Working with dBs

This document presents a thorough introduction to the use of decibels in calculations and laboratory measurements in EE applications. Read this document carefully, for it contains a lot of useful information. It will be necessary to revisit this document several times as you go through your various classes, so you may wish to bookmark it or print it out for ready access. Finally, practice by working through the examples. It is only by practice that this seemingly strange convention will become familiar.

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Decibel (dB) FAQ

1. **What is a decibel (dB), as typically used in electrical engineering?**
   A dB is a means of representing a power ratio R, specifically $R_{dB}=10\log_{10}(R)$. Using dBs to represent such values affords a number of advantages, and is the standard in a number of fields. This document provides a good introduction to the use of dBs. more info
2. **What is a dBm (or dBW)?**
   A dBm is a dB representation of a power level relative to 1 mW. The standard means of representing a power (level) is to represent that power level relative to some known power, such as one mW. Similarly, a dBW is a dB representation of a power level relative to 1 W. more info
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   No. These are all dB representations of values. The dB "unit" represents a power ratio. The dBm "unit" represents a power level relative to 1 mW, and dBW represents a power level relative to 1 W. The latter two "units" are still power ratios! more info
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   The decibel was originally used to represent one tenth (deci) of a "bel", or one tenth of the amount of loss encountered by an audio signal propagating along a standard telephone cable. more info

Introduction

When EEs monitor a signal passing through a system, most often they are concerned with its power level as it propagates through the various components and subsystems. Normally, tracking the power level involves multiplication or division of power gain and loss ratios representing the action of the various components of the system on the signal. Many times, the signal power can vary over a wide range of values - several orders of magnitude - which makes representing the signal power at different points a bit inconvenient. The mathematical computations involving multiplication and division of large numbers can be cumbersome.

In order to make the calculations easier, in a day when calculators had not been invented, a system of logarithmic operations to simplify these multiplicative operations was developed. The decibel itself has its roots in audio engineering, where a decibel (meaning 0.1 "bel") represented the reduction in audio level over one tenth of a mile of standard telephone cable. The decibel representation is still used today and is standard and convenient in many fields of engineering. The way EEs use decibels differs slightly from the original uses of the decibel, and so we will investigate its use in detail in this document.

Logarithmic Mathematics

Decibel representations are based on base 10 logarithms. Logarithms were first introduced by John Napier in the early 1600s, and have a number of uses in mathematics, science, and engineering.

In order to make the math easier, we often use a logarithmic scale to represent values. There are two chief advantages in working with logarithmic scales:

1. Multiplication becomes addition: $\log(a \times b) = \log(a) + \log(b)$ and $\log(a \div b) = \log(a) - \log(b)$).

2. Scales are compressed - if we have values ranging over several orders of magnitude, the plot scale is large for linear representations, while relatively compressed for logarithmic representation of values.

Both of these advantages come into play when working with decibels. It is important to remember that the decibel is a base 10 logarithm, especially when working on a calculator or with software packages. To convert between different base logarithms, use the following relationship:

$$\log_b(x) = \frac{\log_k(x)}{\log_k(b)}$$

Definition of decibel (dB)

The logarithmic scale most often used is one in which the values are represented in *decibels* (dB). Here, the decibel is defined as a logarithmic representation of a *unit-less* quantity, normally a ratio of two powers. The logarithmic base (or radix) used for dBs is 10:

$$N_{dB} = 10\log_{10}\left(N\right),$$

where N refers to the numerical ratio being represented and N_{dB} refers to the value in decibels (dB). Note that other definitions of "dB" exist, but this is the definition used for representing electrical signal powers, and therefore it is the only *valid* definition as far as we are concerned.

There are a number of things to note about this definition. First, there is a multiplier of 10 in front of the logarithm. While it has some historical basis, this is to be considered simply part of the definition. However, it is important to remember that the multiplier is 10 only for quantities involving *power*. Next, the logarithm is base 10 only. Other *radixes* may be used to represent values, but not for dBs. Finally, note that the argument of the logarithm is a unit-less value. This last point is very important in our discussion here.

