ECE300 Hw1 Solutions Winter 07-08

Tuesday, December 02, 2008
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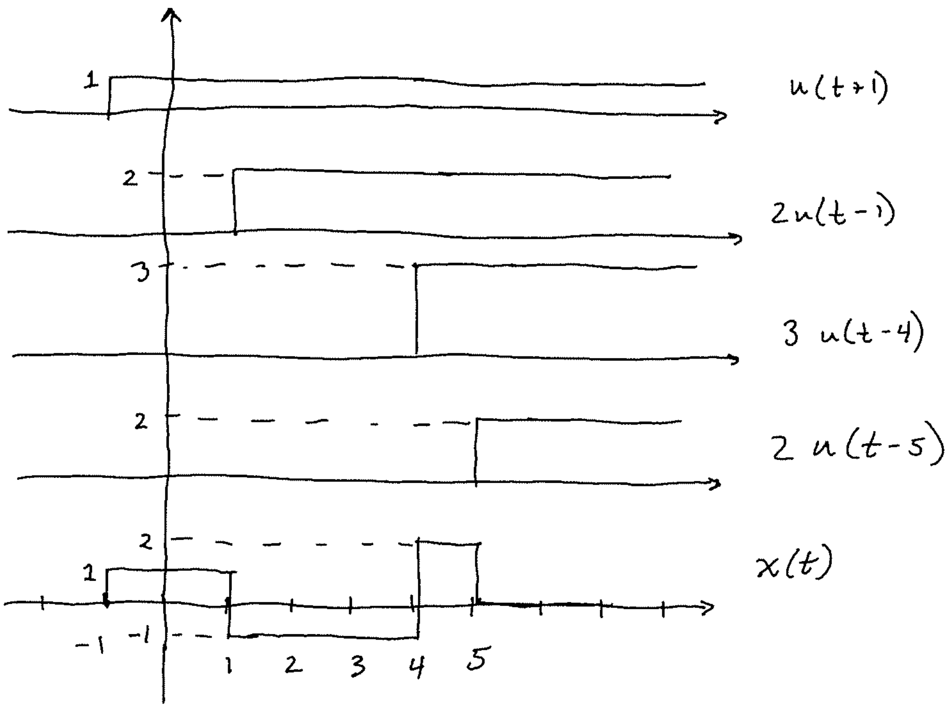
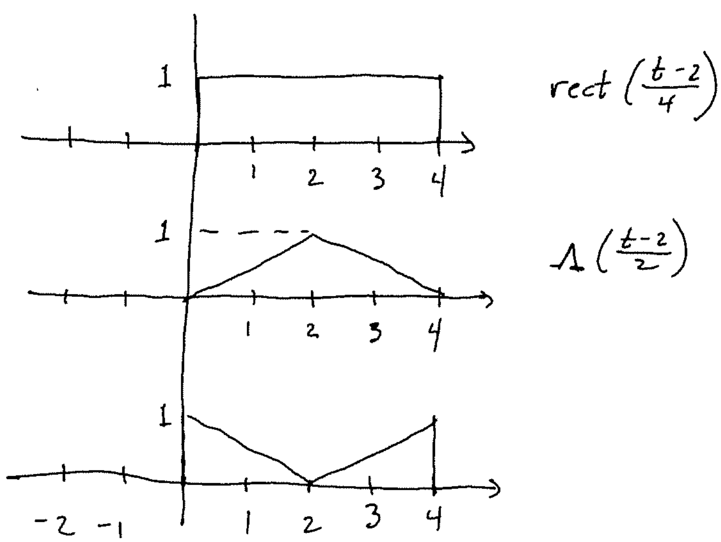
1) Sketch the following fcns.

a) $x(t) = u(t+1) - 2u(t-1) + 3u(t+4) - 2u(t-5)$



(b) $x(t) = rect\left(\frac{t-2}{4}\right) - \Lambda\left(\frac{t-2}{2}\right)$



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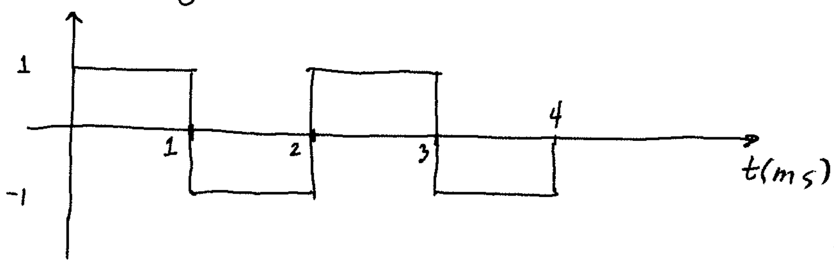
2) Problem 2.40 in Text

