ECE 300
Signals and Systems
Homework 2

Due Date: Tuesday December 16, 2008 *at the beginning of class*

Reading Roberts pages 54-58, 115-140

Problems

1) Consider the following mathematical models of systems:

a $y(t) = e^{-(t+1)}x(t)$ **b** $y(t) = \begin{cases} x(t) & |x(t)| \le 10 \\ 10 & |x(t)| > 10 \end{cases}$ **c** $y(t) = x\left(1 - \frac{t}{2}\right)$

d $y(t) = x\left(\frac{t}{3}\right) + 2$ **e** $y(t) = \sin\left(x(t)\right)$ **f** $y(t) = 1 - e^{-x(t)}$

g $y(t) = \int_{-\infty}^{t/2} x(\lambda)d\lambda$ **h** $\frac{dy(t)}{dt} = 3y(t) + 2x(t)$ **i** $\frac{dy(t)}{dt} = 3ty(t) + 2x(t)$

j $y(t) = \int_{-\infty}^{t} (t-\lambda)x(\lambda+1)d\lambda$

Fill in the following table (Y or N for each question) for each system. You must justify your answers to receive credit. *Assume* t *can be any possible value.*

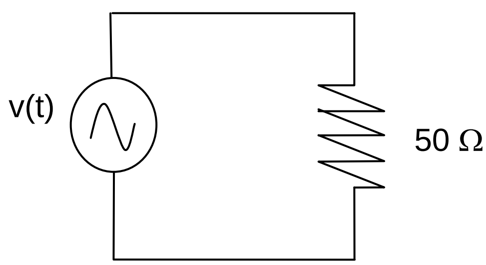
Part	Causal?	Memoryless?	Linear?	Time Invariant?
a				
b				
c				
d				
e- $x(t)$ large				
e- $x(t)$ small				
f- $x(t)$ large				
f- $x(t)$ small				
g				
h				
i				
j				

For parts **e** and **f**, look at the case when $x(t)$ is large, and then when $x(t)$ is assumed to be sufficiently small that you can use a small value (Taylor series) approximation. Even though $x(t)$ is a function, you can still use the same approximations you would use if it was just a small number.

For part **h** you should show $y(t) = y(t_0)e^{3(t-t_0)} + \int_{t_0}^{t} 2e^{3(t-\lambda)}x(\lambda)d\lambda$ in order to determine the system is or is not causal and has memory or is memoryless.

For part **i** you should solve the DE first (see handout about integrating factors) and then determine whether the system is or is not causal and has memory or is memoryless.

2) A circuit consists of a voltage source in parallel with a resistor as shown below.

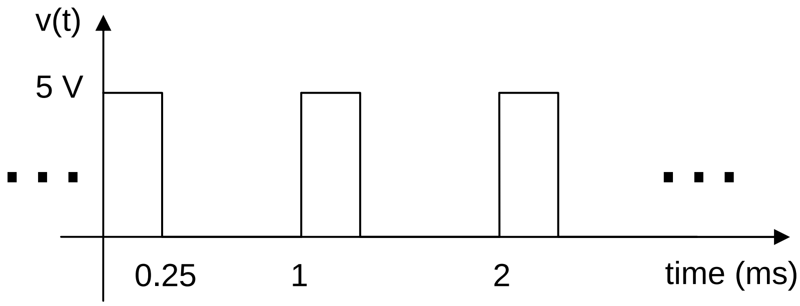


The voltage source can present three different voltage signals to the resistor:

(a) $v(t) = u(t)$ V

(b) $v(t) = \frac{1}{t}u(t-1)$ V

(c) $v(t) = 12\cos(2\pi 1000t)$ V



(d)

For each signal, determine if the signal is an energy or power signal. Depending on the type of signal determine the energy/average power absorbed by the resistor. You must provide the correct units for your result. Note that for periodic signals, you only have to integrate over one period.

3) (**Matlab Problem**) The ***average value*** of a function x(t) is defined as

$$\overline{x} = \frac{1}{b-a}\int_a^b x(t)dt$$

and the ***root-mean-square (rms)*** value of a function is defined as

$$x_{rms} = \sqrt{\frac{1}{b-a}\int_a^b x^2(t)dt}$$

Read the **Appendix (**located at the end of this homework), then

a) use Matlab to find the average and rms values of the function $x(t) = t^2$ for $-1 < t < 1$

b) use Matlab to find the average and rms values of the following functions

$$\begin{array}{cc} x(t) = \cos(t) & 0 < t < \pi \\ x(t) = \cos(t) & 0 < t < 2\pi \\ x(t) = |t| & -1 < t < 1 \\ x(t) = t\cos(t) & -2 < t < 4 \end{array}$$

c) For parts (c) and (d) Problem 2, use Matlab to find the average power that is absorbed by the resistor.

