

Table of Fourier Transform Pairs of Energy Signals

Function name	Time Domain $x(t)$	Frequency Domain $X(\omega)$
FT	$x(t)$	$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \mathcal{F}\{x(t)\}$
IFT	$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \mathcal{F}^{-1}\{X(\omega)\}$	$X(\omega)$
Rectangle Pulse	$rect\left(\frac{t}{T}\right) = \Pi\left(\frac{t}{T}\right) \equiv \begin{cases} 1 & t \leq T/2 \\ 0 & \text{elsewhen} \end{cases}$	$T \operatorname{sinc}\left(\frac{T}{2\pi} \omega\right)$
Triangle Pulse	$\Lambda\left(\frac{t}{W}\right) \equiv \begin{cases} 1 - t /W & t \leq W \\ 0 & \text{elsewhen} \end{cases}$	$W \operatorname{sinc}^2\left(\frac{W}{2\pi} \omega\right)$
Sinc Pulse	$\operatorname{sinc}(Wt) \equiv \frac{\sin(\pi \cdot Wt)}{\pi \cdot Wt}$	$\frac{1}{W} \operatorname{rect}\left(\frac{\omega}{2\pi W}\right)$
Exponential Pulse	$e^{-a t } \quad a > 0$	$\frac{2a}{a^2 + \omega^2}$
Gaussian Pulse	$\exp\left(-\frac{t^2}{2\sigma^2}\right)$	$(\sigma\sqrt{2\pi}) \exp\left(-\frac{\sigma^2 \omega^2}{2}\right)$
Decaying Exponential	$\exp(-at)u(t) \quad \operatorname{Re}\{a\} > 0$	$\frac{1}{a + j\omega}$
Sinc ² Pulse	$\operatorname{sinc}^2(Bt)$	$\frac{1}{B} \Lambda\left(\frac{\omega}{2\pi B}\right)$

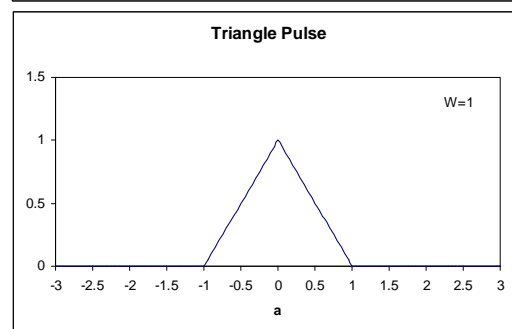
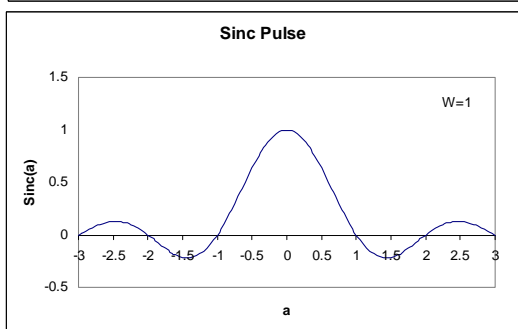
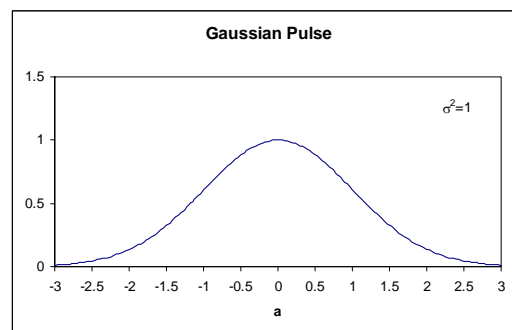
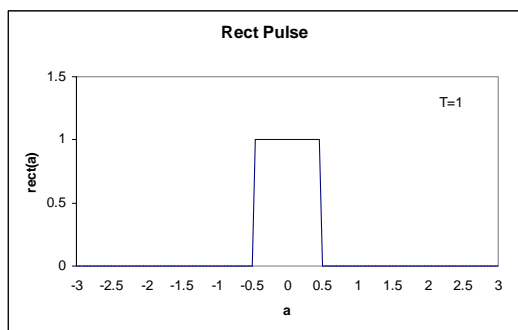


Table of Fourier Transform Pairs of Power Signals

Function name	Time Domain x(t)	Frequency Domain X(ω)
FT	$x(t)$	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt = \mathcal{F}\{x(t)\}$
IFT	$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega = \mathcal{F}^{-1}\{X(\omega)\}$	$X(\omega)$
Impulse	$\delta(t)$	1
DC	1	$2\pi\delta(\omega)$
Cosine	$\cos(\omega_0 t + \theta)$	$\pi [e^{j\theta} \delta(\omega - \omega_0) + e^{-j\theta} \delta(\omega + \omega_0)]$
Sine	$\sin(\omega_0 t + \theta)$	$-j\pi [e^{j\theta} \delta(\omega - \omega_0) - e^{-j\theta} \delta(\omega + \omega_0)]$
Complex Exponential	$\exp(j\omega_0 t)$	$2\pi\delta(\omega - \omega_0)$
Unit step	$u(t) \equiv \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$	$\pi\delta(\omega) + \frac{1}{j\omega}$
Signum	$\text{sgn}(t) \equiv \begin{cases} 1 & t \geq 0 \\ -1 & t < 0 \end{cases}$	$\frac{2}{j\omega}$
Linear Decay	$1/t$	$-j\pi \text{sgn}(\omega)$
Impulse Train	$\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$	$\frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T_s}\right)$
Fourier Series	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$, where $a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0)$