This signal is the product of pos & neg rectangles w/ a cosine fcn

Rectangles



There are many ways to represent this signal mathematically. Here are two:

$x_1(t) = \text{rect}(t-\tfrac{1}{2}) - \text{rect}(t-\tfrac{3}{2}) + \text{rect}(t-\tfrac{5}{2}) - \text{rect}(t-\tfrac{7}{2})$

$x_2(t) = u(t) - 2u(t-0.001) + 2u(t-0.002) - 2u(t-0.003) + u(t-0.004)$

The sine function is given by:

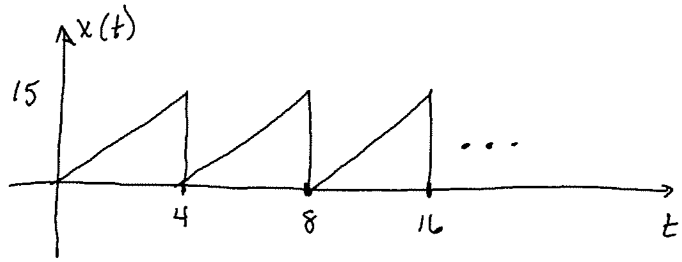
$x_2(t) = \sin(\omega_0 t) = \sin(8000\pi t) = \cos(8000\pi t - \pi/2)$

there are 16 periods in 4 ms so $f = \frac{16}{4ms} = 4\,kHz$

$\omega_0 = 2\pi f = 8000\pi\ r/s$

The given function $\boxed{x(t) = x_1(t) \cdot x_2(t)}$

3) Describe the following signal in 2 ways



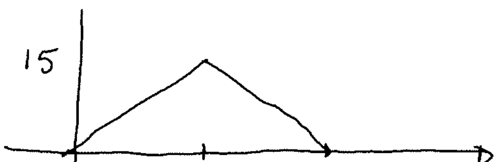
(a) A ramp fcn minus a summation of step fcns.

$$x(t) = \frac{15}{4} r(t) - 15\,u(t-4) - 15\,u(t-8) - \cdots$$

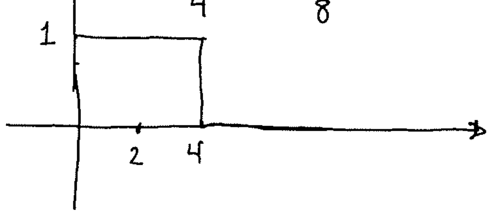
$$= \frac{15}{4} r(t) - 15 \sum_{i=1}^{\infty} u(t-4i)$$

(b) As a SOP of triangle & rect fcns

One triangle in $x(t)$ is equivalent to the product of



$15\,\Lambda\left(\frac{t-4}{8}\right)$



$\text{rect}\left(\frac{t-2}{4}\right)$

The function $x(t)$ can be written as

$$\boxed{x(t) = 15 \sum_{i=1}^{\infty} \Lambda\left(\frac{t-4i}{4}\right) \text{rect}\left(\frac{t-4i-2}{4}\right)}$$