Representing Values as dBs

There are a number of values for which it will be good to know the dB representation in order to make life easier and to understand the "lingo". Simply plug the values in the dB expression to create this table (verify a few in your head):

$$N_{dB} = 10\log_{10}\left(N\right)$$

numerical	dB
1.00E-06	-60.0
0.001	-30.0
0.01	-20.0
0.5	-3.0
0.2	-7.0
0.1	-10.0
1	0.0
2	3.0
5	7.0
10	10.0
100	20.0
1000	30.0
1.00E+06	60.0

Note that values less than 1 produce negative dB values, and values greater than 1 produce positive dB values. The dB value of 1 is zero dB, and the dB value of 0 is undefined (but can be approximated by -99 dB!).

To convert dB values back to linear values, simply invert the definition of decibels as follows:

$$N = 10^{N_{dB}/10}$$

Be sure to verify a few of the table entries using this relationship as well. You must feel very comfortable with both of the relationships between numerical values and dB representations before moving on.

One of the advantages mentioned above was that multiplication was easier using logarithms. Let's try it using the table above. The value 2 converts to 3 dB, and 100 converts to 20 dB. What should the value 200 convert to? Using multiplication, we see that 200 = 2 x 100. Using dBs, we find:

$$\begin{aligned} 200 &\rightarrow 10\log_{10}(200) = 10\log_{10}(100\times 2) \\ &= 10\log_{10}(100) + 10\log_{10}(2) = 20 + 3 = 23\,\text{dB} \end{aligned}$$

Below are some more examples of using this multiplicative effect to quickly find dB values:

$$\begin{aligned} &50 \rightarrow 10\log_{10}(10) + 10\log_{10}(5) = 10 + 7 = 17\text{ dB} \\ &{}^{1}\!/\!{}_{50} \rightarrow -10\log_{10}(10) - 10\log_{10}(5) = -10 - 7 = -17\text{ dB} \\ &4.0\text{E}06 \rightarrow 10\log_{10}\left(10^{6}\right) + 10\log_{10}(4) = 60 + 6 = 66\text{ dB} \end{aligned}$$

Representing Power Levels using dBs

There are two ways we typically use dBs: to represent *powers* (normally average powers), and to represent *power ratios* (gains and losses). We must be fluent in both, and how to combine the two. Lets look first at how we use decibels to represent power values (or express powers in terms of corresponding voltages or currents). Here, we need to recall one important point from the definition of dBs - the value we represent must be a unit-less quantity! To create this unit-less ratio, we express the power relative to some standard or *reference power*. For example, suppose we wish to represent 5 W in terms of deciBels. First, consider the use of a power reference of 1W:

$$P = 10\log_{10}\left({}^{5W}\!\!\diagup\!{}_{1W}\right) = 10\log_{10}(5) = 6.99\,dBW \cong 7\,dBW$$

Here, the appended "W" to the dB unit reminds us that this is a decibel representation of a power *relative to 1 W* (the dBW is called "dB Watts".) The suffix W does not create a new unit! It is still considered a dB. A more common measure is dBm ("dB milliWatts"), or power relative to 1 mW (note that the "W" is missing in this unit):

$$P = 10\log_{10}\left({}^{5W}\!\!\diagup\!{}_{1mW}\right) = \quad 10\log_{10}(5000) = 37\,dBm$$

and we can relate the power in dBW to power in dBm rather simply:

$$P_{dBm} = 10\log_{10}\left(\frac{5W}{1mW}\frac{1000\,mW}{1W}\right) = 10\log_{10}\left(\frac{5W}{1W}\frac{1000\,mW}{1mW}\right) = 10\log_{10}\left(5\cdot 1000\right)$$
$$= 10\log_{10}(5) + 10\log_{10}(1000) = P_{dBW} + 30$$

So 5 W may be represented by 7 dBW or 37 dBm, values separated by 30 dB or a factor of 1000. Do not think of dBW and dBm as different units. They both are dBs - the “W” and “m” suffixes are there to remind us of the power reference.

