

Quiz Solutions

a) $G(\omega) = \text{rect}\left(\frac{\omega}{4}\right)$ For $G(\omega) = \frac{1}{W} \text{rect}\left(\frac{\omega}{2\pi W}\right) \leftrightarrow g(t) = \text{sinc}(Wt)$

$$2\pi W = 4 \quad W = \frac{4}{2\pi} = \frac{2}{\pi}$$

$$G(\omega) = \text{rect}\left(\frac{\omega}{4}\right) \leftrightarrow \boxed{\frac{2}{\pi} \text{sinc}\left(\frac{2}{\pi}t\right) = g(t)}$$

b) $g(t) = \text{sinc}\left(\frac{3}{\pi}t\right)$ For $g(t) = \text{sinc}(Wt) \leftrightarrow G(\omega) = \frac{1}{W} \text{rect}\left(\frac{\omega}{2\pi W}\right)$

$$\text{Here } W = \frac{3}{\pi}$$

$$g(t) = \text{sinc}\left(\frac{3}{\pi}t\right) \leftrightarrow \boxed{G(\omega) = \frac{\pi}{3} \text{rect}\left(\frac{\omega}{6}\right)}$$

c) $G(\omega) = \text{sinc}\left(\frac{\omega}{5}\right)$ For $G(\omega) = T \text{sinc}\left(\frac{T}{2\pi}\omega\right) \leftrightarrow g(t) = \text{rect}\left(\frac{t}{T}\right)$

$$\frac{T}{2\pi} = \frac{1}{5} \quad T = \frac{2\pi}{5}$$

$$G(\omega) = \text{sinc}\left(\frac{\omega}{5}\right) \leftrightarrow \boxed{g(t) = \frac{5}{2\pi} \text{rect}\left(\frac{t}{\frac{2\pi}{5}}\right)}$$

d) $g(t) = \text{sinc}^2\left(\frac{2t-4}{3}\right) = \text{sinc}^2\left(\frac{2}{3}(t-2)\right)$

$$\text{For } g_1(t) = \text{sinc}^2\left(\frac{2}{3}t\right) \leftrightarrow G_1(\omega) = \frac{3}{2} \mathcal{L}\left(\frac{\omega}{4\pi}\right)$$

$$\text{For } g_2(t) = g_1(t-2) \leftrightarrow G_2(\omega) = G_1(\omega) e^{-j2\omega}$$

$$\text{or } \boxed{G(\omega) = \frac{3}{2} \mathcal{L}\left(\frac{3\omega}{4\pi}\right) e^{-j2\omega}}$$

e) $G(\omega) = \delta(\omega-1000) \cos(\omega) = \cos(1000) \delta(\omega-1000)$

$$\text{For } G_1(\omega) = \delta(\omega) \quad g_1(t) = \frac{1}{2\pi}$$

$$\text{For } G_2(\omega) = G_1(\omega-1000) \quad g_2(t) = g_1(t) e^{j1000t}$$

$$\text{For } G_3(\omega) = \cos(1000) \delta(\omega-1000) \leftrightarrow g(t) = \frac{\cos(1000)}{2\pi} e^{j1000t}$$

$$\boxed{g(t) = 0.090 e^{j1000t}}$$

f) $g(t) = 4$ $\boxed{G(\omega) = 8\pi \delta(\omega)}$

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g) $G(\omega) = \left[\text{rect}\left(\frac{\omega}{6}\right) - \text{rect}\left(\frac{\omega}{2}\right) \right] e^{-j3\omega}$

for $G_1(\omega) = \frac{1}{W} \text{rect}\left(\frac{\omega}{2\pi W}\right) \Leftrightarrow g_1(t) = \text{sinc}(Wt)$

$\text{rect}\left(\frac{\omega}{6}\right) \Rightarrow 2\pi W = 6 \quad W = \frac{6}{2\pi} = \frac{3}{\pi}$

$\text{rect}\left(\frac{\omega}{2}\right) \Rightarrow 2\pi W = 2 \quad W = \frac{1}{\pi}$

for $G_2(\omega) = \text{rect}\left(\frac{\omega}{6}\right) - \text{rect}\left(\frac{\omega}{2}\right) \Leftrightarrow g_2(t) = \frac{3}{\pi} \text{sinc}\left(\frac{3}{\pi}t\right) - \frac{1}{\pi} \text{sinc}\left(\frac{1}{\pi}t\right)$

for $G_3(\omega) = G_2(\omega) e^{-j3\omega} \Leftrightarrow g_3(t) = g_2(t-3)$

$$g(t) = \frac{3}{\pi} \text{sinc}\left(\frac{3}{\pi}[t-3]\right) - \frac{1}{\pi} \text{sinc}\left(\frac{1}{\pi}[t-3]\right)$$