Problem 4

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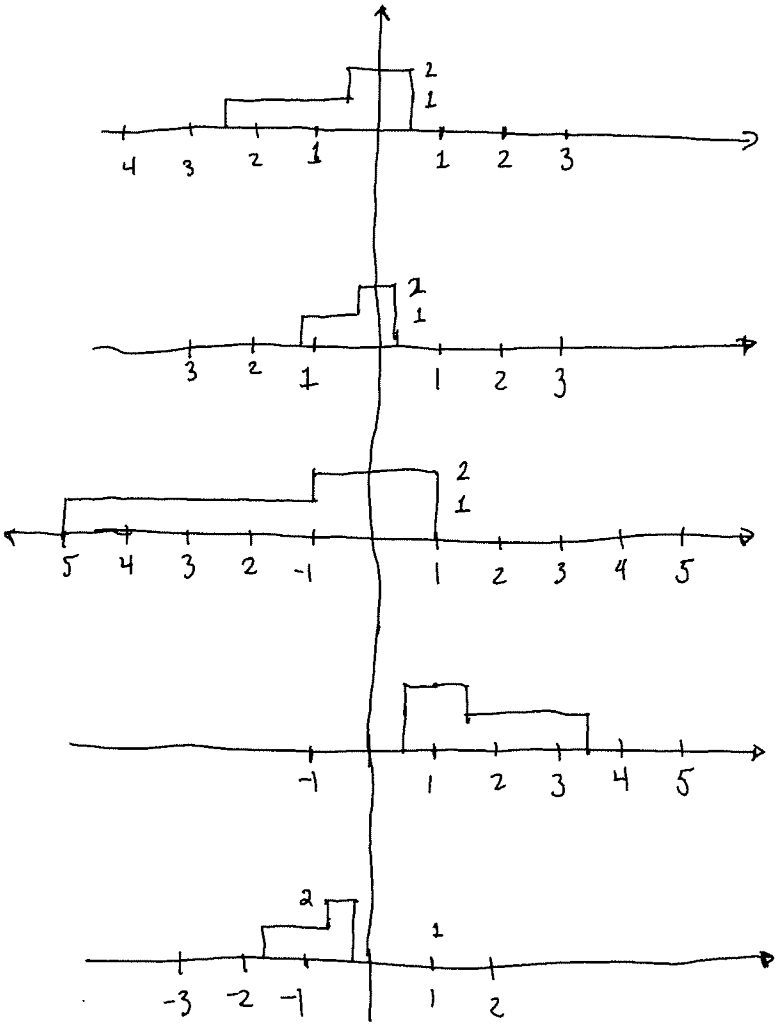
Assume $x(t) = rect\left(\frac{t+1}{3}\right) + rect(t)$



$x(t)$

(a) $x_1(t) = x(2t)$

(b) $x_2(t) = x\left(\frac{t}{2}\right)$

(c) $x_3(t) = x(1-t)$

$= x(-(t-1))$

(d) $x_4(t) = x(1+2t)$

$= x\left(2\left(t+\frac{1}{2}\right)\right)$

Problem 5

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Simplify the following

(a) $\int_{-\infty}^{\infty} e^{-t} u(t-5)\,dt = \int_{5}^{\infty} e^{-t}\,dt = -\left[e^{-\infty} - e^{-5}\right] = e^{-5}$

(b) $\int_{-\infty}^{\infty} t^2\left[u(t-6) - u(t-5)\right] dt = -\int_{5}^{6} t^2\,dt = -\left.\frac{t^3}{3}\right|_5^6 = -\left[72 - 41.66\right] = -30.33$

(c) $\int_{-\infty}^{\infty} t^2 \delta(t-2)\,dt = \int_{-\infty}^{\infty} 4\,\delta(t-2)dt = 4\int_{-\infty}^{\infty} \delta(t-2)dt = 4$

(d) $\int_{5}^{\infty} t^2 \delta(t-2)dt = 4\int_{5}^{\infty} \delta(t-2)\,dt = 4 \cdot 0 = 0$

(e) $\int_{-\infty}^{\infty} \delta(t-3)\,\delta(t-4)dt = 0$

(f) $\int_{-\infty}^{\infty} u(t-3)\delta(t-4)\,dt = \int_{-\infty}^{\infty} u(1)\delta(t-4)dt = 1\int_{-\infty}^{\infty} \delta(t-4)dt = 1$

(g) $\int_{-\infty}^{t} e^{-(t-\lambda-1)}\delta(\lambda-2)d\lambda = \int_{-\infty}^{t} e^{-(t-3)}\delta(\lambda-2)d\lambda = e^{-(t-3)}\int_{-\infty}^{t}\delta(\lambda-2)d\lambda = e^{-(t-3)}\,u(t-2)$

(h) $\int_{-\infty}^{t} e^{-2(t-\lambda)}\,\delta(\lambda+1)\,d\lambda = e^{-2(t+1)}\int_{-\infty}^{t}\delta(\lambda+1)\,d\lambda = e^{-2(t+1)}\,u(t+1)$

(i) $\int_{-\infty}^{t-1} e^{-3(t-\lambda)}\delta(\lambda-1)\,d\lambda = e^{-3(t-1)}\,u(t-2)$

