# Linear System Response to Periodic Inputs
# Lab 07
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## Objectives

In this lab we will examine systems with periodic inputs. We measure the frequency response of a $5^{th}$ order Butterworth filter and determine the relevant parameters. Next we simulate the response of the filter to a periodic waveform, and compare with the actual response using the spectrum analyzer.

## Equipment

Agilent Function Generator                                    Coaxial Cables
Digital Oscilloscope
Orange Butterworth Filter

## Background

A periodic signal can be represented by the complex exponential form of the Fourier series. When a periodic signal is applied to the input of a filter, each of the harmonic components of the input signal experiences an amplitude and phase change caused by the filter. At the filter output the harmonic components add together to produce the output waveform. The amplitude and phase changes experienced by each of the input components combine to make the output signal different from the input signal in a predictable way.

## Pre-Lab: review Labs 5 and 6

## Part 1: Measuring the Frequency Response of a Butterworth Filter

a) Obtain an orange filter from your instructor. Connect the output of function generator to the input of the orange filter and the output of the filter to the input of the oscilloscope using coaxial cable. Be sure to adjust the gain of the oscilloscope input to match the 1:1 of the coax cable.

b) Set the function generator to output a 100 Hz sine wave and adjust the amplitude of the sine wave so that the output of the signal (after passing through the orange filter) is measured as 1V peak peak-to-peak on the oscilloscope. Set the toggle switch on the orange filter to give you the smaller amplitude.

c) Keeping the input amplitude constant, vary the frequency of the function generator according to Table 1 on the data recording sheet. The frequency of the function generator should be adjusted so that the frequency measured on the SCOPE is the same as Table 1. Measure and record the output amplitude on the scope so you can fill in the table on the worksheet at the end of the lab.

d) Adjust the frequency of the function generator to precisely the cutoff frequency of the filter. Remember the cutoff frequency is defined as the frequency where the output POWER is ½ the passband POWER. You should be able to translate this to the appropriate signal AMPLITUDE. Record the measured cutoff frequency in the blank below Table 2 at the end of the lab. As shown in Table 2, measure the amplitude of the output at frequencies $f_c - 200$, $f_c - 100$, $f_c$, $f_c + 100$, and $f_c + 200$ Hz .

e) The orange filter is a fifth order filter. Using your measured cutoff frequency, generate the corresponding Butterworth filter in Matlab using the **butter** command. We are constructing an analog filter, and be sure your cutoff frequency is in the correct units. _**You are to write a new m-file for parts e and f, and turn this code in with your lab. This code should only contain what is necessary for doing parts e and f, nothing else.**_

f) Using Matlab, plot the magnitude of the filter transfer function (in dBV) versus frequency (in Hz), and include your measured points on the graph. Commands that may be helpful here include **linspace**, **logspace**, **freqs**, **log10**, and **semilogx**. An example plot is given in Figure 1.
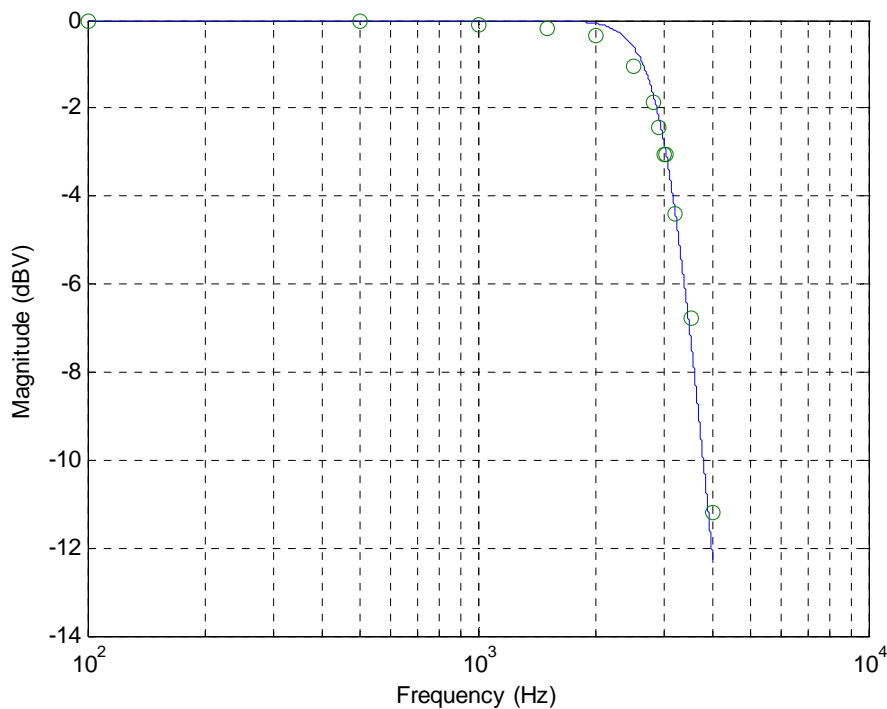


Figure 1: Predicted and measured frequency response.

