ECE300 Hw 5 Solutions W0809

Friday, January 23, 2009
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Problem 1:

1. Show that any function $x(t)$ can be written in terms of and even function and an odd function, i.e. $x(t) = x_e(t) + x_o(t)$, where $x_e(t)$ is an even function, and $x_o(t)$ is an odd function. Determine expressions for $x_e(t)$ and $x_o(t)$ in terms of $x(t)$ (if you can do this than you have shown that $x(t) = x_e(t) + x_o(t)$).

Given $\quad x_e(t) = x_e(-t) \qquad x(t) = x_e(t) + x_o(t)$

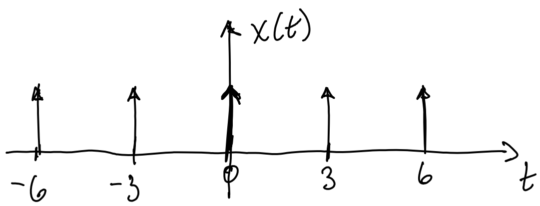
$\quad x_o(t) = -x_o(-t) \qquad x(-t) = x_e(-t) + x_o(-t)$

$$x(t) + x(-t) = 2x_e(t) \qquad x(t) - x(-t) = 2x_o(t)$$

$$x_e(t) = \frac{x(t) + x(-t)}{2} \qquad x_o(t) = \frac{x(t) - x(-t)}{2}$$

2. Find the Fourier series representation for the signal indicated using hand analysis. Clearly indicate the values of ω_0 and the c_k. *Hints:* (1) *Draw the signal, and then use the sifting property to calculate the* c_k. (2) *If you understand how to do this, there is very little work involved.*

$$x(t) = \sum_{p=-\infty}^{\infty} \delta(t - 3p)$$



$T_0 = 3 \qquad \omega_0 = \frac{2\pi}{T_0} = \frac{2}{3}\pi$

$$C_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{3}\int_{-3/2}^{3/2} \delta(t)\, dt = \frac{1}{3}$$

3. Simplify each of the following into the form $c_k = \alpha(k) e^{-j\beta(k)} \mathrm{sinc}(\lambda k)$

a) $c_k = \dfrac{e^{j7k\pi} - e^{-j2k\pi}}{k\pi j}$

$$C_k = \frac{1}{k\pi j} e^{j\frac{5}{2}k\pi}\left[e^{j\frac{9}{2}k\pi} - e^{-j\frac{9}{2}k\pi}\right] = \frac{2}{k\pi} e^{j\frac{5}{2}k\pi} \sin\left(\frac{9}{2}k\pi\right)$$

$$= 9\, e^{j\frac{5}{2}k\pi} \,\mathrm{sinc}\left(\frac{9}{2}k\right)$$

b) $c_k = \dfrac{e^{-j2\pi k} - e^{-j5\pi k}}{jk}$

$$C_k = \frac{1}{jk} e^{-j\frac{7}{2}k\pi}\left[e^{j\frac{3k\pi}{2}} - e^{-j\frac{3k\pi}{2}}\right] = \frac{2}{k} e^{-j\frac{7}{2}k\pi} \sin\left(\frac{3k\pi}{2}\right)$$

$$= 3\pi\, e^{-j\frac{7}{2}k\pi} \,\mathrm{sinc}\left(\frac{3k}{2}\right)$$

c) $c_k = \dfrac{e^{j5k} - e^{j2k}}{k}$

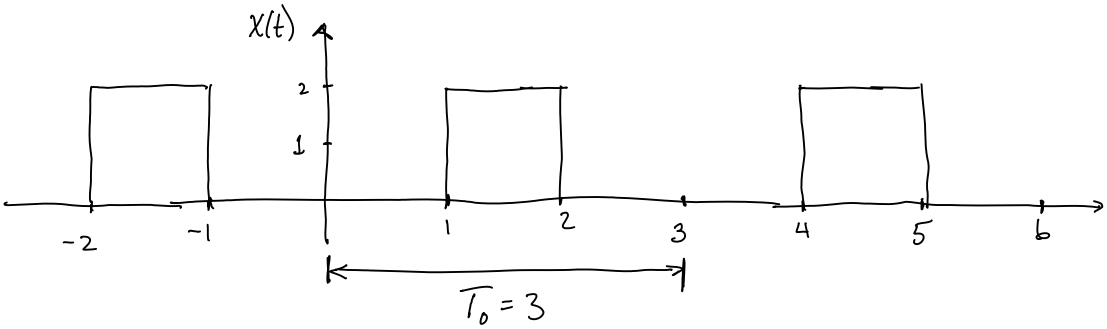
$$C_k = \frac{1}{k} e^{j\frac{7}{2}k}\left[e^{j\frac{3}{2}k} - e^{-j\frac{3}{2}k}\right] = \frac{2j\pi}{\pi k} e^{j\frac{7}{2}k} \sin\left(\frac{3}{2}k\right)$$

$$= \frac{2\pi}{\pi k} e^{j\left(\frac{\pi}{2} + \frac{7}{2}k\right)} \sin\left(\frac{3\pi}{2\pi}k\right) = 3\, e^{j\left(\frac{\pi + 7k}{2}\right)} \,\mathrm{sinc}\left(\frac{3k}{2\pi}\right)$$

Problem 4

For the periodic signal defined as

$$x(t) = \begin{cases} 0 & 0<t<1 \\ 2 & 1<t<2 \\ 0 & 2<t<3 \end{cases}$$



(a) $\omega_0 = 2\pi / T_0 = \frac{2}{3}\pi$

(b) determine the average value:

$$C_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt = \frac{1}{3}\int_1^2 2\,dt = \frac{2}{3}\left[2-1\right] = \boxed{\frac{2}{3}}$$