Hint: You will probably find the **sqrt** function useful. You should write a Matlab m-file for this problem, and turn it in with your homework, as well as the answers.

Appendix

Maple is often used for symbolically integrating a function. Sometimes, though, what we really care about is the numerical value of the integral. Rather than integrating symbolically, we might want to just use numerical integration to evaluate the integral. Since we are going to be using Matlab a great deal in this course, in this appendix we will learn to use one of Matlab's built-in functions for numerical integration. In order to efficiently use this function, we will learn how to construct what are called *anonymous* functions. We will then use this information to determine the average and rms value of a function. Some of this is going to seem a bit strange at first, so just try and learn from the examples.

Numerical Integration in Matlab Let's assume we want to numerically integrate the following:

$$I = \int_0^{2\pi} (t^2 + 2)dt$$

In order to do numerical integration in Matlab, we will use the built-in command **quadl.** The ***arguments*** to quadl, e.g., the information passed to quadl, are

- A function which represents the integrand (the function which is being integrated). Let's call the integrand $x(t)$. This function must be written in such a way that it returns the value of $x(t)$ at each time t. Clearly here $x(t) = t^2 + 2$
- The lower limit of integration, here that would be 0
- The upper limit of integration, here that would be 2π

Note that an optional fourth argument is the tolerance, which defaults to 10^{-6}. When the function value is very small, or the integration time is very small, you will have to change this.

Anonymous Functions Let's assume we wanted to use Matlab to construct the function $x(t) = t^2 + 2$. We can do this by creating what Matlab calls an **anonymous function**. To do this, we type into Matlab

```
x = @(t) t.*t+2;
```

If we want the value of $x(t)$ at $t = 2$, we just type x(2)

Hence, to evaluate the integral $I = \int_0^{2\pi} (t^2 + 2)dt$ in Matlab we would type

```
x = @(t) t.*t+2;
I = quadl(x,0,2*pi)
```

Note that it is important to define x ***before*** it is used by (passed to) quadl

Example 1 To numerically evaluate $I = \int_{-1}^{1} e^{-t} \cos(2t)dt$ we could type

```
x = @(t) exp(-t).*cos(2*t);
I = quadl(x,-1,1);
```

Example 2 To numerically evaluate $I = \int_{-2}^{1} |t| e^{-|t|}dt$ we could type

```
y = @(t) abs(t).*exp(-abs(t));
I = quadl(y,-2,1);
```

Integrating Products of Functions Sometimes we are going to want to integrate the product of functions. While we could just multiply the functions together, it is usually easier to let Matlab do it for us.

Let's assume we want to evaluate the integral $I = \int_0^1 x(t)y(t)dt$, and let's assume that we already have anonymous functions x and y. The function **quadl** needs to be passed a function which is the product of x and y. To do this, we make a new anonymous function z, using the following:

```
z = @(t) x(t).*y(t);
```

and then perform the integration

```
I = quadl(z,0,1)
```

An alternative is to write

```
I = quadl(@(t) x(t).*y(t),0,1);
```

Example 3 To numerically evaluate $I = \int_{-1}^1 e^{-t}\cos(2t)dt$ we could type

```
x = @(t) exp(-t)
y = @(t) cos(2*t);
z = @(t) x(t).*y(t);
I = quadl(z,-1,1);
```

or

```
I = quadl(@(t) x(t).*y(t),-1,1);
```

Example 4 To numerically evaluate $I = \int_{-2}^1 |t| e^{-|t|}dt$ we could type

```
x = @(t) abs(t);
y = @(t) exp(-abs(t));
z = @(t) x(t).*y(t);
I = quadl(z,-2,1);
```

or

```
I = quadl(@(t) x(t).*y(t),-2,1);
```

#1 System Properties

(a) $y(t) = e^{-(t+1)} x(t)$

causal? $y(t)$ only depends on x at the same time, so causal
memoryless? and memoryless

linear? $z_1 = \mathcal{H}\{\alpha_1 x_1 + \alpha_2 x_2\} = e^{-(t+1)}[\alpha_1 x_1(t) + \alpha_2 x_2(t)]$

