ECE 300
Signals and Systems
Homework 5

Due Date: Tuesday January 27. 2009 *at the beginning of class*

1. Show that any function $x(t)$ can be written in terms of and even function and an odd function, i.e. $x(t) = x_e(t) + x_o(t)$, where $x_e(t)$ is an even function, and $x_o(t)$ is an odd function. Determine expressions for $x_e(t)$ and $x_o(t)$ in terms of $x(t)$ (if you can do this than you have shown that $x(t) = x_e(t) + x_o(t)$).

2. Find the Fourier series representation for the signal indicated using hand analysis. Clearly indicate the values of ω_0 and the c_k. *Hints:* (1) *Draw the signal, and then use the sifting property to calculate the* c_k. (2) *If you understand how to do this, there is very little work involved.*

$$x(t) = \sum_{p=-\infty}^{\infty} \delta(t - 3p)$$

3. Simplify each of the following into the form $c_k = \alpha(k) e^{-j\beta(k)} \mathrm{sinc}(\lambda k)$

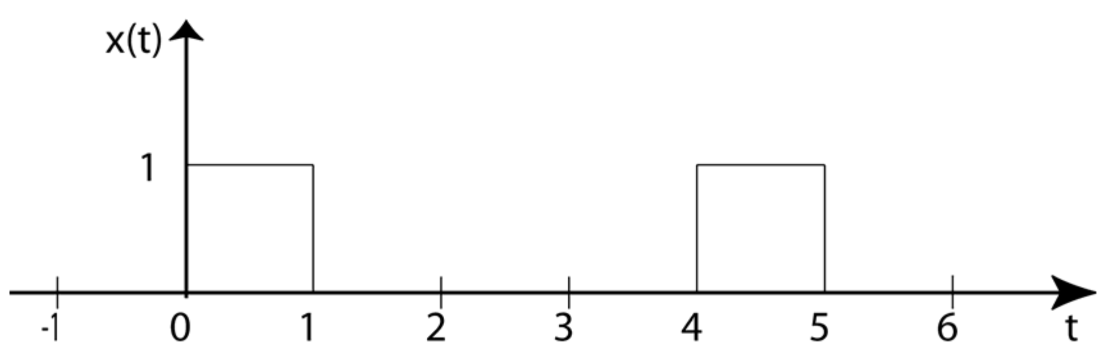
a) $c_k = \dfrac{e^{j7k\pi} - e^{-j2k\pi}}{k\pi j}$

b) $c_k = \dfrac{e^{-j2\pi k} - e^{-j5\pi k}}{jk}$

c) $c_k = \dfrac{e^{j5k} - e^{j2k}}{k}$

Scrambled Answers $c_k = 3\pi e^{-j\frac{7\pi k}{2}} \mathrm{sinc}\left(\dfrac{3\mathrm{k}}{2}\right)$, $c_k = 3e^{j(\frac{7}{2}k + \frac{\pi}{2})} \mathrm{sinc}\left(\dfrac{3k}{2\pi}\right)$, $c_k = 9e^{j\frac{5}{2}k\pi} \mathrm{sinc}\left(k\dfrac{9}{2}\right)$

4. For the periodic signal shown below, with period $T = 4$



a) Determine the fundamental frequency ω_0.

b) Determine the average value.

c) Determine the average power in the DC component of the signal.

d) Determine an expression for the expansion coefficients, c_k. You must write your expression in terms of the **sinc** function, and possibly a leading phase term.

5. (**Matlab/Prelab Problem**) Read the **Appendix** (at the end of this assignment) and then do the following:

a) Copy the file **Trigonometric_Fourier_Series.m** to file **Complex_Fourier_Series.m**.

b) Modify **Complex_Fourier_Series.m** so it computes the average value c_o

c) Modify **Complex_Fourier_Series.m** so it ***directly*** computes c_k for $k=1$ to $k=N$. You are **not** to use the trigonometric Fourier series coefficients for this.

d) Modify **Complex_Fourier_Series.m** so it also computes the Fourier series estimate using the formula

$$x(t) \approx c_o + \sum_{k=1}^{N} 2|c_k|\cos(k\omega_o t + \measuredangle c_k)$$

You will probably need to use the Matlab functions **abs** and **angle** for this.

e) Using the code you wrote in part **d,** find the complex Fourier series representation for the following functions (defined over a single period)

$$f_1(t) = e^{-t}u(t) \quad 0 \le t < 3$$

$$f_2(t) = \begin{cases} t & 0 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

$$f_3(t) = \begin{cases} 0 & -2 \le t < -1 \\ 1 & -1 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

***Turn in your code*.** *Be sure to modify your program so any unnecessary code is eliminated (not just commented out). Note that the values of **low** and **high** will be different for each of these functions!*

Appendix

In the majority of this course we will be using the complex (or exponential) form of the Fourier series, since it is really easier to do various mathematical things with it once you get used to it.

Exponential Fourier Series If $x(t)$ is a periodic function with fundamental period T, then we can represent $x(t)$ as a Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_o t}$$

where $\omega_o = \frac{2\pi}{T}$ is the fundamental period, $\mathrm{c_o}$ is the average (or DC, i.e. zero frequency) value, and

$$\mathrm{c_o} = \frac{1}{T}\int_0^T x(t)dt$$

$$c_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t}dt$$

If $x(t)$ is a real function, then we have the relationships $|c_k| = |c_{-k}|$ (the magnitude is even) and $\measuredangle c_{-k} = -\measuredangle c_k$ (the phase is odd). Using these relationships we can then write

$$x(t) = c_o + \sum_{k=1}^{\infty} 2|c_k|\cos(k\omega_o t + \measuredangle c_k)$$

This is usually a much easier form to deal with, since it lends itself easily to thinking of a phasor representation of $x(t)$. This will be particularly useful when we starting filtering periodic signals