Name Solutions CM

ECE 300
Signals and Systems

Exam 1
12 January, 2009

This exam is closed-book in nature. Credit will not be given for work not shown.
You may not use calculators!

Problem 1-10	________	/ 20
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Exam 1 Total Score:	________	/ 100

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Multiple Choice Questions (20 points, 2 points each)

1) For the system described by the differential equation $\dot{y}(t) - 2y(t) = x(t+1)$, the impulse response is

a) $h(t) = e^{-2(t+1)}u(t)$ **b)** $h(t) = e^{2(t-1)}u(t+1)$ **c)** $h(t) = e^{-2(t+1)}u(t+1)$ $\boxed{\textbf{d)}\ h(t) = e^{2(t+1)}u(t+1)}$

$\dot{h} - 2h = \delta(t+1)$

$\frac{d}{dt}\left(he^{-2t}\right) = e^{-2t}\delta(t+1) = e^{2}\,\delta(t+1)$

$h(t)\,e^{-2t} = \int_{-\infty}^{t} e^{2}\delta(\lambda+1)d\lambda = e^{2}u(t+1)$

$\boxed{h(t) = e^{2(t+1)}u(t+1)}$

2) What is the period of the periodic signal $x(t) = \sin\left(\frac{1}{2}t + \pi\right) + e^{j\frac{t}{3}}$

a) π **b)** 2π **c)** 6π $\boxed{\textbf{d)}\ 12\pi}$ **e)** none of these

$\frac{T}{2} = g(2\pi)$ $\frac{T}{3} = r(2\pi)$ $T = g(4\pi) = r(6\pi)$ g, r integers

$g = 3$ $r = 2$ $\boxed{T = 12\pi}$

3) The signal $x(t) = \cos(t)\left[u(t) - u(t-10)\right]$ is

a) a power signal $\boxed{\textbf{b)}\ \text{an energy signal}}$ **c)** neither a power nor energy signal

4) The signal $x(t) = 2e^{j3t}$ is

$\boxed{\textbf{a)}\ \text{a power signal}}$ **b)** an energy signal **c)** neither a power nor energy signal

$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |2e^{j3t}|^2 dt = \lim_{T\to\infty} \frac{1}{2T} 4\cdot 2T = 4$

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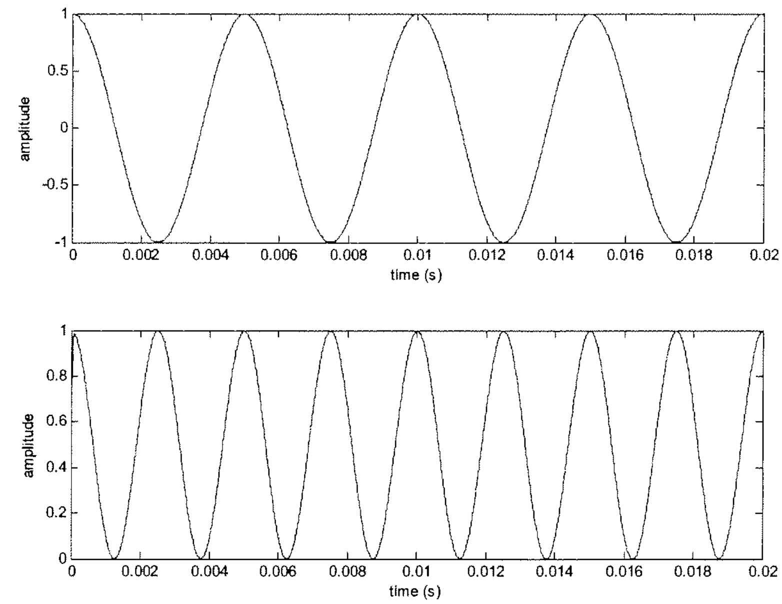
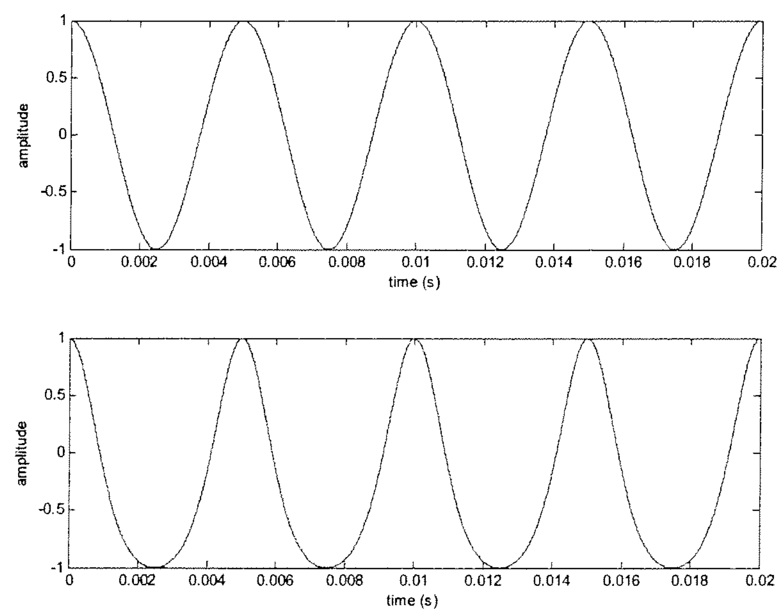
5) Consider a system with the following input (top panel) and output (bottom panel) . Is the system linear?