$z_2 = \alpha_1 \mathcal{H}\{x_1\} + \alpha_2 \mathcal{H}\{x_2\} = \alpha_1[e^{-(t+1)} x_1(t)] + \alpha_2[e^{-(t+1)} x_2(t)]$

$= e^{-(t+1)}[\alpha_1 x_1(t) + \alpha_2 x_2(t)] = z_1$ linear

Time-Invariant? $z_1 = \mathcal{H}\{x(t-t_0)\} = e^{-(t+1)} x(t-t_0)$ $\quad z_1 \neq z_2$

$z_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} = e^{-(t-t_0+1)} x(t-t_0)$ not time-invariant

(b) $y(t) = \begin{cases} x(t) & |x(t)| < 10 \\ 10 & |x(t)| > 10 \end{cases}$

causal? memoryless? $y(t)$ only depends on x at the same time,
so causal and memoryless

linear? assume $x(t) = 5$ then $y(t) = 5$
Next assume $x(t) = 20 = 4 \cdot 5$ (scaled original input by 4)
then $y(t) = 10$ (scaled original input by 2)
not linear

Time-Invariant? $z_1 = \mathcal{H}\{x(t-t_0)\} = \begin{cases} x(t-t_0) & |x(t-t_0)| < 10 \\ 10 & |x(t-t_0)| > 10 \end{cases}$

$z_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} = \begin{cases} x(t-t_0) & |x(t-t_0)| < 10 \\ 10 & |x(t-t_0)| > 10 \end{cases}$

so time-invariant

(c) $y(t) = x\left(1-\frac{t}{2}\right)$

Causal? $y(0) = x(1)$ <u>not causal</u>

memoryless? $y(0) = x(1)$ <u>not memoryless</u>

Linear? $z_1 = \mathcal{H}\{\alpha_1 x_1 + \alpha_2 x_2\} = \alpha_1 x_1\left(1-\frac{t}{2}\right) + \alpha_2 x_2\left(1-\frac{t}{2}\right)$

$z_2 = \alpha_1 \mathcal{H}\{x_1\} + \alpha_2 \mathcal{H}\{x_2\} = \alpha_1 x_1\left(1-\frac{t}{2}\right) + \alpha_2 x_2\left(1-\frac{t}{2}\right)$

$z_1 = z_2$ so <u>linear</u>

Time-Invariant? $z_1 = \mathcal{H}\{x(t-t_0)\} = x\left(1-\frac{t}{2}-t_0\right)$

$z_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} = x\left(1-\frac{(t-t_0)}{2}\right)$

$z_1 \neq z_2$ so <u>not time-invariant</u>

(d) $y(t) = x\left(\frac{t}{3}\right)+2$

causal? $y(-1) = x\left(-\frac{1}{3}\right)+2$ <u>not causal</u>

memoryless? <u>not memoryless</u>

Linear? $z_1 = \mathcal{H}\{\alpha_1 x_1(t) + \alpha_2 x_2(t)\}$

$= \alpha_1 x_1\left(\frac{t}{3}\right) + \alpha_2 x_2\left(\frac{t}{3}\right) + 2$

$z_2 = \alpha_1 \mathcal{H}\{x_1(t)\} + \alpha_2 \mathcal{H}\{x_2(t)\}$

$= \alpha_1\left[x_1\left(\frac{t}{3}\right)+2\right] + \alpha_2\left[x_2\left(\frac{t}{3}\right)+2\right]$

$= \alpha_1 x_1\left(\frac{t}{3}\right) + \alpha_2 x_2\left(\frac{t}{3}\right) + 2(\alpha_1+\alpha_2) \neq z_1$ for all α_1, α_2

<u>not linear</u>

Time-Invariant? $z_1 = \mathcal{H}\{x(t-t_0)\} = x\left(\frac{t}{3}-t_0\right)+2$

$z_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} = x\left(\frac{t-t_0}{3}\right)+2$

$z_1 \neq z_2$ <u>not time-invariant</u>

(e) $y(t) = \sin(x(t))$

$x(t)$ - Large

Causal, memoryless? $y(t)$ only depends on x at time t, so the system is causal and memoryless

