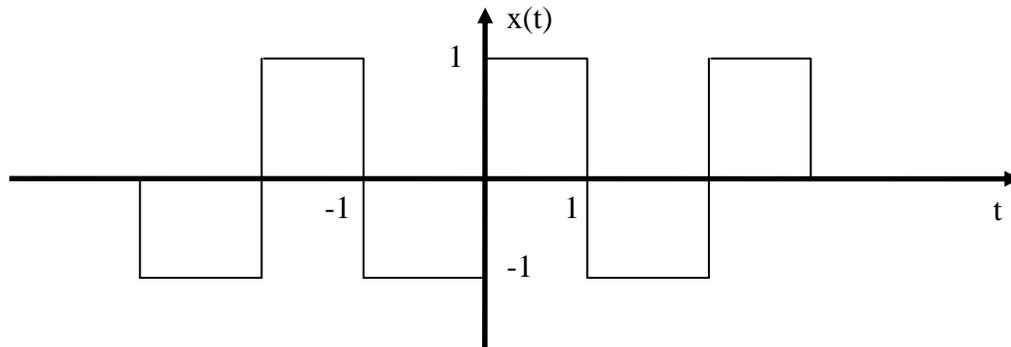Name _____ CM_____

ECE 300
Signals and Systems

Exam 3
17 February, 2009

This exam is closed-book in nature. Credit will not be given for work not shown.___You may not use calculators!___

Problem 1   _____ / 30
Problem 2   _____ / 30
Problem 3   _____ / 30
Problem 4   _____ / 10

Exam 3 Total Score: _____ / 100

1. Fourier Series (30 points) The plot of x(t) can be used to answer parts (a) and (b) below.

(a) Determine the Fourier Series coefficients for x(t). Do NOT express your answer as a sinc function, but do simplify your answer as much as possible.

(b) You found the Fourier Coefficients of a waveform that is different from the one above to be as shown below. Express these coefficients as a sinc function with a possible gain phase term.

$$c_k = \frac{-3}{j2\pi k}\left[e^{-jk\frac{4\pi}{3}} - e^{-jk\frac{2\pi}{3}} \right]$$

2. (30 points) An LTI system has the following transfer function

$$H(\omega) = \begin{cases} 6e^{-j2\omega} & 4 < |\omega| < 6 \\ 0 & \text{otherwise} \end{cases}$$

The input to the system is given by

$$x(t) = \text{sinc}\left(\frac{5t}{\pi}\right)$$

a) What is $X(\omega)$?

b) What is $y(t)$?

c) What is the energy in $x(t)$?

3. (30 points) Determine $X(\omega)$ or $x(t)$. You must show your work. You may leave solutions in terms of integrals, derivatives, or convolutions (i.e. you do not have to explicitly solve them), but you must go further than the basic Fourier/Inverse Fourier Transform Definitions.

a) For $x(t) = e^{-4t^2}$ determine the corresponding Fourier transform $X(\omega)$

b) For $X(\omega) = \dfrac{2}{(1+j\omega)^2}$, determine the corresponding inverse Fourier transform $x(t)$

c) For $x(t) = \dfrac{2}{2+\left[\dfrac{2}{3}t-2\right]^2}$, determine the corresponding Fourier transform $X(\omega)$

4. (10 points) Starting from the Fourier transform (or inverse transform) integrals,

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

If $x(t) \leftrightarrow X(\omega)$ then derive the corresponding Fourier Transform of $x(-(t-a))$

Name _____ CM_____

Some Potentially Useful Relationships

$$E_\infty = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$e^{jx} = \cos(x) + j\sin(x) \qquad j = \sqrt{-1}$$

$$\cos(x) = \frac{1}{2}\left[e^{jx} + e^{-jx}\right] \qquad \sin(x) = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right]$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x) \qquad \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\text{rect}\left(\frac{t - t_0}{T}\right) = u\left(t - t_0 + \frac{T}{2}\right) - u\left(t - t_0 - \frac{T}{2}\right)$$