a) Yes b) No c) it is not possible to determine input and output frequencies are not the same



6) Consider a system with the following input (top panel) and output (bottom panel) . Is the system linear?

a) Yes b) No c) it is not possible to determine output is not a cosine



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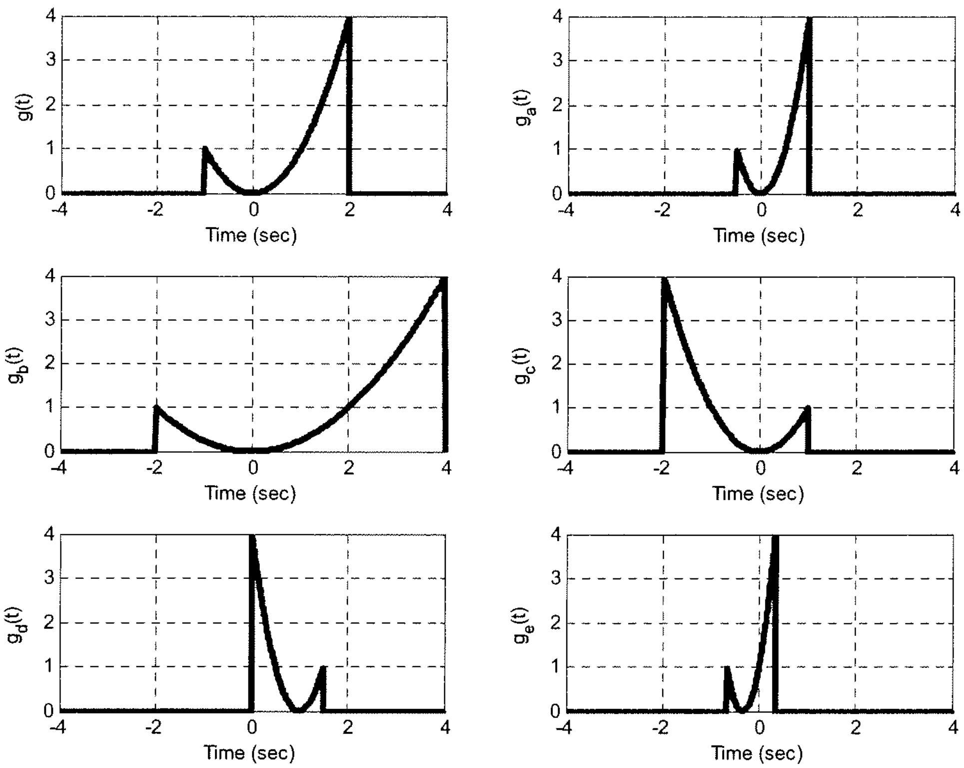
Problems 7-9 refer to the following signals. The original signal, $g(t)$, is in the upper left corner



7) Which signal best represents $g(-t)$?