Linear? $z_1 = H\{\alpha_1 x_1 + \alpha_2 x_2\} = \sin(\alpha_1 x_1(t) + \alpha_2 x_2(t))$

$z_2 = \alpha_1 H\{x_1\} + \alpha_2 H\{x_2\} = \alpha_1 \sin(x_1(t)) + \alpha_2 \sin(x_2(t)) \neq z_1$

not linear

Time-Invariant? $z_1 = H\{x(t-t_0)\} = \sin(x(t-t_0))$

$z_2 = H\{x(t)\}\Big|_{t=t-t_0} = \sin(x(t-t_0)) = z_1$ so time-invariant

$x(t)$ - small so $y(t) = \sin(x(t)) \approx x(t)$ ($\sin(\theta) \approx \theta$ for θ small)

$y(t) = x(t)$

clearly causal, memoryless, linear, and time-invariant

(f) $y(t) = 1 - e^{x(t)}$

$x(t)$ large

Causal, memoryless? The system is clearly causal and memoryless

linear? $z_1 = H\{\alpha_1 x_1(t) + \alpha_2 x_2(t)\} = 1 - e^{-(\alpha_1 x_1(t) + \alpha_2 x_2(t))}$

$z_2 = \alpha_1 H\{x_1\} + \alpha_2 H\{x_2\} = \alpha_1[1 - e^{-x_1(t)}] + \alpha_2[1 - e^{-x_2(t)}]$

$z_1 \neq z_2$ not linear

Time-invariant? $z_1 = H\{x(t-t_0)\} = 1 - e^{-x(t-t_0)}$

$z_2 = H\{x(t)\}\Big|_{t=t-t_0} = 1 - e^{-x(t-t_0)} = z_1$ time-invariant

$x(t)$ small so $y(t) = 1 - [1 - x(t)] = x(t)$ since $e^{-a} \simeq 1 - a$

This system is causal, memoryless, linear, and time-invariant

Problem 1 part (g)

(g) $y(t) = \int_{-\infty}^{t/2} x(\lambda)\, d\lambda$

Causal: If $t = -1$ then $y(-1)$ depends on $x(-\frac{1}{2})$ so y will depend on future values of x. So this is <u>non-causal</u>.

Memory: The system does have memory b/c each value of $y(t)$ depends on all previous values of input up to $t/2$

Linearity:

$$Z_1 = A y_1(t) + B y_2(t) = A \int_{-\infty}^{t/2} x_1(\lambda) d\lambda + B \int_{-\infty}^{t/2} x_2(\lambda) d\lambda$$

$$= \int [A x_1(\lambda) + B x_2(\lambda)]\, d\lambda$$

$$Z_2 = \int_{-\infty}^{t/2} [A x_1 + B x_2]\, d\lambda$$

$Z_1 = Z_2 \longrightarrow$ so <u>Linear</u>

Time Invariance:

$$Z_1 = y(t - t_0) = \int_{-\infty}^{\frac{t-t_0}{2}} x(\lambda)\, d\lambda$$

$$Z_2 = \int_{-\infty}^{t/2} x(\lambda - t_0)\, d\lambda = \int_{-\infty}^{\frac{t}{2} - t_0} x(\sigma)\, d\sigma$$

$\sigma = \lambda - t_0$

$d\sigma = d\lambda$

$Z_1 \neq Z_2 \longrightarrow$ <u>Not Time Invariant</u>

(h) $\dot{y} = 3y + 2x$

$\dot{y} - 3y = 2x \qquad \frac{d}{dt}\left(y(t)\, e^{-3t}\right) = 2x(t)\, e^{-3t}$

$$\int_{t_0}^{t} \frac{d}{d\lambda}\left(y(\lambda) e^{-3\lambda}\right) d\lambda = y(t) e^{-3t} - y(t_0) e^{-3t_0} = \int_{t_0}^{t} 2x(\lambda) e^{-3\lambda} d\lambda$$

$$y(t) = y(t_0) e^{3(t-t_0)} + \int_{t_0}^{t} 2 e^{3(t-\lambda)} x(\lambda) d\lambda$$

Causal? Yes, $y(t)$ only depends on the input up until time t

Memoryless? No, $y(t)$ depends on the input at more than just the current time

Linear? $\dot{y}_1 = 3y_1 + 2x_1 \qquad \dot{y}_2 = 3y_2 + 2x_2$

$\alpha_1 \dot{y}_1 = 3\alpha_1 y_1 + 2\alpha_1 x_1 \qquad \alpha_2 \dot{y}_2 = 3\alpha_2 y_2 + 2\alpha_2 x_2$

