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ECE 300
Signals and Systems

Exam 2
29 January, 2009

This exam is closed-book in nature. Credit will not be given for work not shown. ***<u>You may use calculators for simple calculations, but show what you are calculating!</u>***

Problem 1	________	/ 30
Problem 2	________	/ 25
Problem 3	________	/ 20
Problem 4	________	/ 25
Exam 2 Total Score:	________	/ 100

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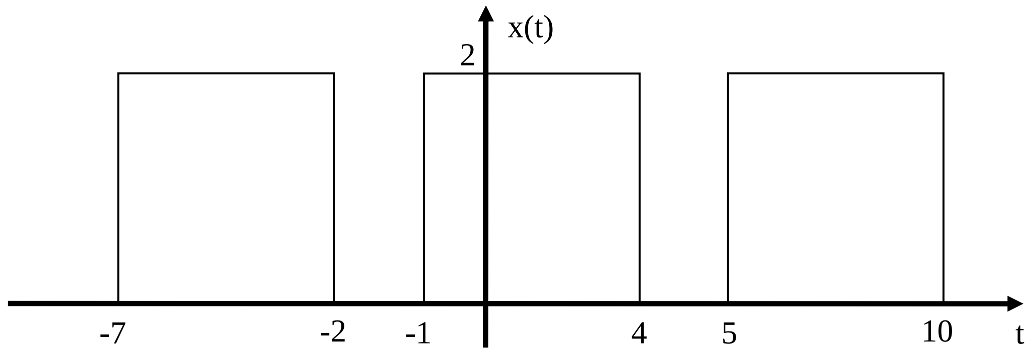
1. (30 points) The next few questions are based on the plot of x(t) below.



(a) What is the fundamental frequency of x(t) in r/s?

(b) Determine the Fourier Series coefficients for x(t). Express your answer in terms of a sinc function with a gain and phase term if necessary.

(c) If a periodic signal x(t) is inserted into a system given by $y(t) = x(t+1.5) - 1$, express the Fourier coefficients of the output y(t), c_k^y, in terms of the input Fourier coefficients of x(t), c_k^x. This problem is independent of part (b).

$c_0^y =$

$c_k^y =$

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2. (25 points) A periodic signal $x(t)$ is the input to an LTI system with output $y(t)$. The signal $x(t)$ has period 2 seconds, and is given over one period as

$$x(t) = e^{-t} \quad 0 < t < 2$$

$x(t)$ has the Fourier series representation

$$x(t) = \sum_k \frac{0.4323}{1 + jk\pi} e^{jk\pi t}$$

The system is an ideal lowpass filter with unit amplitude that eliminates all signals with frequency content higher than 1.25 Hz.

a) Find the average power in $x(t)$.

b) Determine an expression for the output, $y(t)$. Your expression for $y(t)$ must be real, and written in terms of sines and/or cosines.

c) Determine the average power in $y(t)$. *Be sure to show what you are calculating, not just the final answer!*

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3. (20 points) A periodic signal $x(t)$ with period $T_0 = 3$ seconds is the input to an LTI system with transfer function $H(j\omega)$. The relevant non-zero spectra are shown below. Determine an expression for the output of the system $y(t)$. All angles are multiples of 45 degrees. Your final answer must be in terms of sines and/or cosines. *You do not need to simplify you answer.*

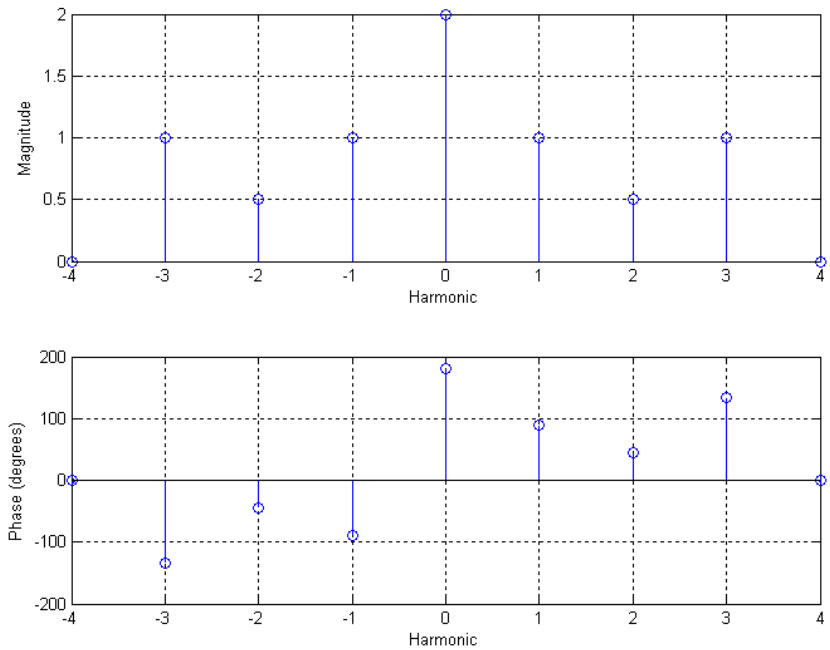

Figure 1. Spectrum of the input signal $x(t)$.