(j) $\int_{-t}^{\infty} e^{-(t-\lambda)}\,\delta(\lambda+2)\,d\lambda = e^{-(t+2)}\int_{-t}^{\infty}\delta(\lambda+2)d\lambda = e^{-(t+2)}\,u(-(t-2)) = e^{-(t+2)}u(t-2)$

(k) $\delta(t)\delta(t-2) = 0$

(l) $\delta(2(t-2))\,\sin(t_y\pi) = \frac{1}{2}\delta(t-2)\,\sin(t_y\pi) = \frac{1}{2}\sin(2\pi_y)\,\delta(t-2)$

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(m) $t\,\delta(t-1) + t^3\,\delta(t-3) = \delta(t-1) + 27\,\delta(t-3)$

(n) $H(\omega)\,\delta(\omega-1) + A(\omega - x + 1)\,\delta(\omega) = H(1)\,\delta(\omega-1) + A(-x+1)\,\delta(\omega)$

Problem 6

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For each of the signals determine if periodic and, if so, the fundamental period.

(a) $x(t) = \sin(2t) + \cos(3t + 30^\circ)$

$$x(t+T) = \sin(2(t+T)) + \cos(3(t+T) + 30^\circ)$$
$$= \sin(2t + 2T) + \cos(3t + 3T + 30^\circ)$$

$2T = 2\pi r$ $\qquad 3T = 2\pi q$

$T_1 = \pi r$ $\qquad T_2 = \frac{2\pi}{3} q$

If $T_1 = T_2$ then $\pi r = \frac{2\pi}{3} q$

$\frac{r}{q} = \frac{2}{3} \longrightarrow$ This is a ratio of integers so the sum is periodic with $T = 2\pi$ seconds

(b) $x(t) = \cos(2t) + \cos(\pi t)$

$$x(t+T) = \cos(2t + 2T) + \cos(\pi t + \pi T)$$

$2T_1 = 2\pi r$ $\qquad \pi T_2 = 2\pi q$

$T_1 = \pi r$ $\qquad T_2 = 2q$

if $T_1 = T_2$ then $\pi r = 2q$

$\frac{r}{q} = \frac{2}{\pi} \longrightarrow$ This is an irrational number so the sum is aperiodic.

(c) $x(t) = e^{-t}\cos(t)$

$$x(t+T) = e^{-(t+T)}\cos(t+T)$$

Is $e^{-t}\cos(t) = e^{-t}e^{-T}\cos(t+T)$? $\rightarrow$ No

The e^{-T} term makes this an aperiodic fcn.

(d) $x(t) = 2e^{j2t} + 3e^{j(3t+2)}$

$x(t+T) = 2e^{j2t}e^{j2T} + 3e^{j(3t+2)}e^{j3T}$

$e^{j2T} = 1$ for every $2T = 2\pi r$

$e^{j3T} = 1$ for every $3T = 2\pi q$

If periodic then $\pi r = \frac{2\pi q}{3}$

or $\frac{r}{q} = \frac{2}{3}$ $\longrightarrow$ This is a ratio of integers so it's <u>periodic</u>