$g(-1) = g(-t) \quad t = 1$

$g(2) = g(-t) \quad t = -2$

a) $g_a(t)$ **b)** $g_b(t)$ **c)** $g_c(t)$ **d)** $g_d(t)$ **e)** $g_e(t)$

8) Which signal best represents $g(2(1-t))$?

$g(-1) = g(2(1-t)) \quad -1 = 2(1-t) \quad t = 3/2$

$g(2) = g(2(1-t)) \quad 2 = 2(1-t) \quad t = 0$

a) $g_a(t)$ **b)** $g_b(t)$ **c)** $g_c(t)$ **d)** $g_d(t)$ **e)** $g_e(t)$

9) Which signal best represents $g\left(\frac{t}{2}\right)$?

$g(-1) = g\left(\frac{t}{2}\right) \quad -1 = \frac{t}{2} \quad t = -2$

$g(2) = g\left(\frac{t}{2}\right) \quad 2 = \frac{t}{2} \quad t = 4$

a) $g_a(t)$ **b)** $g_b(t)$ **c)** $g_c(t)$ **d)** $g_d(t)$ **e)** $g_e(t)$

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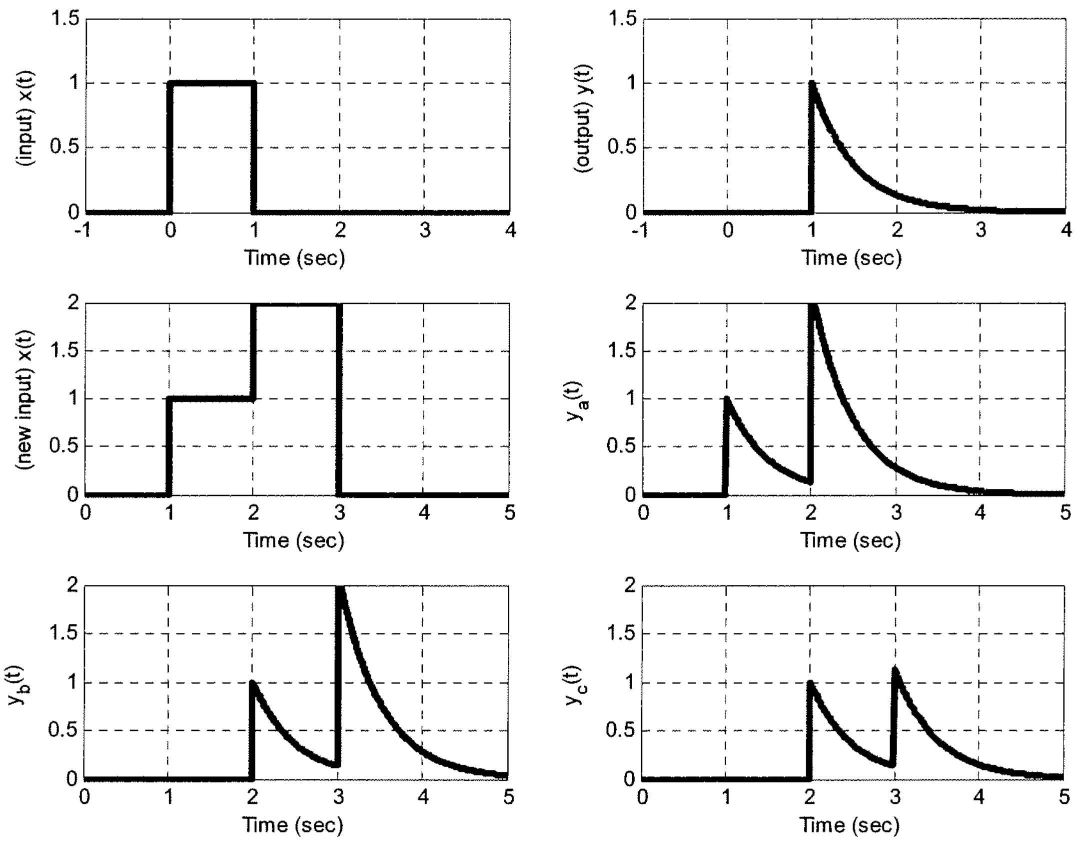
10) Consider an LTI system with known input $x(t)$ and corresponding output $y(t)$ shown in the top row in the figure below. Assume a new input to the system, shown in the left of the middle row. The output of this LTI system will be