$(\alpha_1 \dot{y}_1 + \alpha_2 \dot{y}_2) = 3(\alpha_1 y_1 + \alpha_2 y_2) + 2(\alpha_1 x_1 + \alpha_2 x_2)$

$\dot{Y} = 3Y + 2X \qquad Y = \alpha_1 y_1 + \alpha_2 y_2 \quad X = \alpha_1 x_1 + \alpha_2 x_2$

so linear

Time-invariant? $z_1 = \mathcal{H}\{x(t-t_0)\} \rightarrow \dot{y}(t-t_0) = 3y(t-t_0) + 2x(t-t_0)$

$z_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} \rightarrow \left[\dot{y}(t) = 3y(t) + 2x(t)\right]_{t=t-t_0} = z_1$

so time-invariant

(i) $\frac{dy}{dt} = 3ty(t) + 2x(t)$

$\dot{y} - 3ty = 2x \quad \frac{d}{dt}\left(e^{-\frac{3}{2}t^2}y(t)\right) = e^{-\frac{3}{2}t^2}x(t)$

$$\int_{t_0}^{t} \frac{d}{d\lambda}\left(e^{-\frac{3}{2}\lambda^2}y(\lambda)\right)d\lambda = e^{-\frac{3}{2}t^2}y(t) - e^{-\frac{3}{2}t_0^2}y(t_0) = \int_{t_0}^{t} e^{-\frac{3}{2}\lambda^2}x(\lambda)d\lambda$$

$$y(t) = y(t_0)e^{\frac{3}{2}(t^2 - t_0^2)} + \int_{t_0}^{t} e^{-\frac{3}{2}(t^2-\lambda^2)}x(\lambda)d\lambda$$

Causal? Yes, $y(t)$ only depends on x up until time t

Memoryless? No, $y(t)$ depends on the input at more times than t

Linear? $\dot{y}_1 = 3ty_1 + 2x_1 \quad \dot{y}_2 = 3ty_2 + 2x_2$

$\alpha_1\dot{y}_1 = 3t\alpha_1 y_1 + 2\alpha_1 x_1 \quad \alpha_2\dot{y}_2 = 3t\alpha_2 y_2 + 2\alpha_2 x_2$

$(\alpha_1\dot{y}_1 + \alpha_2\dot{y}_2) = 3t(\alpha_1 y_1 + \alpha_2 y_2) + 2(\alpha_1 x_1 + \alpha_2 x_2)$

$\dot{Y} = 3tY + 2X \quad Y = \alpha_1 y_1 + \alpha_2 y_2 \quad X = \alpha_1 x_1 + \alpha_2 x_2$

so linear

Time-invariant? $z_1 = \mathcal{H}\{x(t-t_0)\} \rightarrow \dot{y}(t-t_0) = 3t\,y(t-t_0) + 2x(t-t_0)$

$z_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} \rightarrow \left[\dot{y}(t) = 3ty(t) + 2x(t)\right]_{t=t-t_0} \neq z_1$

not time-invariant

① $y(t) = \int_{-\infty}^{t} (t-\lambda)\, x(\lambda+1)\, d\lambda$

causal? No, $y(t)$ depends on $x(t+1)$

memoryless? No, $y(t)$ depends on $x(t+1)$

Linear? $z_1 = \mathcal{H}\{\alpha_1 x_1 + \alpha_2 x_2\} = \int_{-\infty}^{t} (t-\lambda)\left[\alpha_1 x_1(\lambda+1) + \alpha_2 x_2(\lambda+1)\right] d\lambda$

$z_2 = \alpha_1 \mathcal{H}\{x_1\} + \alpha_2 \mathcal{H}\{x_2\}$

$= \alpha_1 \int_{-\infty}^{t} (t-\lambda)\, x_1(\lambda+1)\, d\lambda + \alpha_2 \int_{-\infty}^{t} (t-\lambda)\, x_2(\lambda+1)\, d\lambda$

$= \int_{-\infty}^{t} (t-\lambda)\left[\alpha_1 x_1(\lambda+1) + \alpha_2 x_2(\lambda+1)\right] d\lambda = z_1$

<u>linear</u>

Time-invariant?