(c) What is average Power in DC

$$P_{ave} = |C_0|^2 = 4/9 \text{ W}$$

(d) Find C_k:

$$C_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\omega_0 t}\,dt = \frac{2}{3}\int_1^2 e^{-jk\omega_0 t}\,dt = \frac{-2}{3jk\omega_0}\left[e^{-jk2\omega_0} - e^{-jk\omega_0}\right]$$

$$= \frac{2}{2\pi jk}\, e^{-j\frac{(3k\omega_0)}{2}}\left[e^{j\frac{k\omega_0}{2}} - e^{-j\frac{k\omega_0}{2}}\right] = \frac{2}{\pi k}\, e^{-j\pi k}\sin\left(\frac{k\pi}{3}\right)$$

$$\boxed{= \frac{2}{3}\, e^{-j\pi k}\,\mathrm{sinc}\left(\frac{k}{3}\right)}$$

Problem 5

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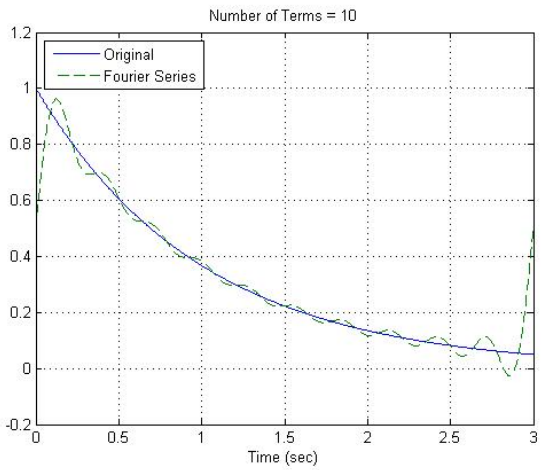
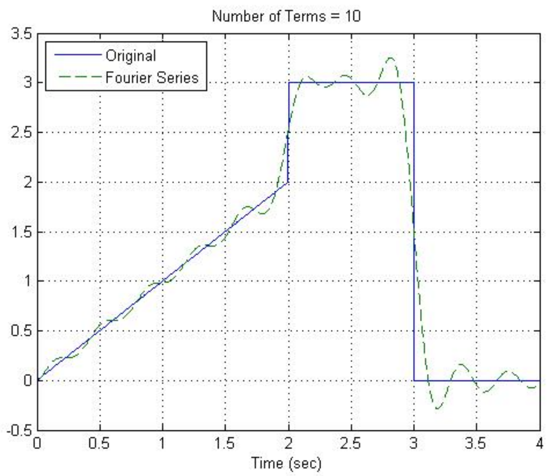
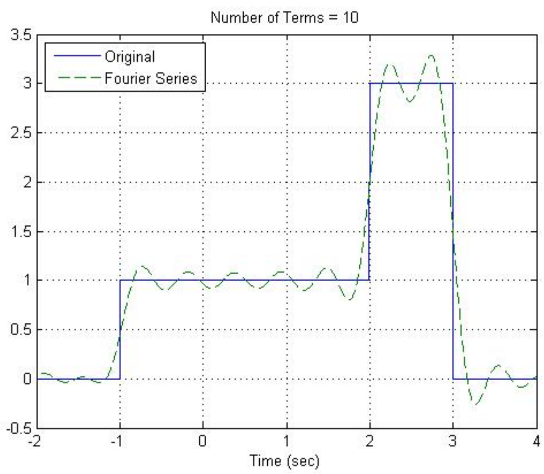
This is the MATLAB problem for which you need to plot the Fourier Series of three different functions.
Solution to f1(t) = exp(-t) u(t):



Solution to f2(t):



Solution to f3(t):



MATLAB scrip for Complex_Fourier_Series function

```
%
%  This routine implements a Complex Fourier Series
%
% Inputs: N is the number of terms to be used in the series
%
function Complex_Fourier_series(N)
%
%  one period of the function goes from low to high
%
  low = -2;
  high = 4;
%
%  the difference between low and high is one period
%
  T = high-low;
  w0 = 2*pi/T;
%
%  the periodic function
%
%  x  = @(t) exp(-t);
%  x  = @(t) t.*((t>=0)&(t<2)) + 3*((t>=2)&(t<3)) + 0*((t>=3)&(t<4));
  x  = @(t) 0.*((t>=-2)&(t<-1)) + 1*((t>=-1)&(t<2)) + 3*((t>=2)&(t<3))
+ 0*((t>=3)&(t<4));
%
%   find b(1) to b(N)
%
  for k = 1:N
      carg = @(t) x(t).*exp(-j*k*w0*t);
      c(k) = (1/T)*quadl(carg,low,high);
  end;
%
%  determine a time vector over one period
%
  t = linspace(low,high,1000);
%
%  Find the Fourier series representation
%
  c0arg = @(t) x(t);
  c0 = (1/T)*quadl(c0arg,low,high);
  est = c0;
  for k = 1:N
      est = est + 2*abs(c(k))*cos(k*w0*t+angle(c(k)));
  end;
%
%  plot the results
%
  plot(t,x(t),'-',t,est,'--'); grid; xlabel('Time (sec)');
  legend('Original','Fourier Series','Location','NorthWest');
  title(['Number of Terms = ', num2str(N)]);
%
```