ECE 300
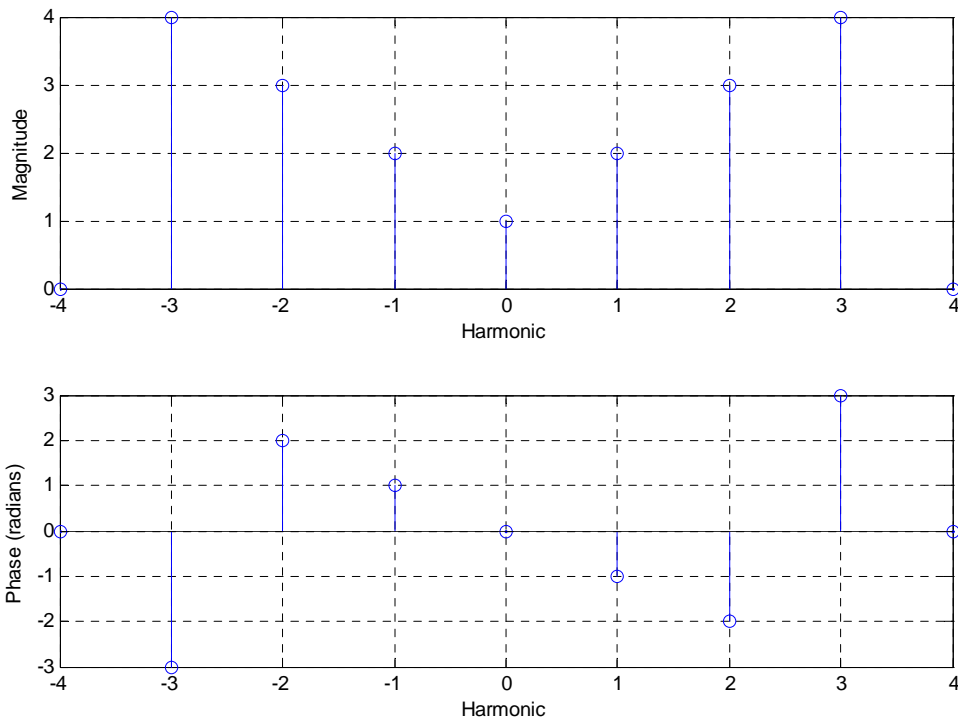**Signals and Systems**
Homework 6

Due Date: Tuesday January 22, 2008 at the beginning of class

Exam 2, Thursday January 24, 2008

Problems:

1. Assume $x(t)$ has the spectrum shown below (the phase is shown in radians) and a fundamental frequency $\omega_o = 2 \text{ rad/sec}$:

Assume $x(t)$ is the input to a system with the transfer function

$$H(\omega) = \begin{cases} e^{-j\omega} & 1 \le |\omega| < 3 \\ 2e^{-j2\omega} & 3 < |\omega| < 5 \\ 0 & else \end{cases}$$

Determine an expression for the steady state output $y(t)$. Be as specific as possible, simplifying all values and using actual numbers wherever possible.

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2. ZTM, Problem 3-16.

3. A periodic signal $x(t)$ is the input to an LTI system with output $y(t)$. The signal $x(t)$ has period 2 seconds, and is given over one period as

$$x(t) = e^{-t} \quad 0 < t < 2$$

$x(t)$ has the Fourier series representation

$$x(t) = \sum_k \frac{0.4323}{1 + jk\pi} e^{jk\pi t}$$

The system is an ideal highpass filter that eliminates all signals with frequency content less than 0.75 Hz.

a) Find the average power in $x(t)$.

b) Determine an expression for the output, $y(t)$. Your expression for $y(t)$ must be real.

(*Answer:* $y(t) = e^{-t} - 0.4323 - 0.2622 \cos(\pi t - 1.2626)$)

c) Determine the average power in $y(t)$.

d) What fraction of the average power in $x(t)$ is contained in the DC and fundamental frequency components?

4. Assume $x(t)$ has the Fourier series representation $x(t) = \sum_k X_k e^{jk\omega_o t}$ and fundamental period T_o. The function $y(t)$ is related to $x(t)$ through the relationship $y(t) = x\left(\frac{t}{b}\right)$.

a) Determine the period of $y(t)$ in terms of T_o (the period of $x(t)$) and fundamental frequency for $y(t)$ in terms of ω_o (the fundamental frequency for $x(t)$)

b) Set up the integral to determine the Fourier series coefficients Y_k in terms of the parameters determined in part a (the integral should be centered at 0), and determine how Y_k is related to X_k

c) Starting from the relationship $x(t) = \sum_k X_k e^{jk\omega_o t}$ and making a simple substitution, show how we can determine the results from part b.

This problem demonstrates that compression or expansion of a signal does not change the Fourier series coefficients, it only changes the fundamental frequency.