$z_1 = \mathcal{H}\{x(t-t_0)\} = \int_{-\infty}^{t} (t-\lambda)\, x(\lambda - t_0 + 1)\, d\lambda$

$z_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} = \int_{-\infty}^{t-t_0} (t - t_0 - \lambda)\, x(\lambda+1)\, d\lambda$

$z_1 = \int_{-\infty}^{t} (t-\lambda)\, x(\lambda - t_0 + 1)\, d\lambda$ let $\sigma = \lambda - t_0$ $\quad d\sigma = d\lambda$
$\sigma + t_0 = \lambda$

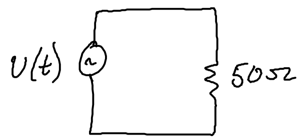
$= \int_{-\infty}^{t-t_0} (t - (\sigma + t_0))\, x(\sigma+1)\, d\sigma = \int_{-\infty}^{t-t_0} (t - t_0 - \sigma)\, x(\sigma+1)\, d\sigma = z_2$

so <u>time-invariant!</u>

ECE300 Hw2 Sol Wint 08-09

Tuesday, December 09, 2008
11:44 AM

2) Circuit Consists of Voltage Source in Parallel w/ Resistor



$$P_R(t) = \frac{v^2(t)}{R}$$

$$P_{ave} = \frac{1}{R}\,\frac{1}{T}\int_{-T/2}^{T/2} v^2(t)\,dt$$

$$E = \int_{-\infty}^{\infty} p(t)\,dt = \frac{1}{R}\int_{-\infty}^{\infty} v^2(t)\,dt$$

a) $v(t) = u(t)$

$$E = \frac{1}{R}\int_{-\infty}^{\infty}[u(t)]^2\,dt = \frac{1}{50\Omega}\int_{-\infty}^{\infty} u(t)\,dt = \frac{1}{50\Omega}\int_0^{\infty} 1\;dt = \infty$$

$\therefore$ this must be a <u>power signal</u>

$$P_{av} = \frac{1}{RT}\int_{-T/2}^{T/2}[u(t)]^2\,dt = \frac{1V^2}{RT}\left[t\,\Big|_0^{T/2}\right] = \frac{1V^2}{R\cancel{T}}\left[\frac{T}{2}\right] = \frac{1V^2}{100\Omega} = 10\text{ mW}$$

(b) $v(t) = \frac{1}{t}\,u(t-1)$

$$E = \frac{1}{R}\int_{-\infty}^{\infty}\frac{1}{t^2}[u(t-1)]^2\,dt = \frac{1V^2}{R}\int_1^{\infty} t^{-2}\,dt = \frac{-1V^2}{R}\left[\frac{1}{t}\right]_1^{\infty} = \frac{-1V^2}{R}\left[\frac{1s}{\infty} - 1s\right]$$

$$= \frac{1V^2}{50\Omega}(1s) = 20\text{m J}$$

$$P_{ave} = \frac{1}{R}\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\frac{1}{t^2}[u(t-1)]^2\,dt = \frac{1V^2}{R}\lim_{T\to\infty}\int_1^{T/2} t^{-2}\,dt = \frac{-1V^2}{R}\lim_{T\to\infty}\frac{1}{T}$$

$$= \frac{-1V^2}{R}\lim_{T\to\infty}\frac{1}{T}\left[\frac{2s}{T} - 1s\right] = 0$$

* This signal has finite energy & zero average power so it is an <u>Energy signal</u>

(c) $v(t) = 12\cos(2\pi 1000 t)$ V

Energy: by observation $\int_{-\infty}^{\infty} \cos^2(t)\,dt = \infty$

Power:

$$P_{ave} = \frac{1}{R}\frac{1}{T}\int_{-T/2}^{T/2} 144\cos^2(2\pi 1000t)\,dt = \frac{1}{R}\frac{144V^2}{1ms}\int_{-\frac{1}{2}ms}^{\frac{1}{2}ms} \cos^2(2\pi 1000t)\,dt$$

