

ECE-320 Linear Control Systems
Homework 3

Due Date: Tuesday September 21

Problems:

1] For the following transfer functions:

$$\begin{aligned} a) H(s) &= \frac{s^2 + 3}{s^4 + 3s^3 + 6s^2 + 7s + 3} \\ b) H(s) &= \frac{3}{s^4 + 3s^3 + 6s^2 + 7s + 3} \\ c) H(s) &= \frac{43s + 5}{s^4 + 14s^3 + 44s^2 + 43s + 5} \\ d) H(s) &= \frac{s^2 + s + 0.0125}{s^5 + 1.75s^4 + 1.475s^3 + 0.625s^2 + 0.1375s + 0.0125} \end{aligned}$$

- 1) Using the pole command in Matlab, find the poles.
- 2) Estimate the settling time (4 time constants) Complex poles generally have the form $-1/\tau \pm j\omega_d = -\sigma \pm j\omega_d$ where τ is the time constant. Compute the settling time associated with each pole. The system settling time is **largest** settling time.
- 3) The pole(s) associated with the largest settling time is (are) the *dominant* poles. Determine the dominant poles for each system.
- 4) Plot the step response of the system using Matlab for 30 seconds. Use the subplot command to plot all four graphs on one page. Using the formulas derived in class estimate the position error for each system and identify the position error on the graphs (draw on the graph to show you know what the position error is.)
- 5) Plot the ramp response of the system using Matlab for 30 seconds. Use the subplot command to plot all four graphs one one page. Using the formulas derived in class estimate the velocity error for each system and identify the velocity error on the graphs (draw on the graph to show you know what the velocity error is).

Turn in your Matlab plots for each system!

Ans. $T_s = 8, 8, 29.9$ and 16 . One of these is not a very good approximation. The formula for T_s ($T_s = 4\tau = 4/\sigma$) assumes an ideal second order system. It is just a guideline., $e_p = 0$ for all. $e_v = 2.333, 2.333, 0, 69.0$

2] For the following transfer functions:

I) determine the characteristic polynomial

II) determine the characteristic modes

III) determine if the system is stable, unstable, or marginally stable.

$$a) H(s) = \frac{s + 1}{(s)(s + 2)(s + 10)}$$

$$b) H(s) = \frac{(s - 1)(s + 2)}{(s + 1)^2(s^2 + s + 1)}$$

$$c) H(s) = \frac{1}{s^2(s + 1)}$$

$$d) H(s) = \frac{s^2 - 2}{(s - 1)(s + 2)(s^2 + 1)}$$

$$e) H(s) = \frac{1}{(s^2 + 2)(s + 1)}$$

3] For each of the following groups of poles for a given transfer function, determine the form of the corresponding impulse response (time domain function) :

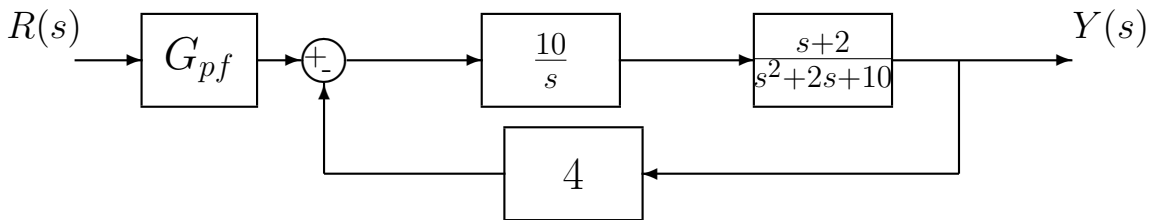
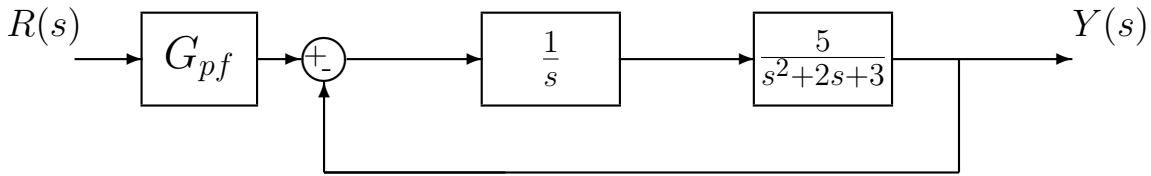
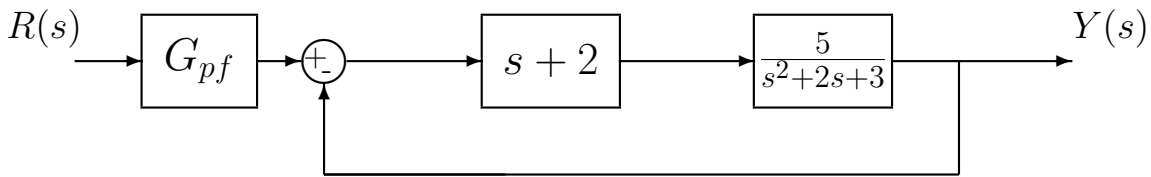
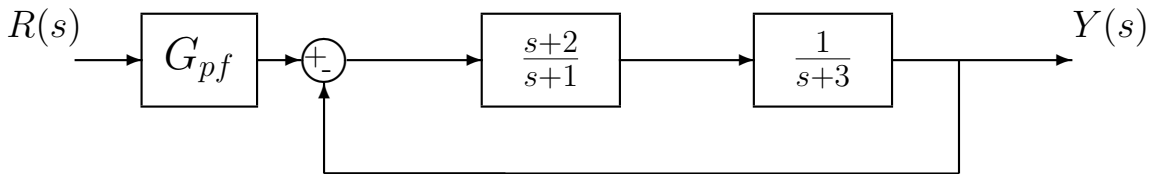
a) $-1, -2, -5 \pm 3j$

b) $1, -2 \pm 2j, -1$ (*repeated*)

c) 0 (*repeated*), $-1 \pm j$ (*repeated*)