## Part 2: Filtering Periodic Signals

a) Write a MATLAB script to determine the complex Fourier series for a square-wave periodic signal with a fundamental period of 1 kHz, a 75% duty cycle, with an amplitude of 1 volt peak-to-peak.

b) Use the subplot command to create a figure with a top and bottom panel. In the top panel, plot the original signal and your Fourier series approximation with 10 harmonics. In the bottom panel, plot the single-sided power spectrum of the input signal in dBmV for the first 10 harmonics. *Turn in your plot.* Have your program print out the values of the spectral components and fill in the Predicted Values in Table 3 on the worksheet at the end of this lab.

c) Adjust your program to filter the square-wave signal using the Butterworth filter you modeled in Part 1 with the measured cutoff frequency. Again use subplot to create a figure with two panels. In the top panel, plot: 1) the original signal, 2) your Fourier series approximation with 10 harmonics, and 3) the filtered signal. In the bottom panel, plot the single-sided power spectrum of the filtered signal in dBmV for the first 10 harmonics Turn in your plot. Have your program print out the values of the spectral components of the output of the filter and fill in Table 4 on the worksheet at the end of this lab.

d) Use the function generator to create the waveform from Part 2(a) as verified on the oscilloscope, then connect the function generator to the spectrum analyzer. *You will need to set the spectrum analyzer to DC coupling (push the Input button, then DC coupling).* Remember to account for the change in input impedance between the scope and spectrum analyzer. The input amplitude to the spectrum analyzer should be 1 V peak-peak. Measure and record the power spectrum from the spectrum analyzer in Table 3 on the worksheet.

e) Disconnect the function generator from the spectrum analyzer and connect it to the input of the orange filter.  Flip the toggle switch on the orange filter and connect the output of the filter to the spectrum analyzer. Measure the spectrum and fill in Table 4 on the worksheet.

## Part 3: Recreating Signals from the Power Spectrum

The spectrum analyzer gives us the one-sided power spectrum of the signal, z(t), which contains information about the Fourier Series coefficients. If we know the Fourier Series coefficients we can recreate a signal. The question is, "is there enough information in the power series as measured from the spectrum analyzer to adequately recreate a signal".

a) Using the measured values from Tables 3 and 4, write a MATLAB script to calculate the corresponding magnitudes of the Fourier Series coefficients and fill in the values in the tables.

b) Adjust your script to use these coefficient magnitudes to create new estimates of the signals
$\tilde{z}(t) = \sum_{k=1}^{10} 2|c_k|\cos(k\omega_0 t)$ . Use subplot again to create another 2-pane figure, and plot these new estimates of the signals, one in each of the panes.

c) Compare these signals to the original input and output signals of the filter. Explain any differences.

**Lab 07**
**Instructor Verification Sheet**

Names_____Date:_____

At the end of this lab you should have attached the following plots:

- *One plot of the measured frequency response and the predicted frequency response of the (orange box) Butterworth filter.*
- *One plot of the time domain input signal, the Fourier series representation of the input signal, and the single sided power spectrum for the input signal*
- *One plot of  the Fourier series representation of the output signal, and the single sided power spectrum for the output signal*

  *In addition you should turn in your code for part 1f. If should be neat and have no unnecessary code.*

Part 1: Measuring the Frequency Response of a Real Filter

Table 1:

| Frequency (Hz) | 100 | 500 | 1000 | 1500 | 2000 | 2500 | 3000 | 3500 | 4000 |
|---|---|---|---|---|---|---|---|---|---|
| Amplitude (peak voltage) | | | | | | | | | |

Table 2:

| Frequency (Hz) | $f_c - 200$ | $f_c - 100$ | $f_c = ?$ | $f_c + 100$ | $f_c + 200$ |
|---|---|---|---|---|---|
| Amplitude (peak voltage) | | | | | |

Measured Cutoff Response of your Butterworth Filter:_____

Part 2: Power Spectrum of Input and Output Signals

Table 3: Summary of Results for Input Waveform

| Harmonic | Predicted Power (dBmV) | Measured Power (dBmV) | Error (dBmV) | Frequency (kHz) | $|c_k|$ from Measured Power (V) |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | | | | | |
| 9 | | | | | |

Table 4: Summary of Results for Output Waveform

| Harmonic | Predicted Power (dBmV) | Measured Power (dBmV) | Error (dBmV) | Frequency (kHz) | $|c_k|$ from Measured Power (V) |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | | | | | |
| 9 | | | | | |
| | | | | | |

Part 3: Estimates from Power Spectrum

Explain any differences between power spectrum estimates and original signals:

If the data from the spectrum analyzer is insufficient to recreate a signal, what purpose does it serve?