w/ $T = 2\pi$ seconds

(e) $x(t) = 5t - e^{-j(t+3)}$

$x(t+T) = 5t + 5T - e^{-j(t+3)}e^{-jT}$

There is no way to get $5t+5T = 5t$ so this fcn is <u>Aperiodic</u>

(f) $x(t) = \sin(2t) + e^{j(0.5t+1)}$

$x(t+T) = \sin(2t+2T) + e^{j(0.5t+1)}e^{j0.5T}$

$2T = 2\pi r$ $\qquad$ $0.5T = 2\pi q$

$T_1 = \pi r$ $\qquad$ $T_2 = 4\pi q$

$\pi r = 4\pi q \longrightarrow \frac{r}{q} = \frac{4}{1}$

This fcn is <u>periodic</u> w/ $T = 4\pi$ seconds

Problem 7

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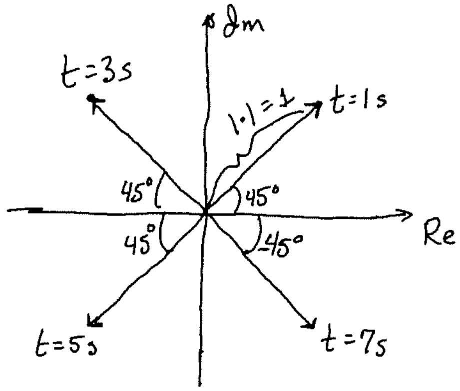
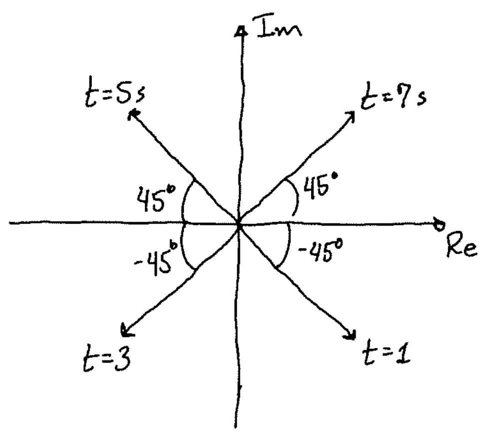
use Euler's Identity in form $e^{j\omega_o t} = \cos(\omega_o t) + j\sin(\omega_o t)$

(a) if $\omega_o = \pi/4$ r/s sketch vector of $e^{j\omega_o t}$ for t = 1, 3, 5, and 7 seconds



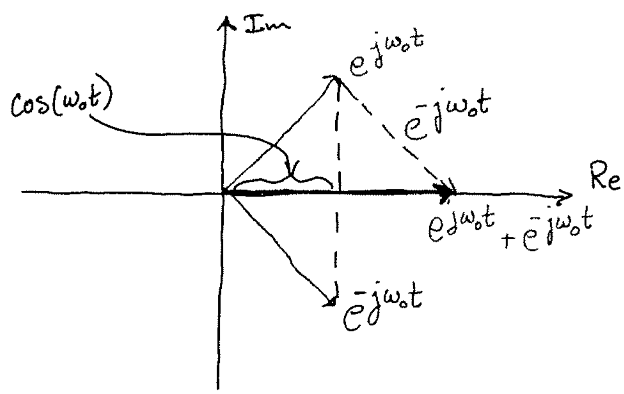
* As t increases the vector is rotating counter clockwise. It makes one complete rotation every 8 seconds.

(b) If $\omega_o = \pi/4$ r/s sketch the vector of $e^{-j\omega_o t}$ for t = 1, 3, 5, 7 seconds



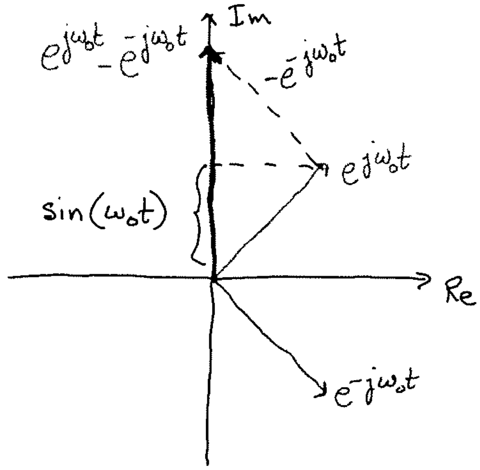
* As t increases this vector rotates in the clockwise direction. It also makes one revolution every 8 seconds.

(c) If we add $e^{j\omega_o t} + e^{-j\omega_o t}$



* These two vectors have same magnitude but opposite angles for all values of t. When summed they make a purely REAL result. The |·| of the result is $2\cos(\omega_o t)$

(d) What property does result have if you subtract the vectors $e^{j\omega_0 t} - e^{-j\omega_0 t}$