Table of Fourier Transforms of Operations

Operation	FT Property Given $g(t) \Leftrightarrow G(\omega)$
Linearity	$af(t) + bg(t) \Leftrightarrow aF(\omega) + bG(\omega)$
Time Shifting	$g(t - t_0) \Leftrightarrow e^{-j\omega t_0} G(\omega)$
Time Scaling	$g(at) \Leftrightarrow \frac{1}{ a } G\left(\frac{\omega}{a}\right)$
Modulation (1)	$g(t)\cos(\omega_0 t) \Leftrightarrow \frac{1}{2}[G(\omega - \omega_0) + G(\omega + \omega_0)]$
Modulation (2)	$g(t)e^{j\omega_0 t} \Leftrightarrow G(\omega - \omega_0)$
Differentiation	If $f(t) = \frac{dg(t)}{dt}$, then $F(\omega) = j\omega \cdot G(\omega)$
Integration	If $f(t) = \int_{-\infty}^t g(\alpha) d\alpha$, then $F(\omega) = \frac{1}{j\omega} G(\omega) + \pi G(0)\delta(\omega)$
Convolution	$g(t) * f(t) \Leftrightarrow G(\omega) \cdot F(\omega)$, where $g(t) * f(t) \equiv \int_{-\infty}^{\infty} g(\alpha) f(t - \alpha) d\alpha$
Multiplication	$f(t) \cdot g(t) \Leftrightarrow \frac{1}{2\pi} F(\omega) * G(\omega)$
Duality	If $g(t) \Leftrightarrow z(\omega)$, then $z(t) \Leftrightarrow 2\pi g(-\omega)$
Hermitian Symmetry	If $g(t)$ is real valued then $G(-\omega) = G^*(\omega)$ ($ G(-\omega) = G(\omega) $ and $\angle G(-\omega) = -\angle G(\omega)$)
Conjugation	$g^*(t) \Leftrightarrow G^*(-\omega)$
Parseval's Theorem	$E = \int_{-\infty}^{\infty} g(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) ^2 d\omega$

Some Notes:

1. There are two similar functions used to describe the functional form $\sin(x)/x$. One is the $\text{sinc}()$ function, and the other is the $\text{Sa}()$ function. We will only use the $\text{sinc}()$ notation in class. Note the role of π in the $\text{sinc}()$ definition:

$$\text{sinc}(x) \equiv \frac{\sin(\pi x)}{\pi x}; \quad \text{Sa}(x) \equiv \frac{\sin(x)}{x}$$

2. The impulse function, aka delta function, is defined by the following three relationships:

- a. Singularity: $\delta(t - t_0) = 0$ for all $t \neq t_0$

- b. Unity area: $\int_{-\infty}^{\infty} \delta(t) dt = 1$

- c. Sifting property: $\int_{t_a}^{t_b} f(t) \delta(t - t_0) dt = f(t_0)$ for $t_a < t_0 < t_b$.

3. Many basic functions do not change under a reversal operation. Other change signs. Use this to help simplify your results.

- a. $\delta(t) = \delta(-t)$ (in general, $\delta(at) \Leftrightarrow \frac{1}{|a|} \delta(t)$)

- b. $\text{rect}(t) = \text{rect}(-t)$

- c. $\Lambda(t) = \Lambda(-t)$

- d. $\text{sinc}(t) = \text{sinc}(-t)$

- e. $\text{sgn}(t) = -\text{sgn}(-t)$

4. The duality property is quite useful but sometimes a bit hard to understand. Suppose a known FT pair $g(t) \Leftrightarrow z(\omega)$ is available in a table. Suppose a new time function $z(t)$ is formed with the same shape as the spectrum $z(\omega)$ (i.e. the function $z(t)$ in the time domain is the same as $z(\omega)$ in the frequency domain). Then the FT of $z(t)$ will be found to be $z(t) \Leftrightarrow 2\pi g(-\omega)$, which says that the F.T. of $z(t)$ is the same shape as $g(t)$, with a multiplier of 2π and with $-\omega$ substituted for t .

An example is helpful. Given the F.T. pair $\text{sgn}(t) \Leftrightarrow 2/j\omega$, what is the Fourier transform of $x(t)=1/t$? First, modify the given pair to $j/2 \text{sgn}(t) \Leftrightarrow 1/\omega$ by multiplying both sides by $j/2$. Then, use the duality function to show that $1/t \Leftrightarrow 2\pi j/2 \text{sgn}(-\omega) = j\pi \text{sgn}(-\omega) = -j\pi \text{sgn}(\omega)$.