A couple of examples would be nice:

$$1\,\mathrm{W} \to 10\log_{10}\left({}^{1\,\mathrm{W}}\!/_{1\,\mathrm{W}}\right)\mathrm{dBW} = 10\log_{10}\left(1\right)\mathrm{dBW} = 0\ \mathrm{dBW}$$
$$1\,\mathrm{W} \to 10\log_{10}\left({}^{1\,\mathrm{W}}\!/_{1\,\mathrm{mW}}\right)\mathrm{dBW} = 10\log_{10}\left(1000\right)\mathrm{dBm} = 30\ \mathrm{dBm}$$
$$20\,\mathrm{mW} \to 10\log_{10}\left({}^{20\,\mathrm{mW}}\!/_{1\,\mathrm{mW}}\right)\mathrm{dBW} = 10\log_{10}\left(20\right)\mathrm{dBm} = 13\ \mathrm{dBm}$$

Another dB unit used to represent a power level is the dBV or dBmV, which is a power level referenced back to an equivalent rms voltage level which would produce that power given a 1 Ω resistance. Note that dB units are for power levels, so we must do some work to represent voltages. Suppose we know a signal’s rms voltage. Given the resistance over which the voltage is developed, the average power would be equal to ${v_{rms}}^2/R$. It turns out that many times we ignore the resistance value in the calculation, calculating the power developed across a 1 Ω resistance. Thus P = v_{rms}^2. We can represent this signal using dB units and a reference of W as follows:

$$P_{dBW} = 10\log_{10}\left({V_{rms}}^2\ W/1\ W\right)\ dBW .$$

Now, we could reason that the 1 W reference is just (1 V_{rms})^2, and rewrite

$$P_{dB\_X} = 10\log_{10}\left({v_{rms}}^2/(1V_{rms})^2\right)\ dBW = 20\log_{10}\left(v_{rms}/1V_{rms}\right)\ dBV .$$

Here, we have used the fact that log(x^2) = 2 log(x). The new unit, dBV, is still a dB measure of power, but the suffix “V” reminds us that we have referred the power back to an equivalent rms voltage. A similar power level unit is dBmV, defined as

$$P_{dBmV} = 20\log_{10}\left(v_{rms}/1\ mV_{rms}\right)\ dBmV .$$

It is important to remember that dBW, dBm, dBV, and dBmV are all dB units of power. Each has a different calculation method and reference, but they are all dB units.

Here are some examples of how the powers corresponding to rms voltage amplitudes can be expressed in dBV and dBmV units:

$$5\,\mathrm{V_{rms}} \rightarrow 20\log_{10}\left(5\,\mathrm{V_{rms}}/1\,\mathrm{V_{rms}}\right)\mathrm{dBV} = 20\log_{10}(5)\,\mathrm{dBV} = 14\,\mathrm{dBV}$$

$$0.1\mathrm{V_{rms}} \rightarrow 20\log_{10}\left(0.1\mathrm{V_{rms}}/1\mathrm{mV_{rms}}\right)\mathrm{dBmV} = 20\log_{10}(100)\,\mathrm{dBmV} = 20\,\mathrm{dBmV}$$

$$0.4\,\mu\mathrm{V_{rms}} \rightarrow 20\log_{10}\left(0.4\mathrm{x}10^{-3}\,\mathrm{mV_{rms}}/1\mathrm{mV_{rms}}\right) = 20\log_{10}\left(0.4\mathrm{x}10^{-3}\right)\mathrm{dBmV} = -68\,\mathrm{dBmV}$$

Representing Power Ratios using dBs

When we wish to represent a power ratio, we simply determine the dB value of that ratio. For example, lets look at *power gain (or loss)* for a circuit:

$$G_p = 10\log_{10}\left(Pout/Pin\right)$$

The symbol G is commonly used to represent gain, and here we use the subscript "p" to indicate that we are calculating the power gain, defined as the ratio of output power to input power. The power gain of a circuit (or system), a unit-less quantity, is often expressed in dBs.

Now, how is this related to voltage gain, which is what we spend much of our time calculating in circuits classes? We need to modify the calculation slightly to deal with voltages (assuming proper *load matching*):

$$G_p = 10\log_{10}\left(Pout/Pin\right) = 10\log_{10}\left(\frac{Vout^2}{R}\Big/\frac{Vin^2}{R}\right) = 10\log_{10}\left(Vout^2/Vin^2\right)\ dB$$

$$= 20\log_{10}\left(Vout/Vin\right)\ dB$$

where the fact that $\log(x^2)=2\log(x)$ has been used.