* The result will be purely IMAGINARY for all t. It will have a magnitude of $2\sin(\omega_0 t)$.

```
%Part (a)
% This function was found to be periodic with T=2*pi seconds
ta=0:6*pi/150:6*pi;
xa=sin(2*ta)+cos(3*ta+pi/6);

%Part (b)
% This function was not periodic but the longer period was multiples of pi
% for the first cos term
tb=0:3*pi/150:3*pi;
xb=cos(2*tb)+cos(pi*tb);

%Part (c)
% This function was not periodic but the cos term was periodic with T=2*pi
% seconds
tc=0:6*pi/150:6*pi;
xc=exp(-tc).*cos(tc);


%Part (d)
% This function was found to be periodic with T=2*pi seconds
td=0:6*pi/150:6*pi;
xd=2*exp(j*2*td)+3*exp(j*(3*td+2));


%Part (e)
% This function was not periodic but the exponential term was periodic with T=2*pi
% seconds
te=0:6*pi/150:6*pi;
xe=5*te-exp(-j*(te+3));


%Part (d)
% This function was found to be periodic with T=4*pi seconds
tf=0:12*pi/150:12*pi;
xf=sin(2*tf)+exp(j*(0.5*tf+1));


subplot(6,2,1), plot(ta, xa)
title('x(t)=sin(2t) + cos(3t+pi/6');
ylabel('Part (a)')
subplot(6,2,3), plot(tb, xb)
title('x(t)=cos(2t) + cos(pi*t)');
ylabel('Part (b)')
subplot(6,2,5), plot(tc, xc)
title('x(t)=exp(-t)*cos(t)');
ylabel('Part (c)')
subplot(6,2,7), plot(td, real(xd))
title('x(t)=Real part of 2exp(j2t)+3exp(j(3t+2))');
ylabel('Part (d)')
subplot(6,2,8), plot(td, imag(xd))
title('x(t)=Imag part of 2exp(j2t)+3exp(j(3t+2))');
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```
subplot(6,2,9), plot(te, real(xe))
title('x(t)=Real part of 5t-exp(-j(t+3))');
ylabel('Part (e)')
subplot(6,2,10), plot(te, imag(xe))
title('x(t)=Imag part of 5t-exp(-j(t+3))}');
subplot(6,2,11), plot(tf, real(xf))
title('x(t)=Real part of sin(2t)+exp(j(0.5t+1))');
ylabel('Part (f)')
subplot(6,2,12), plot(tf, imag(xf))
title('x(t)=Imag Part of sin(2t)+exp(j(0.5t+1))');
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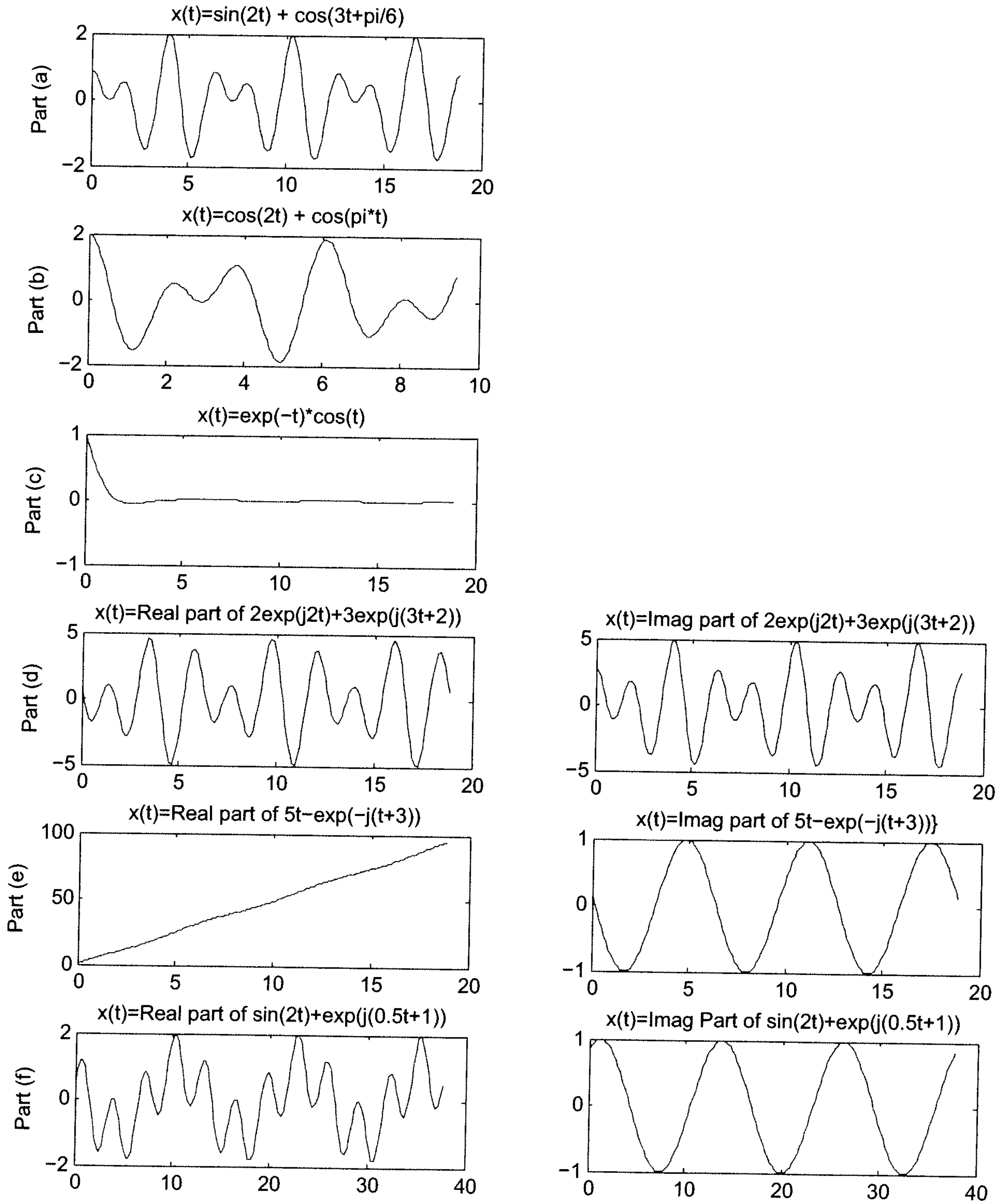