a) $y_a(t)$ **b)** $y_b(t)$ **c)** $y_c(t)$ **d)** none of these



$$x(t) \rightarrow y(t)$$

$$x(t-1) + 2x(t-2) \rightarrow y(t-1) + 2y(t-2)$$

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11. System Properties (25 points)

a) Fill in the following table with a Y (Yes) or N (No). Only your responses in the table will be graded, not any work. Assume $x(t)$ is the system input , $y(t)$ is the system output, and $h(t)$ is the impulse response for an LTI system. Also assume we are looking at all times (positive and negative times).

System	Linear ?	Time-Invariant?	Memoryless?	Causal?
$\dot{y}(t)+y(t)=x(t+1)+t$	N	N	N	N
$y(t)=x\left(\frac{t-1}{2}\right)$	Y	N	N	N

b) Determine whether or not the following system is linear and time-invariant. You will be graded on your procedure, not just the final answer. Be sure to show your work.

$$y(t)=\int_{-\infty}^{t}(x(\lambda)+\lambda)d\lambda$$

Linearity

$$z_1 = ay_1(t) + by_2(t) = a\int_{-\infty}^{t}(x_1(\lambda)+\lambda)d\lambda + b\int_{-\infty}^{t}(x_2(\lambda)+\lambda)\,d\lambda$$

$$= \int_{-\infty}^{t}\left[ax_1(\lambda)+bx_2(\lambda)+\lambda(a+b)\right]d\lambda$$

$$z_2 = \int_{-\infty}^{t}\left[ax_1(\lambda)+bx_2(\lambda)+\lambda\right]d\lambda$$

$z_1 \neq z_2$ so non-linear

Time Invariance

$$z_1 = \int_{-\infty}^{t}(x(\lambda-t_0)+\lambda)\,d\lambda = \int_{-\infty}^{t-t_0}(x(\sigma)+\sigma+t_0)\,d\sigma$$

$\sigma = \lambda - t_0$
$d\sigma = d\lambda$

$$z_2 = y(t-t_0) = \int_{-\infty}^{t-t_0}(x(\lambda)+\lambda)\,d\lambda$$

$z_1 \neq z_2$ so NOT time-invariant

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12. Impulse Response (25 points)

Consider the following system, composed of two subsystems each receiving x(t) as their input. Assume both systems are initially at rest, both systems are causal.

a) Determine the impulse response for each system given below

$$y(t) = \int_{-\infty}^{t-3} x(\lambda - 1)\, d\lambda$$

$$h(t) = \int_{-\infty}^{t-3} \delta(\lambda-1)\, d\lambda = \begin{cases} 0 & t-3 < 1 \\ 1 & t-3 \geq 1 \end{cases} = u(t-4)$$

$$y(t) = 4x(t-3)$$

$$h(t) = 4\delta(t-3)$$

b) Determine the impulse response between input $x(t)$ and output $y(t)$

$$x(t) \longrightarrow \boxed{h_1(t) = -2\delta(t+2)} \xrightarrow{v(t)} \boxed{h_2(t) = \frac{1}{2} e^{-5t} u(t-3)} \longrightarrow y(t)$$

$$H(t) = h_1(t) * h_2(t) = \frac{1}{2} e^{-5t} u(t-3) * -2\delta(t+2)$$

$$= -1 \int_{-\infty}^{\infty} \delta(\lambda+2)\, e^{-5(t-\lambda)} u(t-\lambda-3)\, d\lambda$$

$$= -e^{-5(t+2)} u(t-1) \int_{-\infty}^{\infty} \delta(\lambda+2)\, d\lambda = \boxed{-e^{-5(t+2)} u(t-1)}$$

