**ECE 300**
**Signals and Systems**
Homework 5

**<u>Due Date:</u>** Tuesday January 15. 2008 *at the beginning of class*

1. Using Euler's identity, determine the complex Fourier series coefficients for the following periodic signals.

a) $x(t) = -1 + \cos(2t) + 3\cos(4t + \frac{\pi}{4})$

b) $x(t) = \cos^2(2t)$

c) $x(t) = 2\cos(3t) + 4\sin(6t - \frac{\pi}{3})$

2. Find the Fourier series representation for the signal indicated using hand analysis. Clearly indicate the values of $\omega_0$ and the $c_k$. *Hints:* (1) *Draw the signal, and then use the sifting property to calculate the* $c_k$. (2) *If you understand how to do this, there is very little work involved.*

$$x(t) = \sum_{p=-\infty}^{\infty} \delta(t - 3p)$$

3. For the periodic square wave $x(t)$ with period $T_o = 0.5$ and

$$x(t) \begin{cases} 1 & 0 \le t < 0.25 \\ -1 & 0.25 \le t < 0.5 \end{cases}$$

show that the Fourier series coefficients are given by

$$c_k = \begin{cases} \dfrac{-2j}{k\pi} & k \quad odd \\ 0 & k \quad even \end{cases}$$

where $x(t) = \sum_k c_k e^{jk4\pi t}$

4. Simplify each of the following into the form $c_k = \alpha(k) e^{-j\beta(k)} \mathrm{sinc}(\lambda k)$

a) $c_k = \dfrac{e^{j7k\pi} - e^{-j2k\pi}}{k\pi j}$

b) $c_k = \dfrac{e^{-j2\pi k} - e^{-j5\pi k}}{jk}$

c) $c_k = \dfrac{e^{j5k} - e^{j2k}}{k}$

*Scrambled Answers* $c_k = 3\pi e^{-j\frac{7\pi k}{2}} \mathrm{sinc}\left(\dfrac{3\mathrm{k}}{2}\right)$ , $c_k = 3e^{j(\frac{7}{2}k+\frac{\pi}{2})} \mathrm{sinc}\left(\dfrac{3k}{2\pi}\right)$, $c_k = 9e^{j\frac{5}{2}k\pi} \mathrm{sinc}\left(k\dfrac{9}{2}\right)$

5. Assume $x(t)$ is a periodic signal with period $T_0 = 3$ seconds, and $x(t)$ is defined over one period as

$$x(t) = \begin{matrix} 1 & -1 < t \le 0 \\ 0 & 0 < t \le 2 \end{matrix}$$

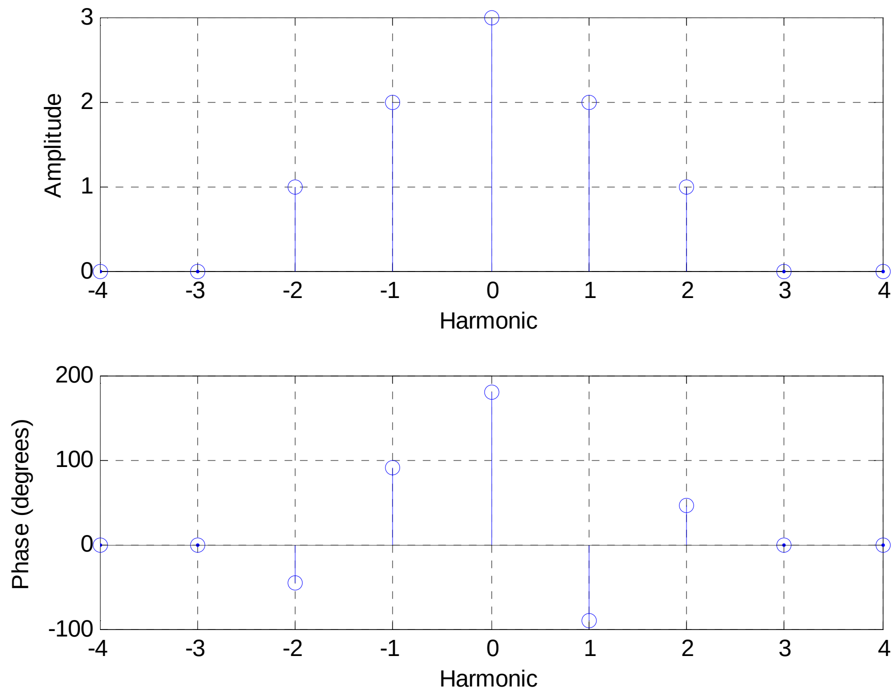
a) Determine the fundamental frequency $\omega_0$.

b) Determine the average value of $x(t)$

c) Determine the average power in the DC component of $x(t)$

d) Determine an expression for the expansion coefficients, $X_k$, where $x(t) = \sum_{k=-\infty}^{k=\infty} X_k e^{jk\omega_0 t}$ .You must write your expression in terms of a **sinc** function, and possibly a leading exponential term.

6. Assume $x(t)$, which has a fundamental period of 2 seconds, has the following spectrum (all phases are multiples of 45




degrees)

a) What is $x(t)$? Your expression must be real.

b) What is the average value of $x(t)$?

c) What is the average power in $x(t)$?

**7**. **(Matlab/PreLab Problem)** Read the **Appendix** and then do the following:

a) Copy the file **Trigonometric_Fourier_Series.m** (you wrote this for homework 4**)** to file **Complex_Fourier_Series.m**.

b) Modify **Complex_Fourier_Series.m** so it computes the average value $c_o$

c) Modify **Complex_Fourier_Series.m** so it also computes $c_k$ for $k=1$ to $k=N$

d) Modify **Complex_Fourier_Series.m** so it also computes the Fourier series estimate using the formula

$$x(t) \approx c_o + \sum_{k=1}^{N} 2\,|\,c_k\,|\cos(k\omega_o t + \measuredangle c_k)$$

You will probably need to use the Matlab functions **abs** and **angle** for this.

e) Using the code you wrote in part **d,** find the complex Fourier series representation for the following functions (defined over a single period)

$$f_1(t) = e^{-t}u(t) \quad 0 \le t < 3$$

$$f_2(t) = \begin{cases} t & 0 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

$$f_3(t) = \begin{cases} 0 & -2 \le t < -1 \\ 1 & -1 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

These are the same functions you used for the trigonometric Fourier series. Use N = 10 and turn in your plots for each of these functions. Also, turn in your Matlab program for one of these. *Note that the values of **low** and **high** will be different for each of these functions!*

# Appendix

In the majority of this course we will be using the complex (or exponential) form of the Fourier series, since it is really easier to do various mathematical things with it once you get used to it.

***<u>Exponential Fourier Series</u>*** If $x(t)$ is a periodic function with fundamental period $T$, then we can represent $x(t)$ as a Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_o t} = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_o t}$$

where $\omega_o = \frac{2\pi}{T}$ is the fundamental period, $c_o$ is the average (or DC, i.e. zero frequency) value, and

$$c_o = \frac{1}{T}\int_0^T x(t)dt$$

$$c_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t}dt$$

If $x(t)$ is a real function, then we have the relationships $|c_k| = |c_{-k}|$ (the magnitude is even) and $\measuredangle c_{-k} = -\measuredangle c_k$ (the phase is odd). Using these relationships we can then write

$$x(t) = c_o + \sum_{k=1}^{\infty} 2|c_k|\cos(k\omega_o t + \measuredangle c_k)$$

This is usually a much easier form to deal with, since it lends itself easily to thinking of a phasor representation of $x(t)$. This will be particularly useful when we starting filtering periodic signals.