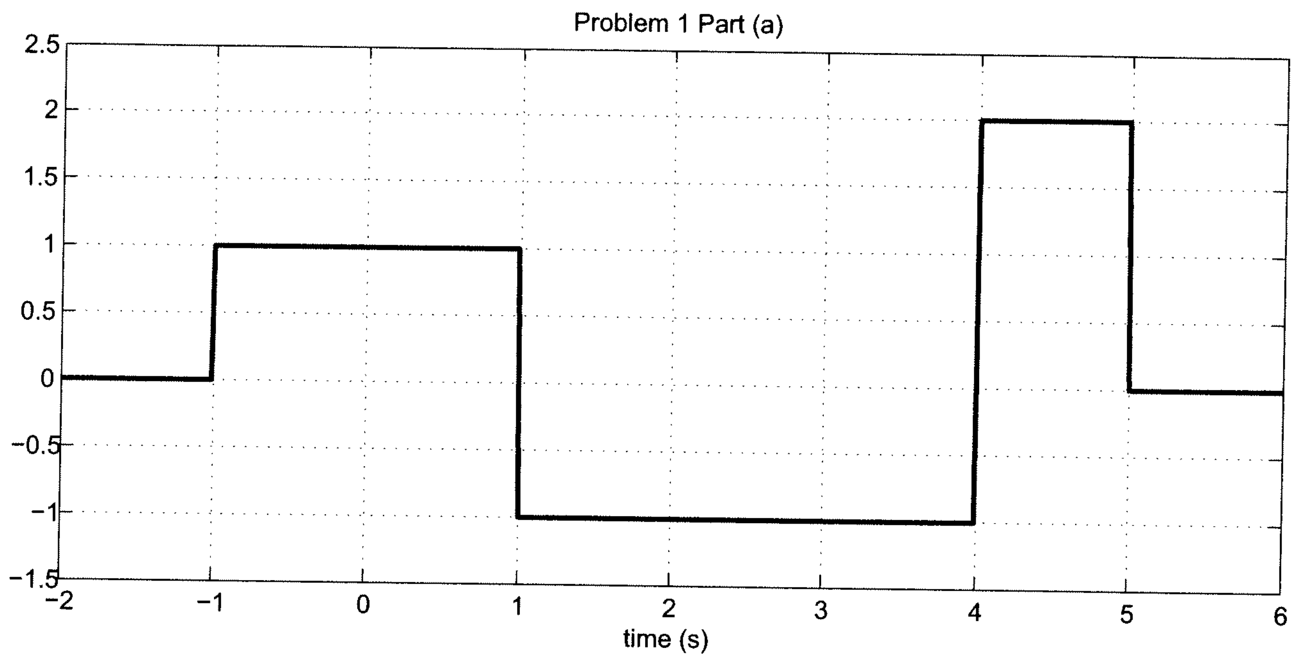
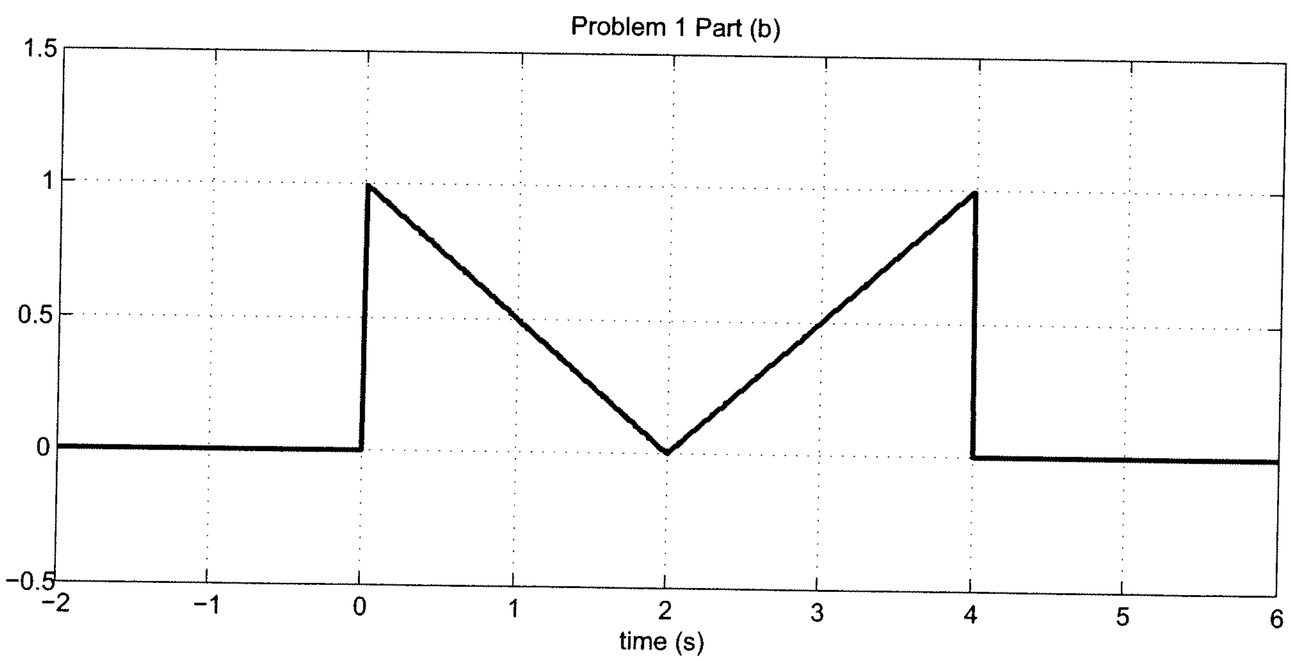

x(t)=sin(2t) + cos(3t+pi/6)
Part (a)
x(t)=cos(2t) + cos(pi*t)
Part (b)
x(t)=exp(−t)*cos(t)
Part (c)
x(t)=Real part of 2exp(j2t)+3exp(j(3t+2))
Part (d)
x(t)=Imag part of 2exp(j2t)+3exp(j(3t+2))
x(t)=Real part of 5t−exp(−j(t+3))
Part (e)
x(t)=Imag part of 5t−exp(−j(t+3))}
x(t)=Real part of sin(2t)+exp(j(0.5t+1))
Part (f)
x(t)=Imag Part of sin(2t)+exp(j(0.5t+1))

```
t=-2:8/1000:6;
x1=unit_step(t+1)-2*unit_step(t-1)+3*unit_step(t-4)-2*unit_step(t-5);
x2=unit_rect((t-2),4)-unit_triangle((t-2),2);

subplot(2,1,1),plot(t,x1)
title('Problem 1 Part (a)')
xlabel('time (s)')
axis([-2 6 -1.5 2.5])
grid on
subplot(2,1,2), plot(t,x2)
title('Problem 1 Part (b)')
xlabel('time (s)')
axis([-2 6 -0.5 1.5])
grid on
```


Problem 1 Part (a)
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−1
0
1
2
3
4
5
6
time (s)


Problem 1 Part (b)
1.5
1
0.5
0
−0.5
−2
−1
0
1
2
3
4
5
6
time (s)