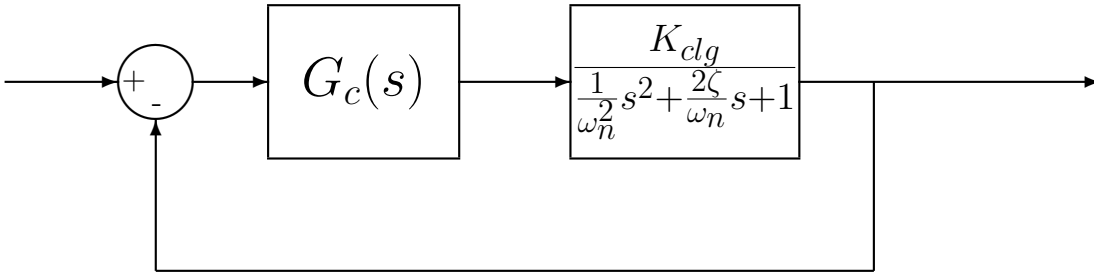
(If a pole is repeated, it is a double pole)

4 For the following systems, determine the value of G_{pf} so that the position error of the closed loop system is zero. (*Hint: Think of the easiest way to do this, you never need to write out the closed loop transfer function as the ratio of two polynomials. You most definitely do not need Maple, and will not have Maple on an exam.*) Then simulate each system (using Matlab) for a step response. Run each simulation until the system comes to steady state. Use **subplot** to put all four plots on one sheet of paper (you may want to type **orient tall** so more of the page is used).



Ans. $G_{pf} = \frac{5}{2}, 1.3, 1, 4$

5] Assume we have the unity feedback system shown below, which models the one degree of freedom systems you have been using in the lab. $G_c(s)$ is a controller that you will be implementing, ω_n and ζ are estimates obtained using either time-domain or frequency domain methods, and K_{clg} is the system closed loop gain. Note that we are modelling the motor as contributing only a gain to the system, and we are lumping both the plant's gain and the motor gain together into one parameter.



Show that for a *proportional controller*,

$$G_c(s) = k_p,$$

the steady state output y_{ss} due to a step input of amplitude Amp is given by

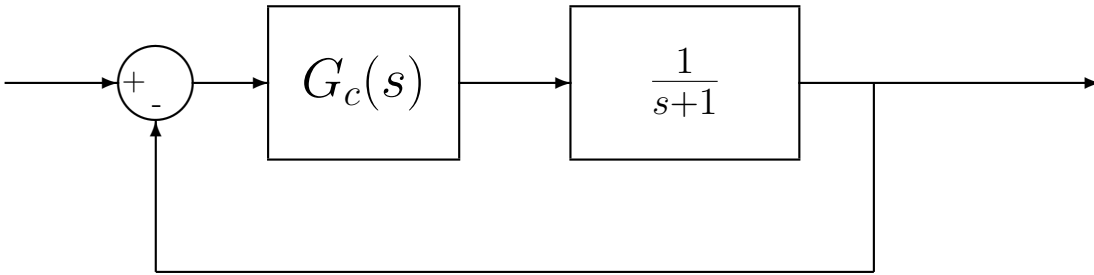
$$y_{ss} = \frac{Amp k_p K_{clg}}{1 + k_p K_{clg}}$$

which can be rewritten as

$$K_{clg} = \frac{y_{ss}}{k_p} \frac{1}{Amp - y_{ss}}$$

This is the expression we will use in lab to estimate the closed loop system gain K_{clg} .

6] Consider the following system, with plant $G_p(s) = \frac{1}{s+1}$ and controller $G_c(s)$.



a) Determine the controller so that the closed loop system matches a second order zero position error ITAE optimal system, i.e., so that the closed loop transfer functions is

$$G_0(s) = \frac{\omega_0^2}{s^2 + 1.4\omega_0 s + \omega_0^2}$$

Ans. $G_c(s) = \frac{\omega_0^2(s+1)}{s(s+1.4\omega_0)}$. Note that there is a pole/zero cancellation between the controller and the plant.

b) Determine the controller so that the closed loop system matches a second order zero velocity error ITAE optimal system, i.e., so that the closed loop transfer functions is

$$G_0(s) = \frac{3.2\omega_0 s + \omega_0^2}{s^2 + 3.2\omega_0 s + \omega_0^2}$$

Ans. $G_c(s) = \frac{(3.2\omega_0 s + \omega_0^2)(s+1)}{s^2}$. Note that there is a pole/zero cancellation between the controller and the plant.

c) Simulate the step response system in part (a) for $\omega_0 = 10, 20, 40$ and 80 for 1 second. Use **subplot** to put all four graphs on one sheet of paper.