plot the step response of this system for $q = 0.01$, $q = 0.1$, and $q = 1$. To use this program (which you'll be using in lab this week), you first need to enter the estimated plant transfer function in the form

$$G_p(s) = \frac{K_{clg}\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

**quadratic.m** has the input arguments

- The amplitude of the step input (assume 1 cm, so enter 1)
- The plant transfer function $G_p(s)$
- The value of $q$.
- The length of time to plot the results (be sure the system has reached steady state, but not too long).
- The filename with data to compare the model to. In this case, type " (two single quotes). In lab you'll generate data for this part.

You should see that as $q$ increases, which means the penalty on the difference between input and output is getting larger, the system should produce a smaller and smaller position error and response more and more quickly. If your final position error is not near 0, you've probably made a scaling mistake.

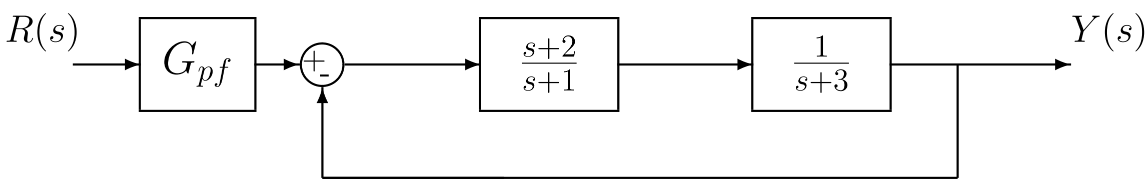
$\boxed{4}$ For the systems on the following page:

a) Determine the system type.

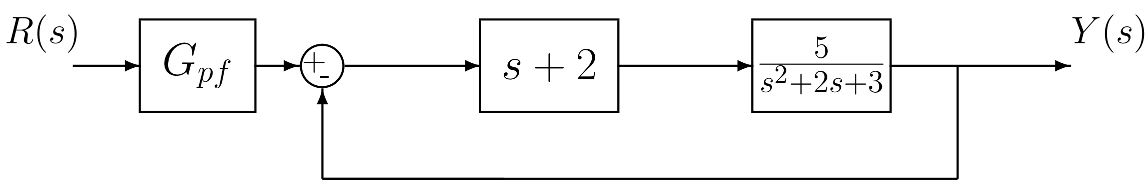
b) If the system is type 0 assume $G_{pf} = 1$ and determine the position error constant $K_p$ and the position error $e_p$. Then determine the value of $G_{pf}$ that makes the position error zero.

c) If the system is type 1, assume $G_{pf} = 1$ and determine the position error, the velocity error constant $K_v$, and the velocity error $e_v$. Is there any *constant* value of $G_{pf}$ that can change the velocity error?

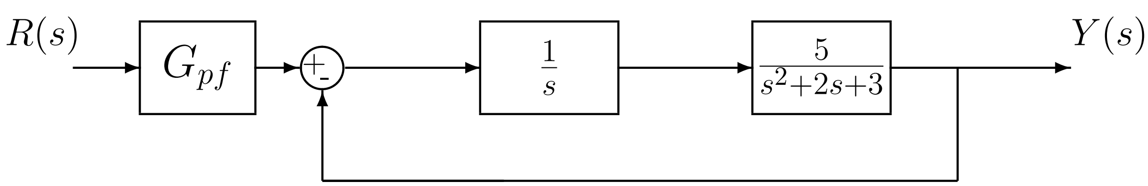
*Ans.* $e_p = \frac{3}{5}$ *and* $G_{pf} = 5/2$, $e_p = 3/13$ *and* $G_{pf} = 13/10$, $e_v = 3/5$, $e_v = 4/5$, $G_{pf}$ *has no effect on* $e_v$.



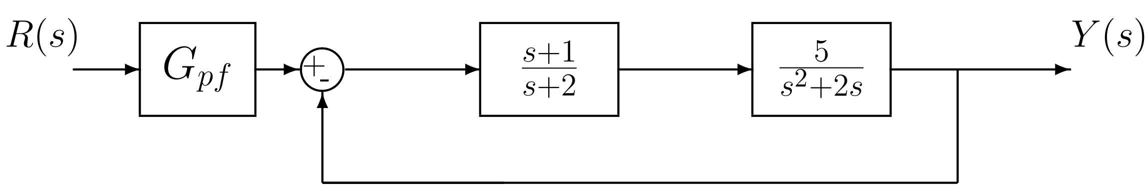
$R(s)$
$G_{pf}$
+
-
$\frac{s+2}{s+1}$
$\frac{1}{s+3}$
$Y(s)$

$R(s)$
$G_{pf}$
+
-
$s+2$
$\frac{5}{s^2+2s+3}$
$Y(s)$

$R(s)$
$G_{pf}$
+
-
$\frac{1}{s}$
$\frac{5}{s^2+2s+3}$
$Y(s)$

$R(s)$
$G_{pf}$
+
-
$\frac{s+1}{s+2}$
$\frac{5}{s^2+2s}$
$Y(s)$