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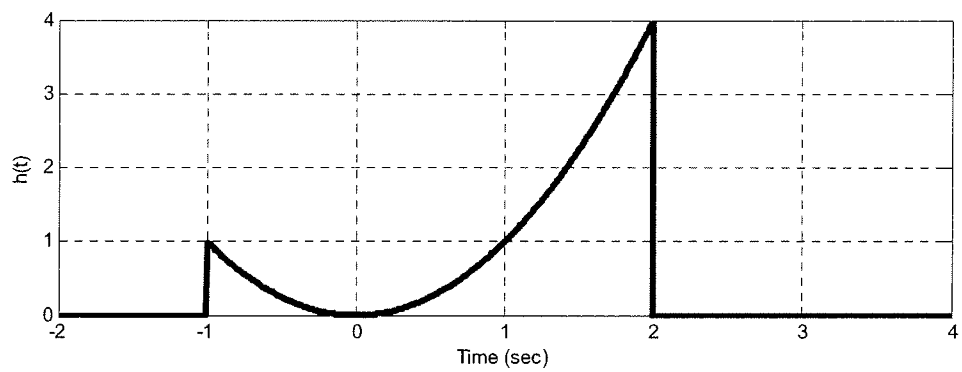
13. (30 points) Consider a causal linear time invariant system with impulse response

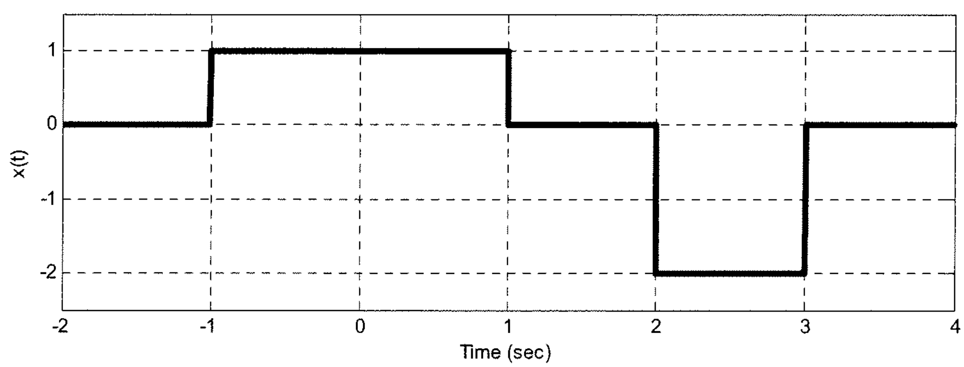
$$h(t) = t^2 \left[u(t+1) - u(t-2)\right]$$

The input to the system is

$$x(t) = u(t+1) - u(t-1) - 2u(t-2) + 2u(t-3)$$

These two functions are plotted below:





Using ***graphical convolution***, set up the integrals to determine the output $y(t)$
Specifically, you must

- Flip and slide $h(t)$
- Show graphs displaying $h(t-\lambda)$ relative to $x(\lambda)$ for each region of interest.
- Determine the ranges of t for which each part of your solution is valid.
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete and simplified as much as possible (no unit step functions)
- ***Do Not Evaluate the Integrals!!***