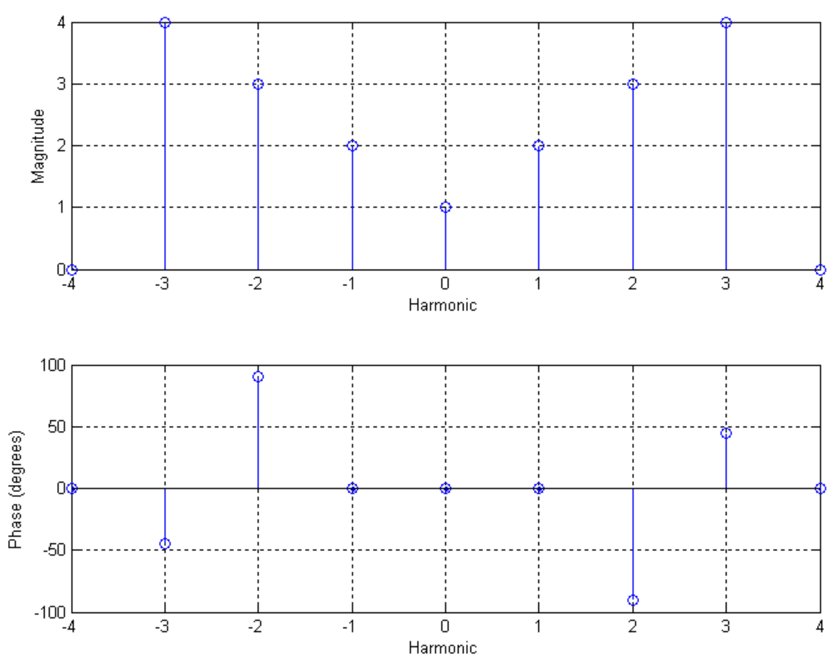

Figure 2. Samples of the transfer function $H(jk\omega_0)$

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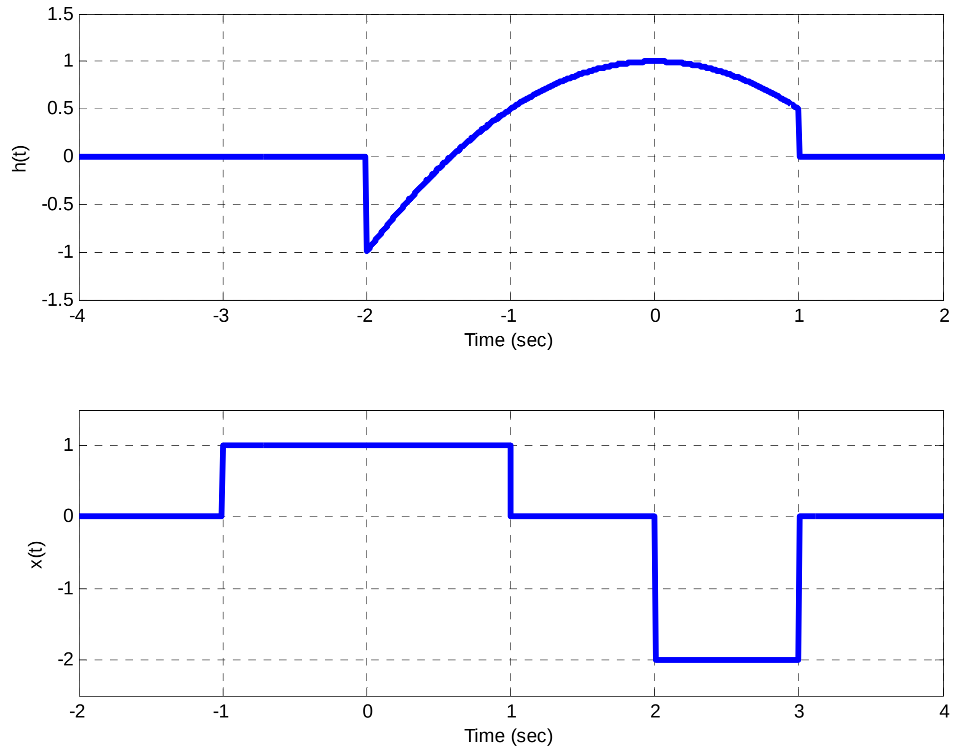
4. (25 points) Consider a causal linear time invariant system with impulse response

$$h(t) = \left[1 - 0.5t^2\right][u(t+2) - u(t-1)]$$

The input to the system is

$$x(t) = u(t+1) - u(t-1) - 2u(t-2) + 2u(t-3)$$

These two functions are plotted below:



Using ***graphical convolution***, set up the integrals to determine the output $y(t)$ **for** $t \le 1$. **Note that we are only interested in a limited range!!**

Specifically, you must

- Flip and slide $h(t)$
- Show graphs displaying $h(t-\lambda)$ relative to $x(\lambda)$ for each region of interest.
- Determine the ranges of t for which each part of your solution is valid.
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete and simplified as much as possible (no unit step functions)
- ***Do Not Evaluate the Integrals!!***

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Some Potentially Useful Relationships

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$e^{jx} = \cos(x) + j\sin(x) \qquad j = \sqrt{-1}$$

$$\cos(x) = \frac{1}{2}\left[e^{jx} + e^{-jx}\right] \qquad \sin(x) = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right]$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x) \qquad \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\text{rect}\left(\frac{t-t_0}{T}\right) = u\left(t - t_0 + \frac{T}{2}\right) - u\left(t - t_0 - \frac{T}{2}\right)$$