h) $G(\omega) = \frac{j\omega}{2-j\omega}$ for $G_1(\omega) = \frac{1}{2+j\omega}$ $g_1(t) = e^{-2t}u(t)$

for $g_2(t) = g_1(-t) = e^{2t}u(-t) \Leftrightarrow G_2(\omega) = G_1(-\omega) = \frac{1}{2-j\omega}$

for $G_3(\omega) = j\omega G_2(\omega) = \frac{j\omega}{2-j\omega} \Leftrightarrow g_3(t) = \frac{d}{dt} g_2(t)$

$\frac{d}{dt} g_2(t) = \frac{d}{dt} [e^{2t}u(-t)] = 2e^{2t}u(-t) + e^{2t}\delta(-t)(-1)$

$$g(t) = 2e^{2t}u(-t) - \delta(t)$$

i) $G(\omega) = \frac{3}{2+j\left(\frac{\omega}{5}+2\right)} = \frac{3}{2+j\left(\frac{1}{5}(\omega+10)\right)}$

for $g_1(t) = 3e^{-2t}u(t)$ $G_1(\omega) = \frac{3}{2+j\omega}$

for $g_2(t) = g_1\left(\frac{t}{5}\right)$ $G_2(\omega) = \frac{1}{5} G_1\left(\frac{\omega}{5}\right) = \frac{\frac{3}{5}}{2+j\frac{\omega}{5}}$

for $g_3(t) = g_2(t)e^{-j10t} \Leftrightarrow G_3(\omega) = G_2(\omega+10) = \frac{\frac{3}{5}}{2+j\left(\frac{\omega+10}{5}\right)}$

for $g_4(t) = 5g_3(t) \Leftrightarrow G_4(\omega) = \frac{3}{2+j\left(\frac{\omega+10}{5}\right)}$

$$g(t) = 5e^{-j10t} e^{-\frac{10t}{5}} u(t)$$

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j) $G(\omega) = j\omega e^{-\frac{\omega^2}{6}}$

for $g_1(t) = e^{-t^2/2}$ $G_1(\omega) = \sqrt{2\pi} e^{-\omega^2/2}$

for $g_2(t) = g_1(at) = e^{-a^2 t^2/2}$ $G_2(\omega) = \frac{\sqrt{2\pi}}{a} e^{-\frac{\omega^2}{2a^2}}$

$a^2 = 3$ $a = \sqrt{3}$

so $g_2(t) = e^{-3t^2/2} \leftrightarrow \sqrt{\frac{2\pi}{3}} e^{-\omega^2/6}$

$g_3(t) = \sqrt{\frac{3}{2\pi}} e^{-3t^2/2} \leftrightarrow e^{-\omega^2/6} = G_3(\omega)$

for $G_4(\omega) = j\omega G_3(\omega) \leftrightarrow g_4(t) = \frac{d}{dt} g_3(t)$

$\frac{d}{dt} g_3(t) = -3t \sqrt{\frac{3}{2\pi}} e^{-3t^2/2} = g(t)$

k) $g(t) = \frac{2}{3+(2t+3)^2} = \frac{2}{3+[2(t+\frac{3}{2})]^2}$

for $g_1(t) = e^{-\sqrt{3}|t|}$ $G_1(\omega) = \frac{2\sqrt{3}}{3+\omega^2}$

by duality $G_1(t) = \frac{2\sqrt{3}}{3+t^2} \leftrightarrow 2\pi g_1(-\omega) = 2\pi e^{-\sqrt{3}|\omega|}$

for $g_2(t) = \frac{2}{3+t^2} \leftrightarrow G_2(\omega) = \frac{2\pi}{\sqrt{3}} e^{-\sqrt{3}|\omega|}$

for $g_3(t) = g_2(2t) = \frac{2}{3+(2t)^2} \leftrightarrow G_3(\omega) = \frac{1}{2} G_2\left(\frac{\omega}{2}\right) = \frac{\pi}{\sqrt{3}} e^{-\sqrt{3}|\frac{\omega}{2}|}$

for $g_4(t) = g_3(t+\frac{3}{2}) = \frac{2}{3+[2(t+\frac{3}{2})]^2} \leftrightarrow G_4(\omega) = G_3(\omega) e^{j\omega \frac{3}{2}}$

$G(\omega) = \frac{\pi}{\sqrt{3}} e^{j\omega \frac{3}{2}} e^{-\sqrt{3}|\frac{\omega}{2}|}$