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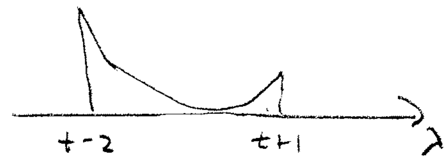
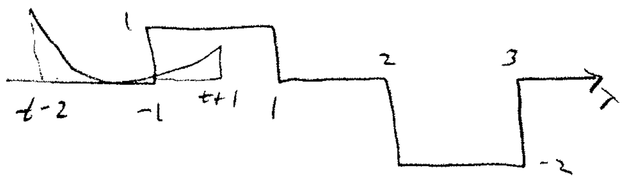
$h(2) = h(t-\lambda) \quad 2 = t-\lambda \quad \lambda = t-2$

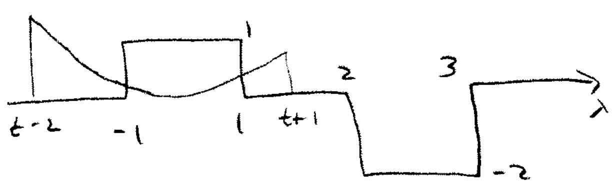
$h(-1) = h(t-\lambda) \quad -1 = t-\lambda \quad \lambda = t+1$

$h(t-\lambda) = (t-\lambda)^2$

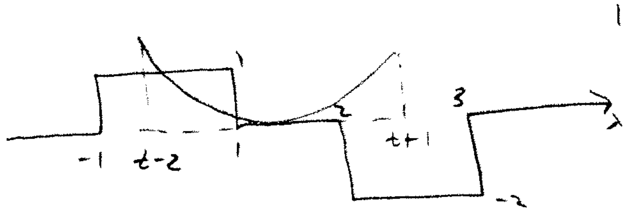




$-2 \le t \le 0 \quad y(t) = \int_{-1}^{t+1} (t-\lambda)^2 (1)\, d\lambda$

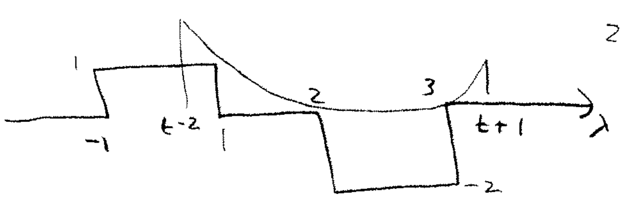


$0 \le t \le 1 \quad y(t) = \int_{-1}^{1} (t-\lambda)^2 (1)\, d\lambda$



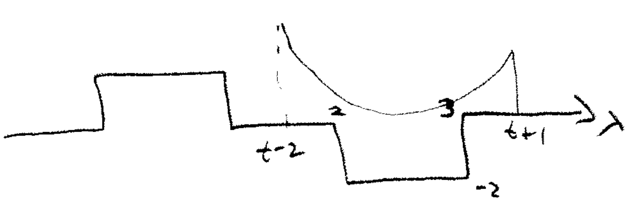
$1 \le t \le 2 \quad y(t) = \int_{t-2}^{1} (t-\lambda)^2 (1)\, d\lambda$

$+ \int_{2}^{t+1} (t-\lambda)^2 (-2)\, d\lambda$



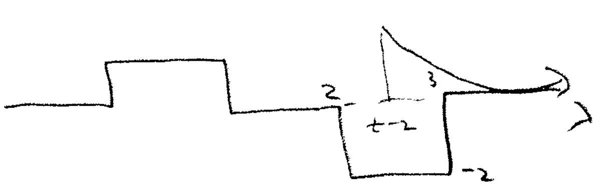
$2 \le t \le 3 \quad y(t) = \int_{t-2}^{1} (t-\lambda)^2 (1)\, d\lambda$

$+ \int_{2}^{3} (t-\lambda)^2 (-2)\, d\lambda$



$3 \le t \le 4 \quad y(t) = \int_{2}^{3} (t-\lambda)^2 (-2)\, d\lambda$

$4 \le t \le 5 \quad y(t) = \int_{t-2}^{3} (t-\lambda)^2 (-2)\, d\lambda$



$y(t) = 0$ for $t \le -2$ or $t \ge 5$