As an example, suppose we measure the voltage at the input to a subsystem as 15 V across 50 ohms. The output voltage, again across a 50 ohm load, is 12 V. Calculate the power gain in dBs.

The power at the input is $V^2/R=4.5$ W, and the power at the output is 2.88 W. This gives a power gain of $G_p=2.88/4.5=0.64$ -> -1.93 dB. (Do not round off the result – this will be explained later.) This is actually a power loss (negative power gain – tricky!). Let's try it with voltage: $G_p = 20\log_{10}(V_{out}/V_{in})=$ -1.93 dB. Why doesn't the resistance make a difference?

Returning to the power gain computation above, we can see how using logarithms to convert multiplication to addition happens:

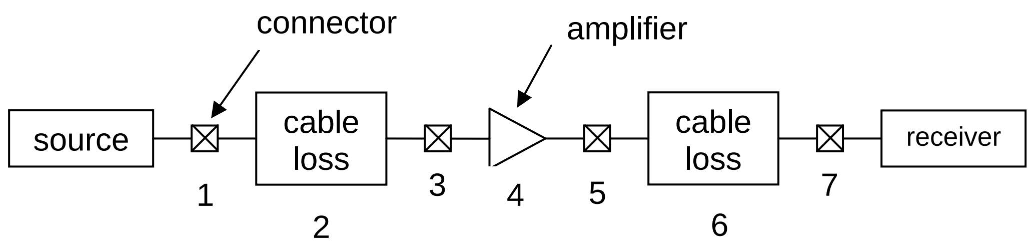
$$
\begin{aligned}
G_p &= 10\log_{10}\left(Pout/Pin\right) = 10\log_{10}\left(\frac{Pout}{P_{ref}}\right) - 10\log_{10}\left(\frac{Pin}{P_{ref}}\right) \\
&= Pout_{dBm} - Pin_{dBm} \\
&= 34.59\ dBm - 36.53\ dBm = -1.93\ dB
\end{aligned}
$$

and multiplication is transformed to addition.

Signal Propagation

Suppose that we wish to determine how much signal power should arrive at a certain location, given the gains and losses along its path. This is where the use of dBs really shows its advantages.

Suppose a system is constructed as shown below. A power source (transmitter) is connected to a series of losses (cables) and gains (amplifiers), and this system finally delivers some power to the destination (receiver).



element #	type	power loss ratio	power gain ratio	power loss dB	power gain dB
1	connector	1/2		3.01	
2	loss	1/100		20	
3	connector	1/2		3.01	
4	amplifier		100		20
5	connector	1/2		3.01	
6	loss	1/100		20	
7	connector	1/2		3.01	

There are a number of questions which might be asked concerning this system. For example, it is important to know the total system gain (loss), that is, the power gain (loss) in propagating from source to destination. The system might represent connectors, cable losses, and an amplifier, as shown.

Working with linear power gains, the total gain is the multiplication of all of the power losses and gains expressed as ratios:

$$G_{sys} = \prod_i G_i \ ,$$

where each term G_i refers to the gain of the stage i. Here, it is important to note that in this context, losses and gains are considered the same sort of term. A gain of less than 1 represents a loss. Using the values from the example diagram, we find the total loss to be

$$G_{sys} = \frac{1}{2}\cdot\frac{1}{100}\cdot\frac{1}{2}\cdot 100\cdot\frac{1}{2}\cdot\frac{1}{100}\cdot\frac{1}{2} = \frac{1}{1600}$$

Working in dBs, the system gains in dBs are added together. However, there is a convention which is important to note. The loss terms are normally reported as a positive quantity. This is so the power relationship can be expressed in the following form:

$$P_{dest,dB} = P_{source,dB} + G - L$$

It is suggested that the system gain first be computed in dBs. For this example:

$$G_{sys,dB} = (-3.01) + (-20) + (-3.01) + (20) + (-3.01) + (-20) + (-3.01) = -32.04\ dB$$

The system has a negative dB value for gain, which indicates an overall system loss. Changing this to a positive quantity in keeping with convention, we would say the total system loss is 32.04 dB. In common practice, factors of 2 are approximated as 3 dB, so the system loss would be 32 dB. This quantity agrees with the quantity calculated above by multiplication (verify this!).