$T = 0.001$ s

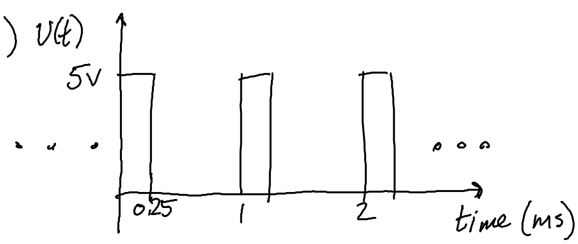
$$= \frac{1}{R}\frac{144V^2}{1ms}\int_{-\frac{1}{2}ms}^{\frac{1}{2}ms} \frac{1}{2} + \frac{1}{2}\cos(4\pi 1000t)\,dt = \frac{144V^2}{R(1ms)}\left[\frac{1}{2}t + \cancel{\frac{1}{8000\pi}\sin(4000\pi t)}^{\,0}\right]_{-\frac{1}{2}ms}^{\frac{1}{2}ms}$$

$$\boxed{= \frac{144V^2}{2R} = \frac{144V^2}{100\Omega} = 1.44W} \longrightarrow \text{Note for sinusoid } V_{RMS} = \frac{|V|}{\sqrt{2}}$$

$$P_{ave} = \frac{V_{RMS}^2}{R} = \frac{\left(\frac{12}{\sqrt{2}}\right)^2}{R} = \frac{144V^2}{2R}$$

* This signal has infinite Energy but finite Power. so it is an Energy Signal

(d)



Energy: By Observation $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \infty$

Average Power: $P_{ave} = \frac{1}{R(1ms)}\int_0^{0.25ms} 25V^2 dt = \frac{25V^2}{R(1ms)}\Big[t\Big]\Big|_0^{0.25ms} = \frac{25V^2}{4(50\Omega)}\frac{(1\cancel{ms})}{(1\cancel{ms})}$

$$\boxed{= \frac{1}{8}W} \longrightarrow \text{Note that this is still equivalent to } \frac{V_{RMS}^2}{R} \text{ but now } V_{RMS} \neq \frac{V_p}{\sqrt{2}}$$

* This signal has infinite energy but finite average power. It is a Power signal

```
%
% script for homework 2
%
% define the function for part a
%
  x = @(t) t.*t;
  y = @(t) x(t).*x(t);
%
% compute the average
%
  ave = (1/2)*quadl(x,-1,1)
%
% compute the rms
%
  rms = sqrt((1/2)*quadl(y,-1,1))
%
% now define the functions for part b
%
  x = @(t) cos(t);
  y = @(t) x(t).*x(t);
%
  ave = (1/pi)*quadl(x,0,pi)
  rms = sqrt((1/pi)*quadl(y,0,pi))
%
  ave = (1/(2*pi))*quadl(x,0,2*pi)
  rms = sqrt(1/(2*pi)*quadl(y,0,2*pi))
%
  x = @(t) abs(t);
  y = @(t) x(t).*x(t);
%
  ave = (1/2)*quadl(x,-1,1)
  rms = sqrt((1/2)*quadl(y,-1,1))
%
  x = @(t)  t.*cos(t);
  y = @(t) x(t).*x(t);
%
  ave = (1/6)*quadl(x,-2,4)
  rms = sqrt((1/6)*quadl(y,-2,4))
```

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To get started, select MATLAB Help or Demos from the Help menu.

ave =

0.3333 $\bar{x}$ for $x(t) = t^2$

rms =

0.4472 x_{rms}

ave =

4.9335e-017 $\bar{x}$ for $x(t) = \cos(t)$ $0 < t < \pi$

rms =

0.7071 x_{rms}

ave =

7.8024e-013 $\bar{x}$ for $x(t) = \cos(t)$ $0 < t < 2\pi$

rms =

0.7071 x_{rms}

ave =

0.5000 $\bar{x}$ for $x(t) = |t|$ $-1 < t < 1$

rms =

0.5774 x_{rms}

ave =

-0.8472 $\bar{x}$ for $x(t) = t\cos(t)$ $-2 < t < 4$

rms =

1.5652

```
>> % This is the solution to Part (c) of Problem 3 on Hw 2 for ECE300 The
% purpose of this problem was to use matlab to solve for the average power
% for parts (c) and (d) of Problem 2.

period = 1e-3;
resistance=50;


% This part computes the average power for part(c) of problem 2.
x1 = @(t) (12*cos(2*pi*1000*t)).^2;
pav1 = (1/(resistance*period))*quadl(x1,0,period)


% This part computes the average power for part(d) of problem 2.

x2 = @(t) 25*(t<=0.25e-3)+0*(t>0.25e-3);
pav2 = (1/(resistance*period))*quadl(x2,0,period)

pav1 =

    1.4400


pav2 =

    0.1250

>>
```