5. Assume two periodic signals have the Fourier series representations

$$x(t) = \sum_k X_k e^{jk\omega_o t} \quad y(t) = \sum_k Y_k e^{jk\omega_o t}$$

For the following system (input/output) relationships:

a) $y(t) = bx(t-a)$

b) $y(t) = b\dot{x}(t-a)$

c) $y(t) = bx(t)\cos(\omega_o t)$ (Answer: $Y_n = \dfrac{b}{2}(X_{n-1} + X_{n+1})$)

d) $\ddot{y}(t) + \dfrac{2\varsigma}{\omega_n}\dot{y}(t) + \dfrac{1}{\omega_n^2}y(t) = Kx(t)$

i)   write Y_k in terms of the X_k

ii) If possible, determine the system transfer function $H(j\omega)$

iii) A system must be both linear and time-invariant to have a transfer function. If you cannot determine the transfer function, indicate which system property is not satisfied (**L** or **TI).**

6. Most microcontrollers are capable of generating pulse width modulation (PWM) signals on one or more output pins. These signals are square waves where both the period and the duty cycle can be programmed in the microcontroller by the use of timers and different reference clocks. These PWM output signals can then be used in conjunction with lowpass filters to produce reasonable approximations to analog output signals. In this problem we will use what we have learned in the course to investigate how to do this. The signal $v(t)$ below is a PWM signal, shown for about one and a half periods. The signal has period T_0, amplitude V (usually fixed at 5 or 3.3 volts), pulse duration τ, and duty cycle $D = \dfrac{\tau}{T_0}$.
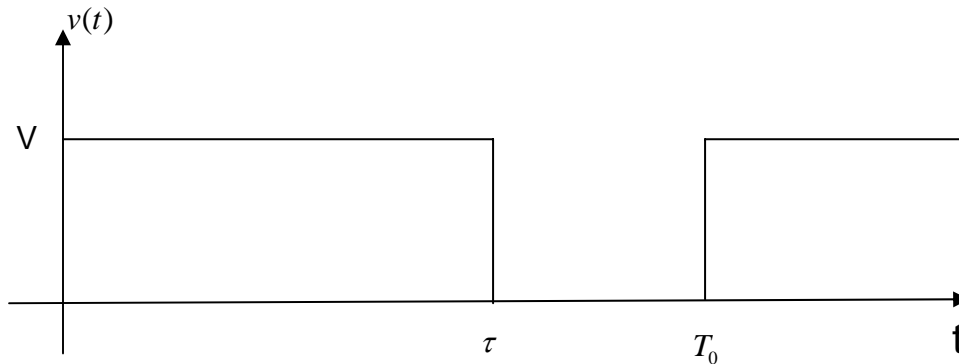
The Fourier series representation is $v(t) = \displaystyle\sum_{k=-\infty}^{k=\infty} e^{-\frac{jk\pi\tau}{T_o}} \left(\frac{V\tau}{T_0}\right) \text{sinc}\left(\frac{k\tau}{T_0}\right) e^{jk\omega_0 t}$

a) Determine an expression for the **average value** of $v(t)$ in terms of T_0, τ, and V.

b) It is the average value of $v(t)$ that we want to use as our analog output. Hence we need to design a lowpass filter that allows us to keep our DC term, and, ideally, remove all of the other harmonics. Let's assume we want to use a simple first order RC lowpass filter with transfer function

$$H(j\omega) = \frac{1}{j\omega RC + 1} = \frac{1}{j\omega \dfrac{\sqrt{\alpha}}{\omega_0} + 1}$$

where we have set $RC = \dfrac{\sqrt{\alpha}}{\omega_0}$ for convenience.

Determine the value of α so that the average power in the first harmonic of the output signal is 20 dB lower than the average power in the DC component the output signal.

Assume here that the fundamental frequency is $f_0 = 100\,Hz$, the duty cycle is $\dfrac{\tau}{T_0} = 0.8$, and $V = 5.0$ volts.

c) For your value of α determined in part **b** and the parameter values given in part **b**, determine an expression for the first two terms (the DC and first harmonic) in the Fourier series representation of the output signal.
(Answer: $y(t) \approx 4 + 0.566\cos(2\pi 100 t - 3.77)$)

7. Review of Logarithms and dB. _Read the documents available online (website, ANGEL, etc) to answer these questions._

a) Express the value of -120dBm in terms of dBmV

b) Express the value of 60dBmV in terms of the amplitude of a cosine function.

c) Express the average power in the signal $x(t) = 4\cos(100\pi t - 1.21)$ V in terms of dB

d) If the average power of a sinusoidal signal into a 1Ω resistor is increased by 12dBm by what factor is the amplitude of the sinusoidal signal increased.

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