Another question we might ask is, given that the receiver requires -20 dBm of average power to operate correctly, what is the required source power? To find this, simply add 32 dB to the -20 dBm required receiver power to find that the required source power is +12 dBm.

dB "Units"

At this point, a confusing issue must be addressed. Many students think that combining dBs and dBms in one equation results in a violation of some units convention. Specifically, let's look at the power relationship:

$$P_{rcvr,dBm} = P_{source,dBm} - L_{sys}$$
$$-20\,dBm = 12\,dBm - 32\,dB$$

Some would say, "The units don't match!" Don't be misled. Each of the terms in the equation is unitless, and is expressed in dBs! On the left hand side of the equal sign is a power level referenced to 1 mW. On the right hand side of the equal sign is a power level referenced to 1 mW and a change in level. Now, intermixing dBW and dBm would be inappropriate – can you explain why?

Rounding Decibel Values

When we first start working with decibels, we are tempted to try to "round off" values as we would with linear values. However, the use of the logarithmic scale (which is nonlinear) distorts the result of rounding. The appropriate number of digits to use to express powers and gains in dBs normally varies a bit with application, but a few examples will serve to demonstrate the effect.

Suppose we have a power gain G_p of 3.50 dB. In linear terms, this is

$$G_{p,lin} = 10^{G_{p,dB}/10} = 10^{3.5/10} = 2.238721$$

What happens if I represent G_p as 3 dB, or 4 dB? The value 3 dB is equivalent to 1.995262, but that is so close to two that we normally say 3 dB is a factor of 2 (or -3 dB is a factor of 1/2). If we choose to write the gain as 3 dB, the error is about 11%, which is usually not acceptable. If we choose to write the gain as 4 dB, which is equivalent to 2.51, the error is over 12%. So we know that we should at least carry the first digit to the right of the decimal to accurately represent a dB value.

What about the second digit? Let's look at the gain value 3.55 dB (2.265), and compare the errors in writing 3.5 and 3.6 dB. The error writing the gain as 3.5 dB (2.239) is a little over 1%, and the error writing the gain as 3.6 dB (2.291) is about 1.2%. So, carrying the second digit to the right of the decimal can give accuracies on the order of 1%. Normally, dB values are not written with more than 2 values to the right of the decimal in practice.

So, we need at least one digit to the right of the decimal to accurately represent dB values, and 2 digits can give more precision. However, when measuring values in dBs, we often find the accuracy of the measurement equipment is on the order of many tenths of a dB, often 0.5 or 1 dB. So, using the second digit may be a bit extreme if we wish to compare our values to measurement. However, certain applications using precision equipment carry the precision of dB estimates to two digits past the decimal, since in that case performance can vary greatly with a power difference of 0.1 dB. Most classroom applications are fine using one digit to the right of the decimal. The exception would be in signal propagation calculation involving a number of gains and losses, where cumulative errors are important.

Decibel Value Error Reporting

When measuring values in the laboratory, it is always important to compare your results to predictions from theory or simulations. Typically, a percent error calculation is used to compare results. But percent error is not appropriate for dB measurements, since the dB conversion is nonlinear. Instead, the appropriate comparison is dB difference. The usual convention is

dB difference = (expected value in dB) – (actual value in dB) .

So how close is good enough, in dBs? Well, 3 dB is a factor of 2 in power, and 6 dB is a factor of 2 in voltage. 10 dB is a factor of 10 in power. A factor of 2 is 100% error if it is multiplied, or 50% error if it is divided. In either case, 3 dB would not be acceptable in most instances. A measurement off by 10 dB is even worse!

We often say that 5, 10, or 20% error is acceptable. Why not examine the dB values associated with those errors? In the following table, assume the following errors refer to measurements of power gains.

Percent Error	dB Difference
+ 5%	+ 0.21
- 5%	- 0.22
+10%	+ 0.41
- 10%	- 0.45
+ 20%	+ 0.79
- 20%	- 0.96
+ 50%	+ 1.76
- 50%	- 3.01

From this table, it is seen that acceptable results are those within ±0.5 or ±1 dB of the predicted (dB) value. This is normally the reading accuracy of equipment such as generic spectrum analyzers and power meters.